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Simulations of $\mathcal{N} = 1$ supersymmetric Yang-Mills theory with three colours

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We report on our recent results regarding numerical simulations of the four dimensional, $\mathcal{N} = 1$ Supersymmetric Yang-Mills theory with SU(3) gauge symmetry and light dynamical gluinos.

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1. Introduction

In this work we explore the supersymmetric Yang-Mills theory (SYM) with gauge group SU(3). This work is a natural continuation of what has been done by our collaboration until now simulating the gauge group SU(2). Our conclusive results have been presented in [1]: we have verified that, in the continuum limit, the degeneracy of the supermultiplet is recovered and we have no sign of a possible spontaneous breaking of supersymmetry (SUSY). Moreover, in [2] we have studied the theory at finite temperature: the most interesting result was the evidence that chiral symmetry is restored near the deconfinement phase transition.

SU(2) SYM has been an interesting test case for the more phenomenological relevant SU(3) theory, which contains the gluons of QCD. On the other hand, there are important new aspects in SU(3) SYM like the new bound states and CP-violating phases. From the computational side SU(3) is much more demanding than QCD and SU(2) SYM. Moreover, this model has been proposed as an attractive candidate for a supersymmetric hidden dark-matter sector that may explain astrophysical observations [3].

2. Properties of $\mathcal{N}=1$ SYM with N_c colors

Let us consider $\mathcal{N}=1$ supersymmetric Yang-Mills theory with a gauge group SU(N_c), where $\mathcal{N}=1$ is the number of SUSY generators and N_c is the number of colours.

It is characterised by two fields: one describing the gluon $A_\mu(x)$, the other one describing its superpartner, the gluino $\lambda(x)$. The latter particle is a Majorana fermion in the adjoint representation. For non-zero fermion mass, SUSY is softly broken. Because there is only one Majorana flavour, the global chiral symmetry is simply U(1)$_\lambda$. It is possible to show that this symmetry is anomalous and that, however, a Z_{2N_c} subgroup of U(1)$_\lambda$ is unbroken. As conjectured in [4] the Z_{2N_c} symmetry is spontaneously broken down to Z_2. The result is a theory with N_c vacua, where the gluino condensate can be used as an order parameter (using Weyl notation):

$$\langle \lambda^\alpha \lambda_\alpha \rangle = \text{const}\, \Lambda^3 e^{\frac{2\pi\, i}{N_c} k}\,, \qquad (2.1)$$

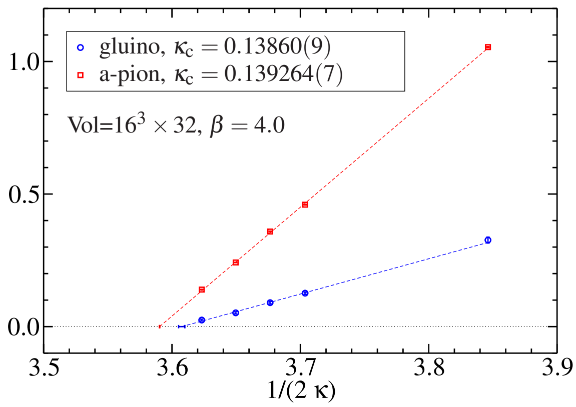

Figure 1: Critical value of κ_c defined as the value of κ where the square of the adjoint pion mass $m^2_{\text{a}-\pi}$ (red square) and the quantity $am_S Z_S^{-1}$, proportional to the renormalised value of the gluino mass, vanish (blue circle), for $N_c = 3$ colours.

where $k = 0,\dots,N_c-1$, and Λ is a scale parameter similar to the Λ parameter in QCD.

This result is supported by arguments in different approaches, see *e.g.* [5, 6]. The coexistence of these N_c vacua implies a first order phase transition (for a massless gluino). We have confirmed this behaviour in the SU(2) theory [7] and we had some evidence [8] also for the SU(3) theory.

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Another feature conjectured for these theories is confinement: the particle spectrum of the theory consists of colourless bound states. In the SUSY limit, the particles are organised in mass-degenerate multiplets. In [9] the authors wrote down an effective action and derived a first supermultiplet of the low-lying spectrum. It consists of a scalar (0^+ gluinoball: a–$f_0 \sim \bar{\lambda}\lambda$), a pseudoscalar ($0^-$ gluinoball: a–$\eta' \sim \bar{\lambda}\gamma_5\lambda$), and a Majorana fermion (spin 1/2 gluino-glueball: $\chi \sim \sigma^{\mu\nu}\mathrm{Tr}\left[F_{\mu\nu}\lambda\right]$). A second supermultiplet was introduced in [10] based on pure gluonic states in the effective action. It consists of a 0^- glueball, a 0^+ glueball, and again a gluino-glueball. According to the authors, this last multiplet should be lighter than the previous one; other authors [11], using different arguments, and hints from ordinary QCD, deduce the opposite order: clarifying this issue is one of the tasks of our project.

