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Non-perturbative matching of HQET heavy-light axial and vector currents in $N_{\mathrm{f}} = 2$ lattice QCD

ALPHA Collaboration

Michele Della Morte
CP^3-Origins & Danish IAS, University of Southern Denmark,
Campusvej 55, 5230 Odense M, Denmark
E-mail: dellamor@cp3-origins.net

Jochen Heitger* **and Christian Wittemeier**
Westfälische Wilhelms-Universität Münster, Institut für Theoretische Physik,
Wilhelm-Klemm-Straße 9, D-48149 Münster, Germany
E-mail:
heitger@uni-muenster.de, christian.wittemeier@uni-muenster.de

Piotr Korcyl
Universität Regensburg, Institut für Theoretische Physik, D-93040 Regensburg, Germany
E-mail: piotr.korcyl@ur.de

Hubert Simma
NIC, DESY, Platanenallee 6, D-15738 Zeuthen, Germany
E-mail: hubert.simma@desy.de

Based on a non-perturbative matching strategy between Heavy Quark Effective Theory (HQET) at $\mathrm{O}(1/m_{\mathrm{h}})$ and finite-volume QCD, we report on our determination of the effective theory parameters of all components of the HQET heavy-light axial and vector currents in two-flavour lattice QCD. These parameters, which can be fixed by matching conditions between suitable QCD and HQET observables evaluated through numerical simulations, are required to absorb the power divergences of lattice HQET, as, for instance, encountered in an effective theory computation of form factors for semi-leptonic decays of B– and $\mathrm{B_s}$–mesons.

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1. Motivation

B-meson decays represent a fruitful ground not only for testing the Standard Model of Particle Physics, but also for putting stringent constraints on the structure and couplings of theories within indirect searches for New Physics beyond the Standard Model. Since the associated experimental rates are related to elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and hadronic matrix elements entering through an effective Hamiltonian, which has contributions from the electroweak sector and — possibly — from New Physics, a theoretically clean and precise quantitative analysis of these decays requires an ab-initio, lattice QCD evaluation of the matrix elements, non-perturbatively in the QCD coupling.

However, the hierarchy of scales

$$L^{-1} \ll m_\pi \approx 140\,\mathrm{MeV} \ll m_\mathrm{B} \approx 5\,\mathrm{GeV} \ll a^{-1}\,, \tag{1.1}$$

to be treated simultaneously, actually forbids direct simulations on a lattice with spatial extent L and lattice spacing a on nowadays computers. Therefore, we here resort to Heavy Quark Effective Theory (HQET) for the heavy quark (i.e., typically the b) [1, 2], which amounts to an expansion of heavy-light QCD observables in inverse powers of the heavy quark's mass, m_h, while the mass scales appearing in the numerator of this expansion parameter are given by the intrinsic QCD scale, $\Lambda_\mathrm{QCD} \sim 400\,\mathrm{MeV}$, and small spatial momenta (in the rest frame of the B-meson). As a consequence, the lattice formulation of this effctive theory only has to cover accurately the scales far below m_B.

HQET contains low-energy constants, also called HQET parameters in the following. In addition to those already present in the action, there are further parameters: one for each composite field that has to be considered when the aforementioned electroweak Hamiltonian is constructed. As explained in [3, 4], for lattice HQET to become a sound computational approach to B-physics phenomenology, these HQET parameters must be determined non-perturbatively, because otherwise any truncation of the perturbative series would leave uncancelled remainders from the inherent power divergences of the effective theory and thereby spoil the existence of the continuum limit. A strategy for such a non-perturbative renormalization programme, based on a matching between HQET at $\mathrm{O}(1/m_\mathrm{h})$ and finite-volume QCD, was devised in [3] and has been used to compute the mass of the b-quark as well as the B-meson decay constants f_B and $f_{\mathrm{B_s}}$ in two-flavour QCD [5 – 7]. Whereas these computations involve the HQET parameters in the action and the temporal component of the axial current only, the natural next step is to extend this strategy to also include the *full* set of temporal and spatial components of the heavy-light vector and axial currents. The corresponding matching procedure has been proposed and worked out in detail in ref. [8] (for a review, see [9]), where also its feasibility at tree-level was demonstrated, later supplemented by a one-loop study [10]. Its implementation on the non-perturbative level is currently in progress, and this contribution summarizes the status of the underlying simulations and their preliminary analysis.

