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Disconnected hadronic contribution to the muon magnetic moment at the physical point

Budapest-Marseille-Wuppertal Collaboration

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We compute the slope and curvature, at vanishing four-momentum transfer squared, of the disconnected contribution of the leading order hadron vacuum polarization function, using lattice QCD. Calculations are performed with $2+1+1$ flavors of staggered fermions directly at the physical values of the quark masses and in volumes of linear extent larger than 6 fm. The continuum limit is carried out using five different lattice spacings.

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1. Introduction

The vacuum expectation value of the product of two electromagnetic currents plays an important role in physics. It describes how virtual particle fluctuations polarize the vacuum as it is traversed by a propagating photon. While the contributions associated with virtual leptons and weak bosons can be computed in perturbation theory, those of quarks require nonperturbative methods for small photon virtuality, because of the confinement of quarks within hadrons. Here we focus on the latter, known as the hadron vacuum polarization (HVP).

The low energy behavior of the HVP is the limiting uncertainty in the standard model (SM) prediction of a number of quantities. It limits the precision with which many electroweak observables are determined [1]. It also represents the leading hadronic uncertainty in the SM prediction for the anomalous magnetic moments of leptons, a_ℓ with $\ell = e, \mu, \tau$ [2, 3]. In fact, it is the limiting factor in the SM prediction [2, 4–6] of the much debated anomalous magnetic moment of the muon that is currently measured to 0.54 ppm [7].

Today the HVP is best determined using dispersion relations and the cross section of e^+e^- to hadrons or the rate of hadronic τ decays [4–6, 8]. However, since the pioneering work of [9], lattice QCD calculations of the leading order (LO) HVP contributions, $a_\mu^{\text{LO-HVP}}$, to a_μ have made significant progress [10–18]. Moreover, in the long run, this approach is likely to represent the most cost-effective way to increase the precision of the HVP to the levels that will soon be required by the new round of measurements of a_μ [19, 20] and, more generally, by particle physics phenomenology.

We calculate the first two derivatives of the HVP function at zero, euclidean virtuality and in the isospin limit. As shown in [21, 22], the slope of the polarization function provides an upper bound on the HVP contribution to the anomalous magnetic moment of all three leptons. It also determines the whole of $a_\ell^{\text{LO-HVP}}/m_\ell^2$ in the limit that the lepton mass, m_ℓ, vanishes [21]. Moreover, together with curvature, the slope gives $a_\mu^{\text{LO-HVP}}$ to within less than 2%. This fraction is obtained by applying a dispersion relation to a phenomenological model of the e^+e^- data compiled in [23] that we have devised, and using Padé approximants [14].

In the present contribution, we focus on the results on the quark-disconnected contribution to the first two derivatives of the hadron vacuum polarization scalar at zero squared-momentum. We perform a full lattice QCD calculation with 2 + 1 + 1 flavors of staggered fermions, at the physical values of the quark masses. Our lattices have spatial extents larger than 6 fm. We take the continuum limit using five different lattice spacings.

2. Simulations

We employ a tree-level improved Symanzik gauge action [24] and a fermion action for four flavors of stout-smeared [25], staggered quarks. The up and down quark masses are treated as degenerate, their ratio to the strange quark mass is tuned to the vicinity of the physical point, which is defined from the Goldstone pion and kaon masses. The charm quark mass is fixed in units of the strange mass to $m_c/m_s = 11.85$ [26]. More information on the action together with simulation and algorithmic details can be found in [27].

To set the physical mass point we use the isospin corrected pion and kaon masses, $M_\pi = 134.8\,\text{MeV}$ and $M_K = 494.2\,\text{MeV}$, from [28]. To convert the lattice results into physical units,

β	a [fm]	$T \times L$	#conf
3.7000	0.134	64×48	1000
3.7500	0.118	96×56	1500
3.7753	0.111	84×56	1500
3.8400	0.095	96×64	1500
3.9200	0.078	128×80	1000

Table 1: List of β, lattice spacings, sizes and number of configurations used for the disconnected correlators.

we use the pion decay constant $f_\pi = 130.50(1)(3)(13)\,\text{MeV}$ [23] which is free of electromagnetic corrections and, to very good accuracy, equals to the decay constant in the $m_d = m_u$ limit [29]. This makes our definition of the physical point well defined in the isospin limit. In intermediate steps of the analysis we use the Wilson-flow-based [30] w_0-scale [31]. For our finest lattice spacing the root mean squared pion mass is about 15% larger, than the Goldstone pion mass.

Table 1 lists the ensembles and the number of configurations used for quark-disconnected measurements. A configuration corresponds to 10 unit length Rational Hybrid Monte Carlo (RHMC) [32] trajectories. The integration over the trajectory is improved with the gradient of the RHMC force [33, 34]. The topological charge undergoes sufficient number of tunnelings even on the finest lattices.