3. The theory on the lattice and simulations

The idea that it is possible to study supersymmetric gauge theories on the lattice goes back to [12]. The proposal was that, instead of trying to have some remnant of SUSY on the lattice, one should only require that SUSY is recovered in the continuum limit, in the same way as it happens for chiral symmetry. The formulation we employ in our simulations is an improved version of what was first proposed in [12]: the gauge fields are described by the Wilson action but with a tree-level Symanzik improvement; the gluinos are described by Wilson fermions in the adjoint representation. To reduce the lattice artifacts we apply one or three levels of stout smearing to the link fields in the Wilson-Dirac operator. The configurations have been obtained mainly by a two-step polynomial hybrid Monte Carlo (TS-PHMC) algorithm [13, 14]. Some results, on 6^4 lattices, have been obtained with a Rational Hybrid Monte Carlo (RHMC) algorithm. Integrating out the Majorana fermions yields a Pfaffian. It can have a negative sign, in particular for small gluino masses near κ_c, but the significance of the negative contributions is reduced towards the continuum limit. If necessary, the sign is taken into account by reweighting. Part of the work was done including a clover term: we have already verified a great improvement in the case of SU(2) [1]. We have simulated the theory mainly on a $16^3 \times 32$ volume, with two values of the inverse gauge coupling β, 4.0 and 4.3, and different values of the hopping parameter κ.

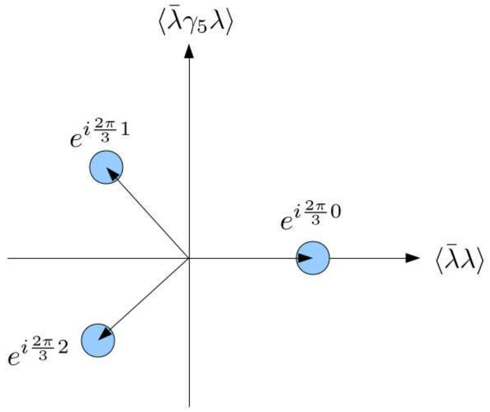

Figure 2: Scatter plot expected for $\kappa = \kappa_c$.

On the lattice we can identify two sources of explicitly SUSY breaking: the first one is due to the introduction of a non-zero gluino mass; the second one is a consequence of the breaking of the translational invariance for non-zero lattice spacing. Moreover we have verified that finite volume effects can drastically increase the mass splitting [15]; in the case of SU(2), using a box size of about 1.2 fm (in QCD units) or larger, the effect is negligible. Note that periodic boundary conditions in spatial directions are compatible with SUSY [4]. It is an interplay between having a finite lattice spacing and a finite volume that leads to the measured larger mass splitting.

4. Tuning towards the SUSY limit

As discussed in [12], and clarified in [16], the chiral symmetric limit implies the supersymmetric limit. More precisely, both the $\mathrm{U}(1)_\lambda$ Ward-Takahashi identity (WTI) (with the axial anomaly) and the SUSY WTI are restored by a single fine-tuning of the bare gluino mass. This gives a solid theoretical basis for lattice formulations of $\mathcal{N}=1$ SYM theories.

In practice, one has to tune the bare gluino mass so that the renormalised gluino mass vanishes. As discussed in [9] using the OZI approximation, or in [17] using a partially quenched setup, the square of the adjoint pion mass $m^2_{\text{a}-\pi}$ is proportional to the mass of the gluino. This relation is apparently well satisfied, as shown in Figure 1 (the red dashed line is a fit to the five data points), where $m^2_{\text{a}-\pi}$ is linear in $1/(2\kappa)$. Given the previous relation, we can determine the critical hopping parameter κ_c, where the renormalised mass of the gluino vanishes. A more direct tuning approach is, of course, to determine the renormalised gluino mass using the WTI. The procedure has already been described in [18]; in Figure 1 we show directly the result: the quantity $am_S Z_S^{-1}$ is plotted against $1/(2\kappa)$. m_S is called subtracted mass and can be identified with the renormalised mass of the gluino; Z_S is a multiplicative renormalisation coefficient. We see a clear linear dependence between $am_S Z_S^{-1}$ and $1/(2\kappa)$: fitting the data (the blue dashed line) we can determine the critical value of κ. Comparing the results obtained using the two methods we see that they are compatible only in 7.5σ. Terms in the WTI proportional to the lattice spacing of course contribute to the difference between the results and there might be also some finite size effects. In future work a better control of systematic errors is needed. Because the determination of the adjoint pion mass is much simpler and numerically not expensive, it will be used as our standard method to tune the SUSY limit in this theory.