Finally, to highlight a prominent physical application of the HQET parameters for the heavy-light currents, let us point out that QCD matrix elements of (e.g., the spatial components of) the vector current get particularly relevant, when it comes to semi-leptonic decays, such as $\mathrm{B} \to \pi\ell\nu$ or $\mathrm{B}_s \to \mathrm{K}\ell\nu$. These allow for an extraction of the CKM matrix element $|V_\mathrm{ub}|$ from a combination of the (experimental) differential decay rate with a theoretical prediction of the form factor $f_+(q^2)$,

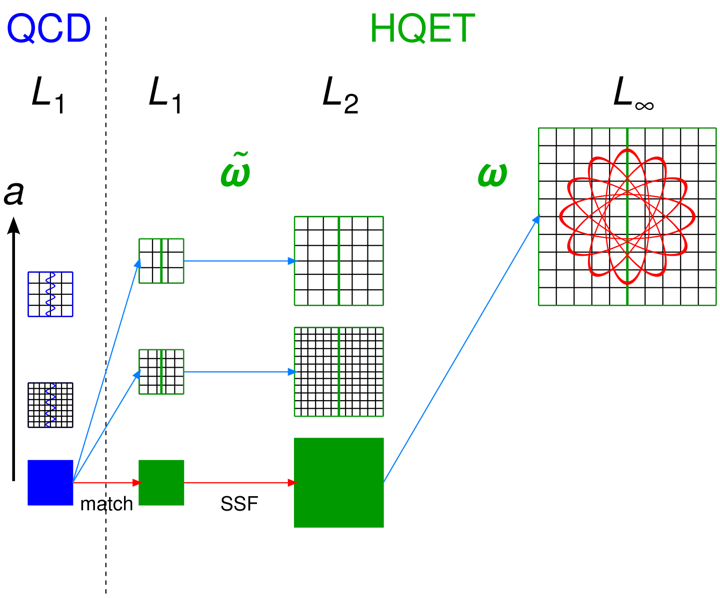

Figure 1: *Sketch of the ALPHA Collaboration strategy to perform lattice HQET computations for B-physics phenomenology via a non-perturbative determination of HQET parameters from small-volume QCD simulations. It ensures that matching and renormalization are performed simultaneously* and *non-perturbatively. Contact to physically large volumes $L_\infty \gtrsim 4/m_\pi$ is made by the step scaling method, while the whole construction is such that the continuum limit can be taken at all stages.*

the latter being a crucial piece in the parameterization of the associated vector current matrix element [11 – 13]. Owing to the persisting $\sim 3\sigma$ tension in the mean values of $|V_{\rm ub}|$ among extractions from inclusive and different exclusive decays ($B \to \pi\ell\nu$ and $B \to \tau\nu$) [14], lattice QCD computations of the required hadronic matrix elements are expected to have significant impact on resolving this tension by discriminating between possible systematic uncertainites or hints at beyond the Standard Model physics. In the framework of $N_f = 2$ QCD and HQET at leading order (LO, i.e., in the static limit), the form factors $f_+(q^2)$ and $f_0(q^2)$ of the semi-leptonic $B_s \to K\ell\nu$ decay at a single value of squared momentum transfer $q^2 = 21.22\,{\rm GeV}^2$ were recently obtained in ref. [15], where for the first time the continuum limits of HQET matrix elements of the vector current were taken. For more details on the methodology and the results of this calculation we refer to this reference as well as to the contributions [16, 17] to this conference. With the present, non-perturbative matching-based $N_f = 2$ QCD determination of the HQET parameters in the action and all heavy-light current components — including their $1/m_h$–terms —, the so far static $B_s \to K$ form factor computation will advance to the next-to-leading order (NLO) in the fully non-perturbatively renormalized effective theory such that two dominant sources of systematic errors of the result in [15] will eventually be eliminated.