3. Observables

The hadron vacuum polarization is derived from the electromagnetic current j_μ, which is defined as $j_\mu/e = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c$, where e is the unit of electromagnetic charge. From this we build the current-current correlator $\langle j_\mu(x) j_\nu(0)\rangle$, in which we use the conserved lattice current at the source and sink. No renormalization is therefore necessary.

We split up the correlator in two different ways. First $\langle j_\mu j_\nu\rangle = \frac{e^2}{9}(5C^l_{\mu\nu} + C^s_{\mu\nu} + 4C^c_{\mu\nu} + C^{\rm disc}_{\mu\nu})$, where the first three terms contain the connected contractions for the light, strange and charm flavors, and the last contains the disconnected contractions. Flavor mixing terms arise only in the latter. In this proceedings, we focus on the disconnected contribution. We can also separate the correlator according to isospin symmetry, which is exact in our simulations: $\langle j_\mu j_\nu\rangle = \langle j_\mu j_\nu\rangle_{I=0} + \langle j_\mu j_\nu\rangle_{I=1}$. Here the isospin singlet contribution is $\langle j_\mu j_\nu\rangle_{I=0} = \frac{e^2}{18}(C^l_{\mu\nu} + 2C^s_{\mu\nu} + 8C^c_{\mu\nu} + 2C^{\rm disc}_{\mu\nu})$, whereas the isospin triplet one is $\langle j_\mu j_\nu\rangle_{I=1} = \frac{e^2}{2}C^l_{\mu\nu}$. The lowest energy state contains three/two pions in the isospin singlet/triplet channel. This fact determines the behavior of the correlator for large separations.

We calculate the quark-disconnected contributions to the correlators using the all-mode-averaging technique (AMA) of [35]. Also we separate the quark propagator into high and low-mode contributions [36]: $\mathrm{Tr}(D^{-1}) = \sum_n \frac{1}{\lambda_n} + \mathrm{Tr}(D^{-1}_{\rm high})$ where the eigenvalues λ_n are calculated explicitly and the high-mode contribution is estimated stochastically. We exploit the approximate SU(3) flavor symmetry on around 6000 stochastic sources [15, 37]. We use random spatial wall sources, so only zero-momentum, time propagators are available. Here a noise reduction can be expected by using the same random vectors for light and strange quarks, because isospin symmetric masses are used [37]. For the disconnected contribution of the charm we apply a hopping parameter expansion.

The n-th coefficient of a Taylor expansion of the vacuum polarization scalar, $\Pi(Q^2)$, around $Q^2 = 0$ (i.e. $[\partial^n \Pi(Q^2)/(\partial Q^2)^n]_{Q^2=0}/n!$) can be written as $\Pi_n = \frac{1}{9}(5\Pi_n^l + \Pi_n^s + 4\Pi_n^c + \Pi_n^{\text{disc}})$, where each term is related to the respective, configuration-space correlator through moments [14, 38]

$$\Pi_{n,i4}^f = (-)^{n+1} \sum_t \frac{\hat{t}^{2n+2}}{(2n+2)!} C_{ii}^f(x) \tag{3.1}$$

for $f = \{l, s, c, \text{disc}\}$, with $i = 1,2,3$ and $\hat{t}$ defined as $\hat{t} = \min(t, T-t)$ and where T is the temporal size of the lattice. We average over time moments of correlators of spatial currents $i = 1,2,3$.

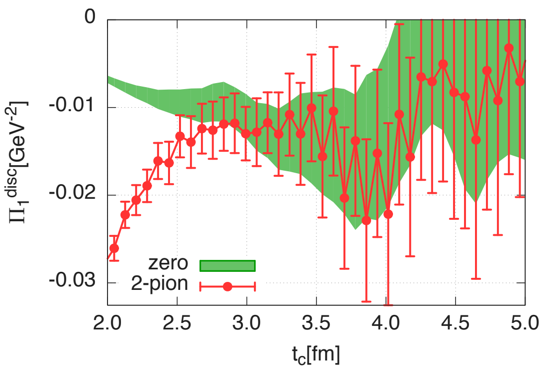

Figure 1: The upper/lower bound on Π_1^{disc} is obtained by setting the correlator to zero/to the connected correlator with a two-pion decay. Results are for an ensemble at $\beta = 3.9200$.

In the case of the disconnected correlators the signal deteriorates quickly with increasing distance. For the time-moment sums, which can extend up to ca. 6 fm, we introduce a cut t_c in time. For times greater than t_c we replace the correlator by an upper and a lower bound.