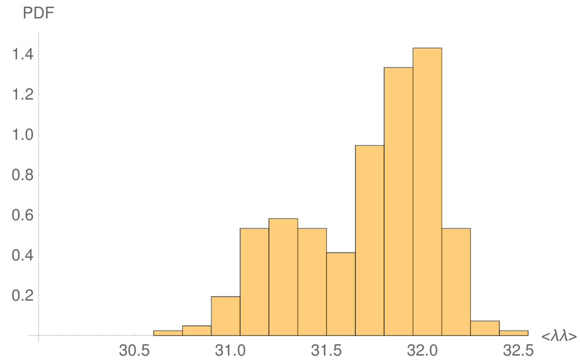

Figure 3: Scalar condensate distribution for $\kappa \lesssim \kappa_c$ obtained using the RHMC algorithm with $\beta = 5.6$, $\kappa = 0.1658$ and $c_{sw} = 1.587$ on a 6^4 volume.

5. The vacuum of the theory

Eq. 2.1 when translated into the Dirac representation, gives rise to two distinct condensates: a scalar condensate $\langle\bar{\psi}\psi\rangle$ and a pseudoscalar condensate $\langle\bar{\psi}\gamma_5\psi\rangle$. As discussed in Sec. 2, we expect that our $\mathcal{N}=1$ SYM theory with gauge group SU(3) is characterised by $N_c = 3$ vacua. The three vacua lie in a Cartesian plane, according to Eq. 2.1, where we put on the abscissa the scalar condensate and on the ordinate the pseudoscalar condensate, see Figure 2.

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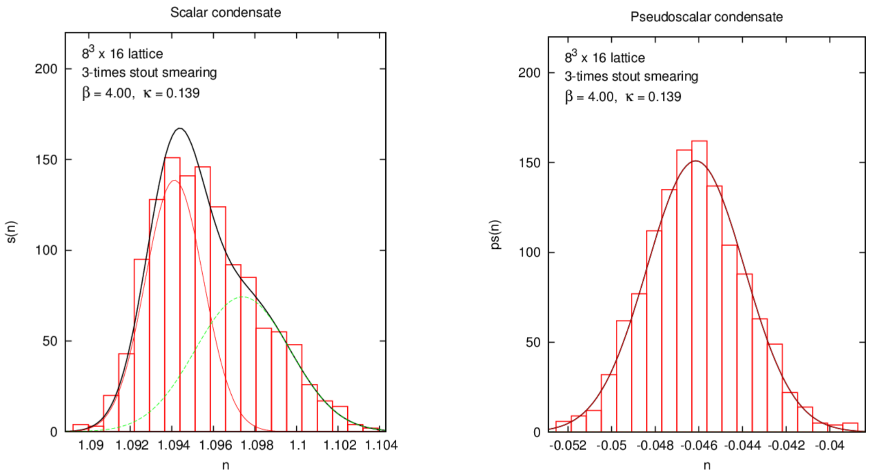

Figure 4: (Left) Scalar condensate distribution: this case corresponds to $\kappa \simeq \kappa_c$ where the peak on the left (in black) is about two times higher than the one on the right (in green). **(Right)** Pseudoscalar condensate distribution: we see only one peak, while two smaller peaks at the two sides should appear.

A first order phase transition should show up as a jump in the expectation value of the scalar gluino condensate at $\kappa = \kappa_c$. Looking at the distribution of this quantity, in relatively small volumes, one expects to see a two peak structure. This is possible only in a small volume; increasing it, the tunnelling between the three ground states becomes less probable and above a certain value practically impossible.

For $\kappa < \kappa_c$ we expect $\langle \bar{\psi}\psi \rangle > 0$ and only one peak should appear (corresponding to the distribution labelled with $e^{\mathrm{i}\frac{2\pi}{3}0}$ in Figure 2). When $\kappa \to \kappa_c$ a second peak should emerge (corresponding to the sum of the distributions labelled with $e^{\mathrm{i}\frac{2\pi}{3}1}$ and $e^{\mathrm{i}\frac{2\pi}{3}2}$ in Figure 2). This is exactly what appears in Figure 3. This result was obtained on a small volume 6^4 using our RHMC algorithm. For $\kappa = \kappa_c$ we should see two peaks, with the left one two times higher the right one: this can be seen in Figure 4 (Left) where data had to be fitted as a sum of two Gaussian distributions.