2. Survey of the strategy

In our B-physics computations, we work with HQET at NLO, i.e., including terms up to order $O(1/m_h)$ in the inverse heavy quark mass. The underlying strategy [3] is also described in, e.g., refs. [4, 5] and illustrated in figure 1. It splits into two parts: *(i)* the determination of the HQET parameters appearing in the Lagrangian and in the full set of components of the heavy-light axial and vector currents via a non-perturbative matching of HQET to QCD in small volume, and *(ii)* the calculation of HQET energies and matrix elements in large volume, to be combined with the once determined HQET parameters, in order to extract the desired physical hadronic observables (such as the semi-leptonic decay matrix element through its form factor contributions f_+ and f_0).

As for the matching part *(i)*, it is performed in a small volume of extent $L_1 \approx 0.5\,\mathrm{fm}$, where thanks to $am_{\rm b} \ll 1$ numerical simulations with a relativistic b-quark are viable. The bare parameters ω_i in the HQET expansions of the action (resp. the Lagrangian $\mathscr{L}_{\rm HQET} = \overline{\psi}_{\rm h} D_0 \psi_{\rm h} - \omega_{\rm kin}\mathscr{O}_{\rm kin} - \omega_{\rm spin}\mathscr{O}_{\rm spin}$ [1, 2]) and the temporal and spatial components of the heavy-light axial and vector currents are fixed by imposing a set of matching conditions:

$$\Phi_i^{\rm HQET}(L,m_{\rm h},a) \stackrel{!}{=} \Phi_i^{\rm QCD}(L,m_{\rm h},0)\;. \tag{2.1}$$

Here, the $\Phi_i^{\rm QCD}$ (where $i=1,\ldots,19$ refers to the alltogether 19 HQET parameters ω_i, see below) are finite-volume, renormalized, QCD quantities defined in the continuum,

$$\Phi_i^{\rm QCD}(L,m_{\rm h},0) = \lim_{a\to 0}\Phi_i^{\rm QCD}(L,m_{\rm h},a)\;, \tag{2.2}$$

whereas the $\Phi_i^{\rm HQET}$ are understood to be expanded up to NLO in $1/m_{\rm h}$ and calculated in HQET at a finite lattice spacing. On the HQET side, the $1/m_{\rm h}$–expansion of the observables Φ_i results from expanding the Lagrangian and the composite fields $O^{\rm QCD}(x)$ in QCD, viz.,

$$O^{\rm HQET}(m_{\rm h}) = Z_O\left\{O^{\rm stat} + \textstyle\sum_n c_n O_n\right\} + {\rm O}(1/m_{\rm h}^2)\;, \tag{2.3}$$

where the linearly independent operators on the r.h.s. are needed for the renormalization and ${\rm O}(a)$ improvement of the effective theory and their appearance is restricted by mass dimension and the common set of symmetries of QCD and HQET. Note that O may particularly include a (axial or vector) current J with associated renormalization factor $Z_J^{\rm HQET}$. The parameters Z_O and c_n, and thus the full set of ω_i, inherit their $m_{\rm h}$–dependence from the quark mass dependence of (renormalized) QCD. In practice, it enters through the dimensionless variable $z \equiv L_1 M$, where M is the renormalization group invariant quark mass, non-perturbatively known for the two-flavour theory [18, 19].

As outlined in the next section, the matching observables Φ_i are built as suitable combinations of (Schrödinger functional) correlators. After a proper grouping, all HQET parameters appearing in the NLO Lagrangian and axial and vector currents can be assembled into a 19-component vector

$$\omega = \Big(m_{\rm bare}, \omega_{\rm kin}, \omega_{\rm spin}, c_{A_{0,1}}, c_{A_{0,2}}, Z_{A_0}^{\rm HQET}, c_{A_{k,1}}, c_{A_{k,2}}, c_{A_{k,3}}, c_{A_{k,4}}, Z_{A_k}^{\rm HQET}, [\text{"}A\text{"} \leftrightarrow \text{"}V\text{"}]\Big)^T \tag{2.4}$$

such that the HQET expansion (2.1) of the observables takes the form

$$\Phi_i^{\rm HQET}(L,M,a) = \eta_i(L,a) + \varphi_i^j(L,a)\,\omega_j(M,a) + {\rm O}(1/m_{\rm h}^2)\;, \quad i=1,\ldots,19\;, \tag{2.5}$$

with the vector η representing the contribution of the static-order terms in the correlators involved. ($m_{\rm bare}$ denotes the additive quark mass renormalization of the static theory.) Apart from a few additional non-zero entries, the matrix φ_i^j emerging from the bare HQET correlators has a simple block structure [8]. Therefore, the linear system (2.5) can always be solved by block-wise backward substitution for the HQET parameters $\omega_i(M,a)$ absorbing the logarithmic and power divergences of HQET, once the various QCD and HQET correlators have been evaluated through numerical simulations[1]. Eventually, a recursive finite-size scaling step $L_1 \to L_2 = 2L_1$ is used to reach larger volumes and lattice spacings a, by which connection with phenomenology in L_∞ can be made.

