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Absence of bilinear condensate in three-dimensional QED

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There are plausibility arguments that QED in three dimensions has a critical number of flavors of massless two-component fermions, below which scale invariance is broken by the presence of bilinear condensate. We present numerical evidences from our lattice simulations using dynamical overlap as well as Wilson-Dirac fermions for the absence of bilinear condensate for any even number of flavors of two-component fermions. Instead, we find evidences for the scale-invariant nature of three-dimensional QED.

34th annual International Symposium on Lattice Field Theory
24-30 July 2016
University of Southampton, UK

*Speaker.

PoS(LATTICE2016)245

1. Introduction

Parity-invariant QED_3 with $2N_f$ flavors of massless two-component fermions coupled to three-dimensional non-compact Abelian gauge-fields has been studied in the past as a quantum field theory which can be tuned to be conformal or to have a mass-gap by changing N_f. The question is the following – is there a critical number of flavors of two-component fermions $2N_f$ below which massless QED_3 in a finite box of length ℓ generates other low-energy length scales which are independent of ℓ as $\ell \to \infty$? One such low-energy length scale that is of interest is the bilinear condensate Σ which, if non-zero, governs the following scaling of the low-lying eigenvalues λ_i of the massless Dirac operator:

$$\lambda_i = \frac{z_i}{\Sigma}\frac{1}{\ell^3}, \tag{1.1}$$

where z_i are universal numbers depending only on the symmetries of the Dirac operator, and can be obtained from a random matrix model with the same symmetries (refer [1] for such a model corresponding to QED_3). In this talk, based on our publications [2, 3], we primarily address the existence of Σ for small N_f $(= 1,2,3,4)$ by asking if $\lambda \sim \ell^{-(1+p)}$ with $p = 2$. We summarize the status of the understanding of the critical N_f before our studies in Figure 1 (see [2] and references therein, for a complete literature survey). The analytical computations, each with their own limitations, suggested that the critical N_f lie between 0 and 4. The previous lattice studies suggested that it could be 1 or 2.

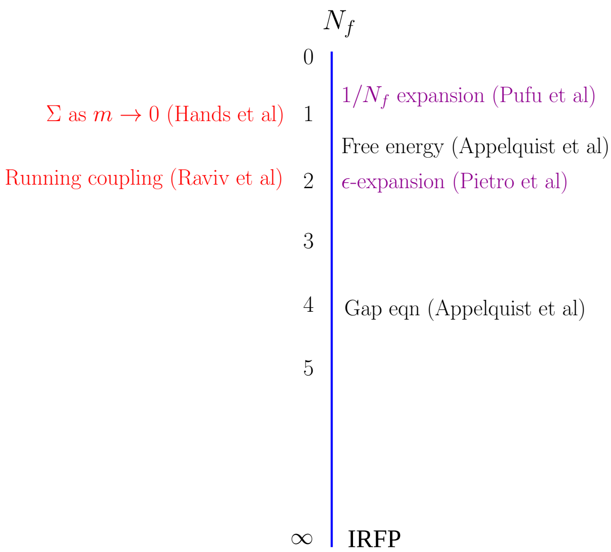

Figure 1: Few representative older calculations [4, 5, 6, 7, 8, 9] of the critical value of N_f below which bilinear condensate exists. The large-N_f computation points to an infra-red fixed point. Various perturbative calculations as well as approximate solutions to the gap equation have been carried out to investigate the stability of the infra-red fixed point. These calculations suggest the critical value might lie anywhere between 0 and 4. The previous non-perturbative lattice studies of QED_3 suggest this critical value might be 1 or 2.

2. Lattice details

We regulated QED_3 in a finite box of physical volume ℓ^3 using L^3 lattices. The lattice coupling appearing in the gauge action is $\beta = L/\ell$; the continuum limit at a fixed physical length ℓ is taken by extrapolating to $L \to \infty$. We regulated the two flavors of massless two-component fermions in a parity-invariant way using the Wilson-Dirac as well as overlap fermions. The fermion propagator G for the parity-preserving Wilson-Dirac fermion is

$$G^{-1} = \begin{bmatrix} 0 & X \\ -X^\dagger & 0 \end{bmatrix} \quad ; \quad X = C_n + B - m_t. \tag{2.1}$$