When we look at $\langle \bar{\psi}\gamma_5\psi \rangle$ we expect to see only one peak for $\kappa < \kappa_c$ (corresponding to the distribution labelled with $e^{\mathrm{i}\frac{2\pi}{3}0}$ in Figure 2), but for $\kappa \to \kappa_c$ three peaks should appear: one in the center and two, with the same height, at the two sides (corresponding to the two distributions labelled with $e^{\mathrm{i}\frac{2\pi}{3}1}$ and $e^{\mathrm{i}\frac{2\pi}{3}2}$ in Figure 2). The reason because they have the same height is simple: we are populating two vacua which have the same probability to be occupied. So far we have never observed the double peak structure, but only one symmetric peak, as in Figure 4 (Right). This happen even for $\kappa > \kappa_c$ where the peak, corresponding to the distribution labelled with $e^{\mathrm{i}\frac{2\pi}{3}0}$ in Figure 2, should disappear and only the two peaks at its sides should remain. The reason for this phenomenon is still under investigation.

6. Mass spectrum: numerical results

As discussed in Sec. 4 the renormalised mass of the gluino is proportional to the square of the adjoint pion mass. As a consequence we extrapolate the masses of the bound states to the chiral limit fitting their mass against the square of the adjoint pion. We present here only the results obtained for $\beta = 4.0$.

The mass spectrum of this theory has been determined using the techniques already discussed in [1]. In Figure 5 (Left) we plot the channel 0^{++}, showing both the glueball and the a–f_0, for different values of $m^2_{\text{a-}\pi}$. It is well known that this channel is particularly noisy; in our case the glueball determination looks somewhat better than the meson one. The masses of both states are perfectly compatible when extrapolated to the chiral limit.

In Figure 5 (Right) we plot the mass of the a–η' and of the gluino-glueball. Their error bars are considerably smaller than those in the channel 0^{++}; the gluino-glueball is by far the state with the best results. The masses of these two states, when extrapolated to the chiral limit, are roughly compatible with those in the 0^{++} channel, taking the errors in that channel into account. But their masses are not compatible with each other. This is probably due to discretisation effects, which should be already reduced at $\beta=4.3$ and that we are currently simulating.

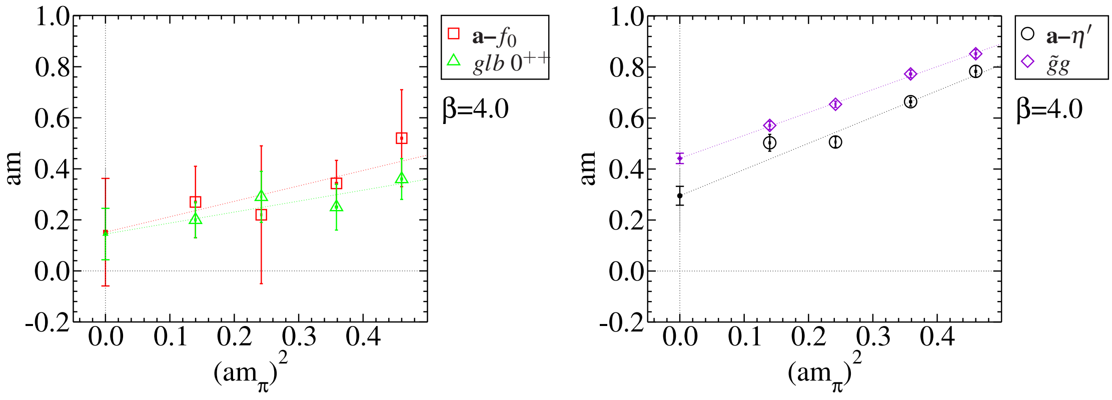

Figure 5: (Left) Mass of the a–f_0 and of the glueball 0^{++} for different values of the square of the adjoint pion mass $m_{\text{a-}\pi}$. **(Right)** As for (Left) but for the mass of the a–η' and of the gluino-glueball $\tilde{g}g$.

7. Conclusions and outlook

We have presented our first results on $\mathcal{N}=1$ supersymmetric Yang-Mills theory with three colours. We have shown results on the use of the Ward-Takahashi identities and of the adjoint pion to tune the theory to supersymmetry. The structure of vacua has been investigated and some preliminary results have been obtained: we see a clear first order transition in the scalar condensate but not in the pseudoscalar one. This issue is still under investigations. We started to measure the particle spectrum of the theory. At the moment we presented the results with only one lattice spacing but the results are promising. We are currently simulating the theory with a second value of β to estimate the discretization effects and, hopefully, to extrapolate the results to the continuum limit.

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