[1] The existence of a continuum limit is only guaranteed for the combination of HQET quantities on the r.h.s. of (2.5).

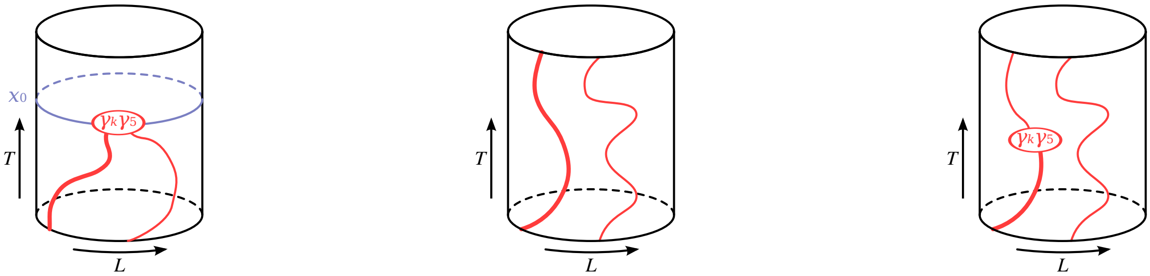

Figure 2: *Pictorial examples of (from left to right: boundary-to-bulk, boundary-to-boundary and three-point) SF correlation functions with single current insertions. Upon expanding them in HQET, all correlators are to be computed in the static approximation, where the $\mathrm{O}(1/m_\mathrm{h})$ terms are treated as local space-time insertions in static correlators, with an extra insertion of the $1/m_\mathrm{h}$–terms $\mathcal{O}_\mathrm{kin}$ or $\mathcal{O}_\mathrm{spin}$ from $\mathcal{L}_\mathrm{HQET}$.*

3. Definition of the matching observables

As an example let us briefly sketch the construction of a matching observable for the NLO HQET expansion of the spatial components of the (renormalized) vector current

$$V_k^\mathrm{HQET}(x) = Z_{\vec{\mathrm{V}}}^\mathrm{HQET}\left[V_k^\mathrm{stat}(x) + \textstyle\sum_{i=1}^4 c_{\mathrm{V}_{k,i}} V_{k,i}(x)\right] , \tag{3.1}$$

with (derivatives being defined from symmetric nearest-neighbour differences):

$$\begin{aligned} V_{k,1}(x) &= \overline{\psi}_\ell(x)\tfrac{1}{2}(\nabla_i^\mathrm{S} - \overleftarrow{\nabla}_i^\mathrm{S})\gamma_i\gamma_k\psi_\mathrm{h}(x) \,, \quad V_{k,2}(x) = \overline{\psi}_\ell(x)\tfrac{1}{2}(\nabla_k^\mathrm{S} - \overleftarrow{\nabla}_k^\mathrm{S})\psi_\mathrm{h}(x) \,, \\ V_{k,3}(x) &= \overline{\psi}_\ell(x)\tfrac{1}{2}(\nabla_i^\mathrm{S} + \overleftarrow{\nabla}_i^\mathrm{S})\gamma_i\gamma_k\psi_\mathrm{h}(x) \,, \quad V_{k,4}(x) = \overline{\psi}_\ell(x)\tfrac{1}{2}(\nabla_k^\mathrm{S} + \overleftarrow{\nabla}_k^\mathrm{S})\psi_\mathrm{h}(x) \,. \end{aligned}$$