C_n is the two-component naive Dirac operator, B is the Wilson term and m_t is tuned such that the lowest eigenvalue λ_1 of iG^{-1} is minimum. We further improved it by adding a Sheikholeslami-Wohlert term and by using HYP smeared links in the Dirac operator. The fermion propagator G for the overlap fermion, which has the full $\mathrm{U}(2N_f)$ symmetry even at finite lattice spacing, is given in terms of a unitary matrix $V = (X^\dagger X)^{-1} X$ as [1]

$$G^{-1} = \begin{bmatrix} 0 & \frac{1-V}{1+V} \\ \frac{1-V}{1+V} & 0 \end{bmatrix}. \tag{2.2}$$

We define the "eigenvalues of the Dirac operator" in either case to be the eigenvalues λ_i of iG^{-1} which are real. We used standard HMC for generating $\sim 500-1000$ independent gauge configurations at all the simulation points ($4 \le \ell \le 250$). Using Wilson-Dirac fermions we studied $N_f = 1, 2, 3$ and 4. With the overlap fermion, we studied $N_f = 1$. At each ℓ, we used multiple L^3 lattices ($12 \le L \le 24$) in order to take the continuum limits.

3. Evidence from ℓ-scaling of the low-lying eigenvalues of Dirac operator

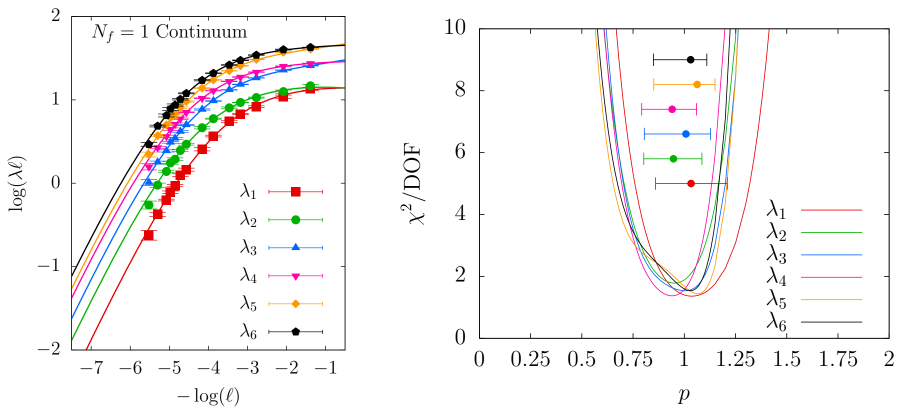

Figure 2: On the left panel, the ℓ dependence of the six low-lying, continuum extrapolated, eigenvalues of the overlap operator is shown. The Padé approximations to their ℓ dependence with $p = 1$ are shown as the solid curves. On the right panel, the likelihood of different values of the exponent p, measured using the χ^2/DOF for the best fit of the Padé approximation with various values of p to the finite ℓ data, is shown.

[1]The Wilson mass $m_t = 1$ in overlap simulations

In a finite physical box, the spectrum of the Dirac operator is discrete. Thus, one can talk about the ℓ-dependence of the individual low-lying eigenvalues. As we noted in the introduction, the i-th low-lying eigenvalue λ_i will scale as ℓ^{-3} when there is a condensate Σ. If ℓ^{-3} scaling is not found, we can conclude that a bilinear condensate is absent and instead we can obtain the mass anomalous dimension of the scale-invariant theory; since λ has an engineering dimension of mass, the mass anomalous dimension γ_m is p if $\lambda \sim \ell^{-p-1}$ and $p < 1$.

In the left panel of Figure 2, we show the dependence of the continuum extrapolated values of $\lambda_i \ell$ as a function of $1/\ell$ for the six low-lying eigenvalues of the overlap operator in a log-log plot. At any finite ℓ that we studied, the slope $\frac{d\log(\lambda\ell)}{d\log(1/\ell)}$ is less than 2, the value that is expected if $\Sigma \neq 0$. In fact, it is less than 1. We estimate the exponent of the power-law that would be seen as $\ell \to \infty$ by describing the ℓ-dependence of our data by

$$\lambda\ell = \ell^{-p} F(1/\ell), \tag{3.1}$$

with an unknown scaling correction F. We approximate F by a $[1/1]$ Padé approximant. We find it numerically stable to write the Padé approximant in terms of $\tanh(1/\ell)$. The best fits of the above ansatz with $p = 1$ to the data are shown by the solid curves in the left panel of Figure 2. In the right panel, we show the χ^2/DOF for such fits to the six low-lying eigenvalues as a function of the exponent p. The value $p = 2$ is clearly ruled out, which implies the absence of a condensate. Assuming the theory does not generate other length scales as well, we can estimate the mass anomalous dimension $\gamma_m = p$ of the theory to be 1.0(2) from the same plot. Further, we support the correctness of our result by comparing the ℓ-dependence of the continuum extrapolated low-lying eigenvalues of the two different lattice Dirac operators in Figure 3. A perfect agreement between the Wilson-Dirac and the overlap formalisms is seen. Due to such an agreement, we study the $N_f = 2, 3, 4$ cases using only the Wilson-Dirac fermion.

