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# Kaon semileptonic decays with $N_f = 2+1+1$ HISQ fermions and physical light-quark masses

**E. Gámiz[*a], A. Bazavov[b], C. Bernard[c], C. DeTar[d], D. Du[e], A.X. El-Khadra[f], E.D. Freeland[g], Steven Gottlieb[h], U.M. Heller[i], J. Komijani[j], A.S. Kronfeld[j,k], J. Laiho[e], P.B. Mackenzie[k], E.T.Neil[l], T. Primer[m], J.N. Simone[k], R. Sugar[n], D. Toussaint[m], R.S. Van de Water[k], and Ran Zhou[k]**

[a] *CAFPE and Departamento de Física Teórica y del Cosmos, Universidad de Granada,[†]Granada, Spain*
[b] *Physics Department, Brookhaven National Laboratory, Upton, NY, USA*
[c] *Department of Physics, Washington University, St. Louis, MO, USA*
[d] *Physics Department, University of Utah, Salt Lake City, UT, USA*
[e] *Department of Physics, Syracuse University, Syracuse, NY, USA*
[f] *Department of Physics, University of Illinois, Urbana, IL, USA*
[g] *Liberal Arts Department, School of the Art Institute of Chicago, Chicago, Illinois, USA*
[h] *Department of Physics, Indiana University, Bloomington, IN, USA*
[i] *American Physical Society, One Research Road, Ridge, NY, USA*
[j] *Institute for Advanced Study, Technische Univerität München, Garching, Germany*
[k] *Fermi National Accelerator Laboratory,[‡] Batavia, IL, USA*
[l] *Department of Physics, University of Colorado, Boulder, CO, USA*
[m] *Physics Department, University of Arizona, Tucson, AZ, USA*
[n] *Department of Physics, University of California, Santa Barbara, USA*

*E-mail:* megamiz@ugr.es

**Fermilab Lattice and MILC Collaborations[§]**

We discuss the reduction of errors in the calculation of the form factor $f_+^{K\pi}(0)$ with HISQ fermions on the $N_f = 2+1+1$ MILC configurations from increased statistics on some key ensembles, new data on ensembles with lattice spacings down to 0.042 fm and the study of finite-volume effects within staggered ChPT. We also study the implications for the unitarity of the CKM matrix in the first row and for current tensions with leptonic determinations of $|V_{us}|$.

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[*]Speaker.

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## 1. Introduction

There exist several tensions involving the CKM matrix element $|V_{us}|$ as extracted from semileptonic kaon decays. First of all, there is a $\sim 2\sigma$ disagreement with unitarity in the first row of the CKM matrix, measured as the deviation from 0 of the quantity

$$\Delta_u \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00122(37)_{V_{us}}(41)_{V_{ud}}\,, \tag{1.1}$$

where we have used the last published Fermilab Lattice/MILC result for the vector form factor at zero momentum transfer, $f_+(0) = 0.9704(\pm 0.33\%)$ [1, 2], the last experimental average of neutral and charged semileptonic $K$ decays in Ref. [3], $|V_{us}|f_+^{K\pi}(0)\big|_{exp} = 0.2165(\pm 0.18\%)$, $V_{ud} = 0.97417(21)$ from superallowed nuclear $\beta$ decays [4], and we have neglected $|V_{ub}|$. The deviation does not change if one uses instead the values of the vector form factor from other recent lattice-QCD calculations [5, 6].

In addition, there is a $\sim 2\sigma$ tension between the value of $|V_{us}|$ as obtained from leptonic and semileptonic decays [7]. The picture is complicated even further by including determinations of $|V_{us}|$ from hadronic $\tau$ decays, which traditionally yielded smaller values of $|V_{us}|$ than $K$ decays (and thus were in even more disagreement with unitarity) —see Fig. 1. More recent results using lattice QCD for the hadronic vacuum polarization correlators and updated (preliminary) BaBar data, however, point towards values more compatible with unitarity [8].

Checking the unitarity of the CKM matrix and the internal consistency of the Standard-Model description in the light sector is crucial for trying to unveil new-physics effects and put constraints on the scale of the allowed new physics [9]. In order to perform even more stringent tests, it is necessary to reduce the error both on the experimental and lattice-QCD inputs entering determinations of $|V_{us}|$. For semileptonic decays, the lattice error on the vector form factor $f_+(0)$ is still the largest uncertainty. We summarize here our progress on reaching the objective of reducing the lattice-QCD error on $f_+^{K\pi}(0)$ to the $\sim 0.2\%$ level, which would match the current experimental uncertainty.