In general, the solution of the complete matching problem involves three types of Schrödinger functional (SF) correlation functions. Some representatives are depicted in figure 2; for instance [8]:

$$k_{\vec{\mathrm{V}}}(x_0,\theta_\ell,\theta_\mathrm{b}) = -\tfrac{a^6}{6}\textstyle\sum_k\sum_{\mathbf{y},\mathbf{z}}\left\langle (V_\mathrm{I})_k(x)\,\overline{\zeta}_\mathrm{b}(\mathbf{y})\gamma_k\zeta_\ell(\mathbf{z})\right\rangle , \tag{3.2}$$

$$F_1(\theta_\ell,\theta_\mathrm{b}) = -\tfrac{a^{12}}{2L^6}\textstyle\sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}}\left\langle \overline{\zeta}'_\ell(\mathbf{u})\gamma_5\zeta'_\mathrm{b}(\mathbf{v})\,\overline{\zeta}_\mathrm{b}(\mathbf{y})\gamma_5\zeta_\ell(\mathbf{z})\right\rangle , \tag{3.3}$$

$$F_{\mathrm{V}_0}(x_0,\theta_\ell,\theta_{\ell'},\theta_\mathrm{b}) = -\tfrac{a^{15}}{2L^6}\textstyle\sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z},\mathbf{x}}\left\langle \overline{\zeta}'_{\ell'}(\mathbf{u})\gamma_5\zeta'_\ell(\mathbf{v})(V_\mathrm{I})_0(x)\overline{\zeta}_\mathrm{b}(\mathbf{y})\gamma_5\zeta_{\ell'}(\mathbf{z})\right\rangle , \tag{3.4}$$

where the label "b" denotes heavy *relativistic* quarks of mass close to the b, and the subscript "I" indicates $\mathrm{O}(a)$ improvement. To gain flexibility in the sensitivity of the matching conditions to the HQET parameters, we allow for accessing various kinematical situations by employing generalized periodic boundary conditions for the fermions, $\psi(x+L\hat{k}) = \mathrm{e}^{\mathrm{i}\theta_k}\psi(x), \overline{\psi}(x+L\hat{k}) = \overline{\psi}(x)\mathrm{e}^{-\mathrm{i}\theta_k}$; this corresponds to injecting a momentum $|\theta_\mathrm{b}-\theta_\ell|/L$ in the correlators via the phases θ_k.

To come back to V_k, a possible choice of matching observable composed of SF correlators is

$$\Phi_{15}^\mathrm{QCD} \equiv \ln\left(\frac{k_{\vec{\mathrm{V}}}(\frac{T}{2},\theta_1,\theta_1)}{k_{\vec{\mathrm{V}}}(\frac{T}{2},\theta_2,\theta_2)}\right) \stackrel{!}{=} \Phi_{15}^\mathrm{HQET} = \Phi_{15}^\mathrm{stat} + \omega_\mathrm{kin}\Phi_{15}^\mathrm{kin} + \omega_\mathrm{spin}\Phi_{15}^\mathrm{spin} + c_{\mathrm{V}_{k,1}}\Phi_{15,1} + c_{\mathrm{V}_{k,2}}\Phi_{15,2} \,,$$

which has sensitivity to $c_{\mathrm{V}_{k,\{1,2\}}}$, given other ω's. Explicit definitions of the Φ's along the decomposition (2.5) are found in [8]. Moreover, matching conditions for A_k and V_0 can also be formulated in terms of three-point functions such as (3.4) [20]. Perturbative studies suggest that, whenever possible, observables built from them exhibit a weaker $1/m_\mathrm{h}$–dependence and thus are favourable.