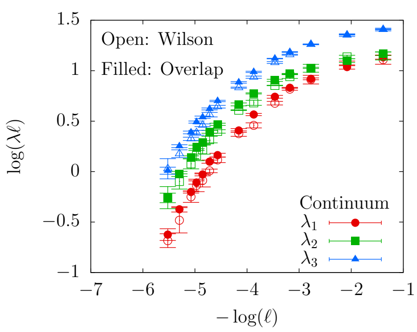

Figure 3: The plot compares the ℓ-dependence of the first three low-lying eigenvalues, after taking the continuum limit, using Wilson fermions (open symbols) and overlap fermions (filled symbols) for the $N_f = 1$ case.

In Figure 4, we show the ℓ-dependence of the continuum extrapolated smallest eigenvalue for different number of flavors $N_f = 1, 2, 3$ and 4. The eigenvalues scale with a smaller exponent p as N_f increases, consistent with the expectation that if $N_f = 1$ does not have a bilinear condensate, the $N_f = 2, 3, 4$ also would not. Thus QED_3 does not have a bilinear condensate for all non-zero N_f. Again, assuming this means that QED_3 is scale-invariant for all N_f, we estimate the mass

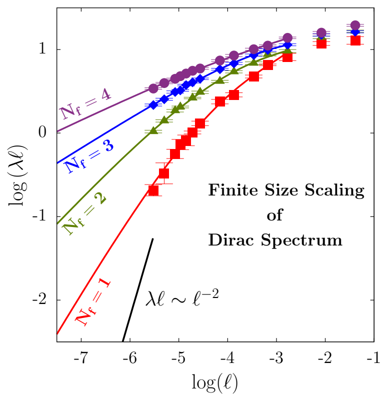

Figure 4: The ℓ-dependence of the smallest eigenvalue of the Wilson-Dirac operator for $N_f = 1,2,3$ and 4. The expected scaling when a bilinear condensate is present, $\lambda\ell \sim \ell^{-2}$, is shown by the black straight line in this log-log plot. The exponent p for the asymptotic ℓ-scaling seems to decrease as $1/N_f$.

anomalous dimension to be $\gamma_m = 1.0(2), 0.6(2), 0.37(6)$ and $0.28(6)$ for $N_f = 1,2,3,4$ respectively. Surprisingly, this agrees with an analytical calculation [10] of γ_m to $\mathcal{O}(1/N_f^2)$ where no assumption about bilinear condensate is made; the analytical values are $\gamma_m = 1.19, 0.56, 0.37$ and 0.28 for $N_f = 1,2,3,4$ respectively.

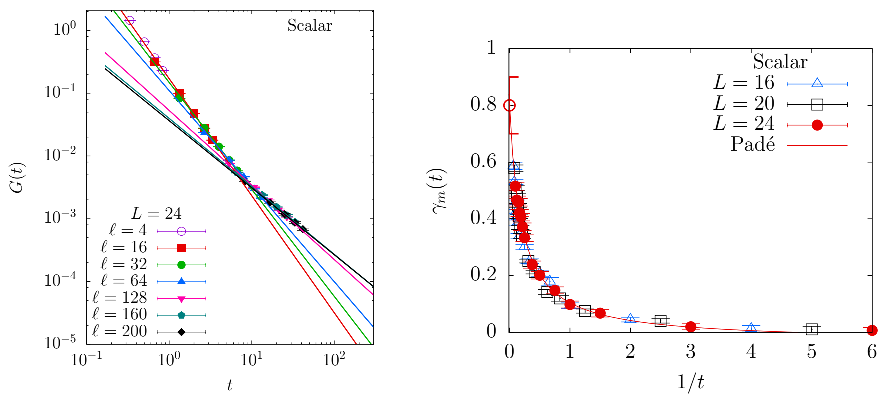

Figure 5: (Left) The zero spatial momentum scalar correlator $G(t) = \langle\Sigma(0)\Sigma(t)\rangle$ as a function of temporal separation t. The different lines are tangents to the correlator, with slope $k(t)$, at various t on the log-log plot. (Right) The mass anomalous dimension given by $\gamma_m(t) = 1 - k(t)/2$ is plotted as a function of the scale t.