The disconnected contribution only enters the isospin singlet channel, whose lowest-energy contribution comes from three-pion states. The isospin singlet channel can be neglected compared to the triplet, dominated by two-pion states. Therefore up to exponentially suppressed corrections in T the correlator satisfies

$$0 \geq [2C^s + 8C^c + 2C^{\text{disc}}](t) \geq -C^l(t_c) \frac{\varphi(t)}{\varphi(t_c)} \tag{3.2}$$

where $\varphi(t) = \cosh[E_{2\pi}(T/2 - t)]$ and $E_{2\pi}$ is the energy of two pions, each with the smallest non-vanishing lattice momentum, for which we use $2\pi/L$. Eq. (3.2) gives an upper and a lower bound on $\Pi^s + 4\Pi^c + \Pi^{\text{disc}}$, which can be used to determine the time t_c after which the two bounds agree within errors. At large t, the connected strange and charm contributions in (3.2) are exponentially suppressed, and their presence does not make a difference when determining t_c so we neglect them. In Fig. 1 we show the upper and lower bounds on Π^{disc}. In our analyses, we take $t_c = 2.7$ fm on the disconnected timelike correlators and average the two bounds to get the final result.

	$\Pi_1\,[\mathrm{GeV}^{-2}]$	$\Pi_2\,[\mathrm{GeV}^{-4}]$
disconnected $\times 10^2$	-1.5(2)(1)	4.4(1.0)(0.4)

Table 2: Final results on the first and second derivative of the disconnected contribution of the hadron vacuum polarization scalar at zero squared-momentum. The first error is statistical and the second is systematic.

4. Results

To obtain our final results in the continuum limit and at the physical point, we fit the lattice results to a function which depends on the pion and kaon masses and on the lattice spacing squared a^2. Since the simulations were done around the physical point, a linear pion/kaon mass dependence is always sufficient. For the disconnected contributions, reasonable fit qualities can be achieved with a linear a^2 dependence.

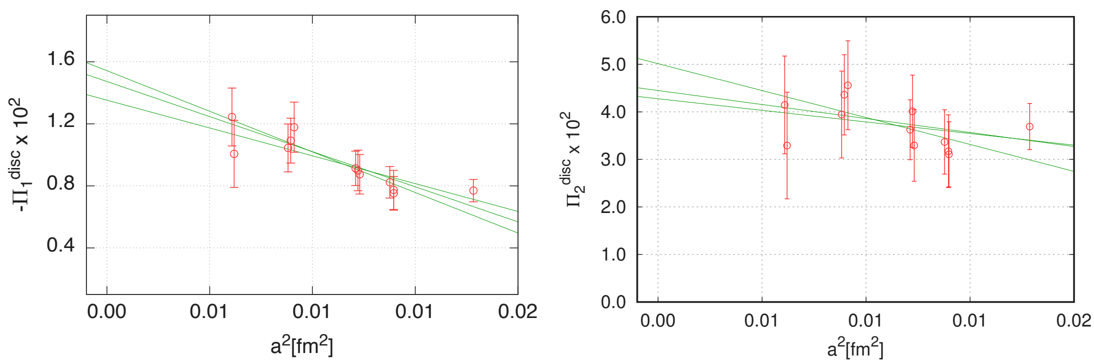

Figure 2: Continuum extrapolations of the light, strange, charm and disconnected contributions to Π_1 and Π_2. The lines are fits linear in a^2, the different lines correspond to different cuts in the lattice spacing.

The lattice spacing dependences of the disconnected Π_1 and Π_2 are shown in Fig. 2. We calculate the charm quark contribution to the disconnected term on the coarsest lattice: we find it to be 0.1% of the total disconnected result, i.e. much smaller than the total disconnected statistical error. We therefore discard the charm from the disconnected term at all lattice spacings. Π_1^{disc} has large lattice artefacts. The coarsest lattice gives a value about 50% smaller than the continuum limit. The final central value and systematic error on the continuum limit are the mean and standard error of the Akaike-Information-Criterion-weighted distribution obtained by imposing three different cuts on lattice spacing (no cut, $a \leq 0.118$, 0.111 fm) in the extended frequentist approach of [39, 40]. The results for the first and the second moments are given in Table 2.

5. Conclusion

We have presented a calculation of the slope and curvature of the disconnected contribution to the leading order hadron vacuum polarization function at vanishing four-momentum transfer squared, using lattice QCD simulations with $2+1+1$ flavors of staggered fermions at the physical values of the quark massees and in volumes of linear extent larger than 6 fm. The continuum limit is taken by using five different lattice spacings. For the moment sums, we introduce a time cut and compute them using an upper and lower bound, in order to improve the signal-to-noise ratio. We obtain 15% and 23% total uncertainties on the disconnected contribution to Π_1 and Π_2, respectively. Results for the connected contributions to Π_1 and Π_2 were presented at this conference [41] and, together with work detailed here, appear in article form in [18].

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