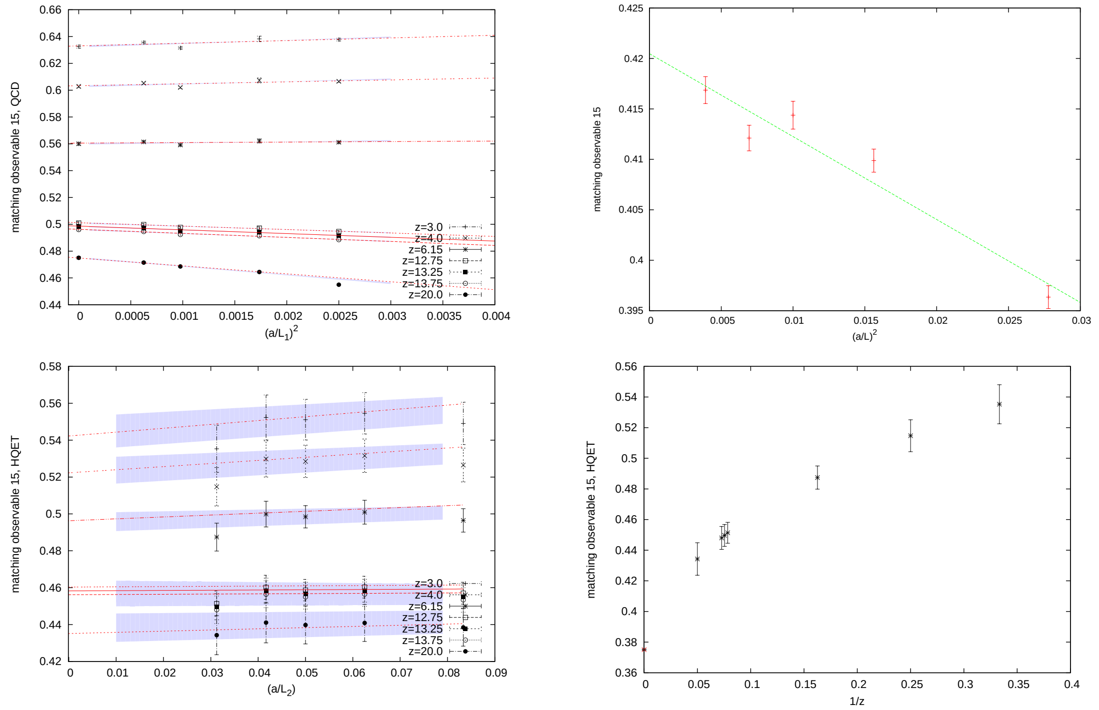

Figure 3: *Preliminary results. Top: Continuum extrapolation for L_1 of QCD observable $\Phi_{15}^{\rm QCD}$ (left) and of its static part $\Phi_{15}^{\rm stat}$ (right). Bottom: Continuum extrapolation for L_2 of $\Phi_{15}^{\rm HQET}$, after inserting the HQET parameters determined for L_1 (left), and heavy quark mass ($z \equiv L_1 M$) dependence of $\Phi_{15}^{\rm HQET}$ for L_2 (right).*

4. Status of (preliminary) results and outlook

For our finite-volume matching computations to non-perturbatively determine the 19 HQET parameters, we re-used the $\mathrm{O}(a)$ improved two-flavour Wilson-QCD SF ensembles available from earlier simulations [5] (employed to fix the subset of parameters in $\mathscr{L}_{\rm HQET}$ and A_0 [5]). From the two- and three-point QCD and HQET correlators, evaluated on these configurations for renormalized masses heavy quark masses $z \equiv L_1 M \in \{3.0, 4.0, 6.15, 12.75, 13.25, 13.75, 20.0\}$ and various combinations of θ's for the light and heavy quarks to support several sets of matching observables and strategies, the quantities filling the linear systems (2.5) — to be solved in L_1^4 and, after step scaling, in L_2^4 — have been constructed. Exemplary results from elements of this *preliminary* analysis for observables sensitive to HQET parameters in the spatial vector current are displayed in figure 3, where we followed the kinematic settings originally proposed in [8]. Note that, by virtue of non-perturbative $\mathrm{O}(a)$ improvement, all continuum extrapolations in QCD and static-order HQET are performed linearly in a^2, while $\mathrm{O}(1/m_{\rm h})$ HQET-contributions still have leading cutoff effects $\propto a$.

In addition, the freedom in choices of θ–angles and observables built from two-point versus three-point functions can be exploited to develop alternative matching strategies that may lead to combinations, by which the matrix φ (to be inverted) receives an optimal condition number and higher $\mathrm{O}(1/m_{\rm h}^2)$ terms in individual observables are rather suppressed. Studies of these aspects are in progress, as well as the remaining steps to *(i)* solve the full linear system in L_2 for $\omega(M,a)$, including a careful error analysis, and *(ii)* to interpolate the resulting parameters to the β's of the large-volume simulations used for the NLO HQET computation of the $\mathrm{B}_s \to \mathrm{K}\ell\nu$ form factors.

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