The other way to obtain the mass anomalous dimension is to study the scalar correlator $G(t) = \langle\Sigma(0)\Sigma(t)\rangle$ projected to zero spatial momentum. The correlator is shown as a function of the temporal separation t in the left panel of Figure 5. The first thing to notice is the concave-up nature of the correlator. This indicates the absence of a mass-gap, thereby ruling out the presence of another length scale in addition to a bilinear condensate. The slope on the log-log plot, $k(t) = \frac{d\log(G(t))}{d\log(t)}$, is related to a scale dependent mass anomalous dimension $\gamma_m(t)$ as $\gamma_m(t) = 1 - k(t)/2$. This is shown as a function of $1/t$ in the right panel of Figure 5. The mass anomalous dimension at the IR fixed point to which QED$_3$ with $N_f = 1$ flows to, is $\gamma^* = \lim_{t\to\infty} \gamma_m(t)$. We estimate by an extrapolation that $\gamma^* = 0.8(1)$. This is consistent with the estimate 1.0(2) from the eigenvalues described above. The agreement between two different approaches to γ^* serves as a cross-check.

4. Evidence from Inverse Participation Ratio and number variance

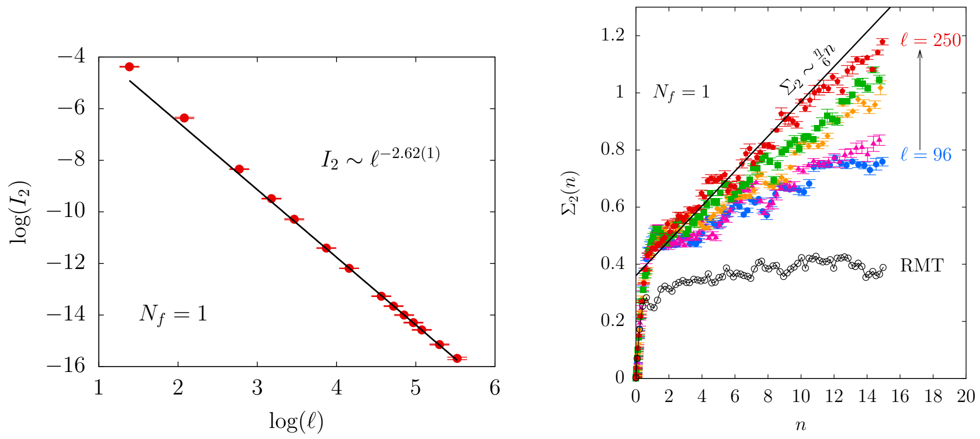

Figure 6: (Left) The ℓ-scaling of the inverse participation ratio I_2 for $N_f = 1$. The critical exponent of the scaling is $\eta = 0.38(1)$. (Right) The number variance Σ_2 is shown as a function of n. A disagreement with nonchiral random matrix model (black points) is seen. Instead, a critical linear rise is seen, whose slope approaches $\eta/6$ shown as the black solid line.

The Inverse Participation Ratio (IPR) is defined as

$$I_2 \equiv \left\langle \int \left(\psi_\lambda^*(x)\psi_\lambda(x)\right)^2 d^3x \right\rangle, \tag{4.1}$$

where ψ_λ is the normalized eigenvector corresponding to the eigenvalue λ. In random matrix models, which are ergodic, $I_2 \sim \ell^{-3}$. Thus, if the theory has a condensate, the low-lying eigensystem of the Dirac operator would be described by a random matrix model. Thus the IPR corresponding to the low-lying eigenvalues should show a ℓ^{-3} scaling. This is another test for the presence of Σ. Instead, if the theory is scale-invariant, the finite size scaling of IPR would be $I_2 \sim \ell^{-3+\eta}$, where η is a critical exponent. The exponent η is related to a quantity called number variance Σ_2 which measures correlations between the eigenvalues. The number variance $\Sigma_2(n)$ is defined as the variance of the number of eigenvalues below a value λ which on the average contains n eigenvalues. In ergodic random matrix models, $\Sigma_2(n) \sim \log(n)$. For a critical theory, $\Sigma_2(n) \sim (\eta/6)n$, where η is the critical exponent from the IPR [11].

In the left panel of Figure 6, we have shown the ℓ-scaling of IPR for $N_f = 1$. For large ℓ, the onset of scaling is clearly seen. The scaling is $I_2 \sim \ell^{-2.62(1)}$. Firstly, this rules out the ergodic ℓ^{-3} scaling. The theory has a non-zero critical exponent $\eta = 0.38(1)$. As explained above, in a critical theory, η should satisfy a critical relation to the slope of number variance. In the right panel of Figure 6, we have shown $\Sigma_2(n)$ as a function of n. Again, clearly there is a disagreement with the expectation from the nonchiral random matrix theory thereby ruling out condensate in another way. We see a linear rise in $\Sigma_2(n)$ indicating a critical behavior. As ℓ is increased, the slope of the linear rise seems to approach $\eta/6$, as shown by the black line in the figure. Thus, both the IPR and Σ_2 show critical behavior, and also they satisfy the critical relation between the two.

5. Conclusions

In this talk, we presented convincing numerical evidences for the absence of a bilinear condensate for all N_f. Instead, we found evidences for QED$_3$ to be scale-invariant, and we estimated the mass anomalous dimension at the infra-red fixed point at various N_f. In another work [12], we established the presence of a condensate in the 't Hooft limit using the same methods we described here. This suggests an interesting phase diagram in the (N_f, N_c) plane whose one side is conformal while the other side has a mass-gap, providing a powerful system to understand the generation of mass in QFTs. We aim to present results on this in a future Lattice meeting.

The authors acknowledge partial support by the NSF under grant number PHY-1205396 and PHY-1515446.

References

[1] J. J. M. Verbaarschot and I. Zahed, *Random matrix theory and QCD in three-dimensions*, *Phys. Rev. Lett.* **73** (1994) 2288–2291, [hep-th/9405005].

[2] N. Karthik and R. Narayanan, *Scale-invariance of parity-invariant three-dimensional QED*, *Phys. Rev.* **D94** (2016) 065026, [1606.04109].

[3] N. Karthik and R. Narayanan, *No evidence for bilinear condensate in parity-invariant three-dimensional QED with massless fermions*, *Phys. Rev.* **D93** (2016) 045020, [1512.02993].

[4] T. W. Appelquist, M. J. Bowick, D. Karabali and L. C. R. Wijewardhana, *Spontaneous Chiral Symmetry Breaking in Three-Dimensional QED*, *Phys. Rev.* **D33** (1986) 3704.

[5] T. Appelquist, A. G. Cohen and M. Schmaltz, *A New constraint on strongly coupled gauge theories*, *Phys. Rev.* **D60** (1999) 045003, [hep-th/9901109].

[6] L. Di Pietro, Z. Komargodski, I. Shamir and E. Stamou, *Quantum Electrodynamics in d=3 from the Îţ Expansion*, *Phys. Rev. Lett.* **116** (2016) 131601, [1508.06278].

[7] S. M. Chester and S. S. Pufu, *Anomalous dimensions of scalar operators in QED$_3$*, *JHEP* **08** (2016) 069, [1603.05582].

[8] S. J. Hands, J. B. Kogut, L. Scorzato and C. G. Strouthos, *Non-compact QED(3) with N(f) = 1 and N(f) = 4*, *Phys. Rev.* **B70** (2004) 104501, [hep-lat/0404013].

[9] O. Raviv, Y. Shamir and B. Svetitsky, *Nonperturbative beta function in three-dimensional electrodynamics*, *Phys. Rev.* **D90** (2014) 014512, [1405.6916].

[10] J. A. Gracey, *Electron mass anomalous dimension at O(1/(Nf(2)) in quantum electrodynamics*, *Phys. Lett.* **B317** (1993) 415–420, [hep-th/9309092].

[11] V. Chalker, J.T. amd Kravtsov and I. Lerner, *Spectral rigidity and eigenfunction correlations at the Anderson transition*, *Pis'ma v ZhETF* **64** (1996) 355–360.

[12] N. Karthik and R. Narayanan, *Bilinear condensate in three-dimensional large-N_c QCD*, *Phys. Rev.* **D94** (2016) 045020, [1607.03905].