## 2. New simulation data

The setup of our new work is the same as that used in our previous calculation [1, 2]. We obtain $f_+(0)$ from the relation $f_+(0) = f_0(0) = \frac{m_s - m_l}{m_K^2 - m_\pi^2}\langle \pi(p_\pi)|S|K(p_K)\rangle$, where the vector form factor, $f_+$, and the scalar form factor, $f_0$, are defined by $(q = p_K - p_\pi)$

$$\langle \pi | V^\mu | K \rangle = f_+(q^2)\left[p_K^\mu + p_\pi^\mu - \frac{m_K^2 - m_\pi^2}{q^2} q^\mu\right] + f_0(q^2)\frac{m_K^2 - m_\pi^2}{q^2} q^\mu\,. \tag{2.1}$$

We simulate directly at zero momentum transfer, $q^2 \approx 0$, by tuning the external momentum of the $\pi$ using partially twisted boundary conditions. We use the HISQ action for sea and valence quarks, simulating on the HISQ $N_f = 2+1+1$ MILC configurations [12].

Our previous calculation [1, 2] used the HISQ action for both the sea and valence sectors, analyzed small lattice spacings down to $a \approx 0.06$ fm and included ensembles with physical quark masses. The total error was dominated by statistics —see Table 1.

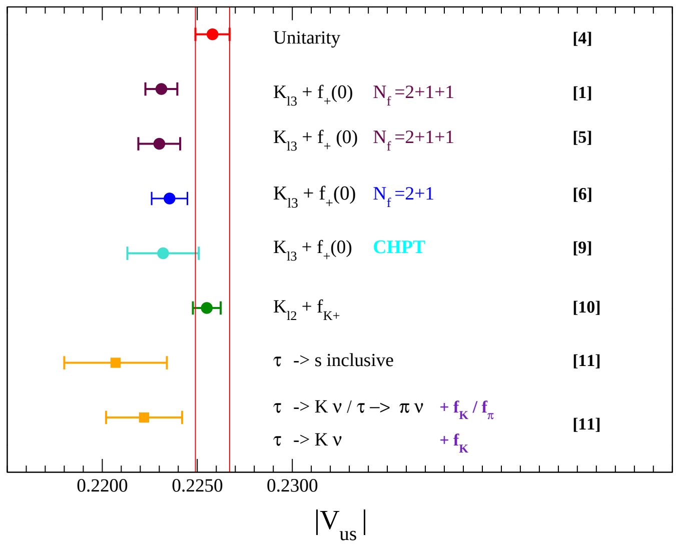


**Figure 1:** Summary of recent $|V_{us}|$ determinations.

| Source of uncertainty | Error $f_+(0)$ (%) Fermilab Lattice-MILC 2014 [1] | Error $f_+(0)$ (%) Preliminary 2016 |
|---|---|---|
| Stat. + disc. + chiral inter. | 0.24 | $\leq 0.2$ |
| $m_s^{\rm val} \neq m_s^{\rm sea}$ | 0.03 | 0.03 |
| Scale $r_1$ | 0.08 | 0.08 |
| Finite volume | 0.2 | $\sim 0$ |
| Isospin | 0.016 | 0.016 |
| Total Error | 0.33 | $\leq 0.22$ |

**Table 1:** Error budget for $f_+(0)$ in percent from our previous calculation in Ref. [1] and error budget estimate from this work.

In order to reduce the statistical error, in the last two years we have generated new data on a number of key ensembles. Table 2 lists the final set of data as well as the data included in our 2014 analysis. We have more than doubled our statistics in one of the key ensembles for the chiral-continuum interpolation/extrapolation, the ensemble with $a \approx 0.09$ fm and physical quark masses (last line in Table 2 with $a \approx 0.09$ fm), and the ensemble with smallest lattice spacing in our previous analysis, the one with $a \approx 0.06$ fm and $m_l = 0.2 m_s$ (first line in Table 2 with $a \approx 0.06$ fm). In addition, in our new analysis we include the $a \approx 0.06$ fm ensemble with physical quark masses and a smaller lattice spacing, the ensemble with $a \approx 0.042$ fm and $m_l = 0.2 m_s$. These are the two last lines in Table 2. Moreover, we have generated extra data in order to further investigate finite volume corrections as explained in the next section.

We block the data by four in all ensembles to avoid autocorrelations and follow the fit strategy already discussed in Refs. [13] and [14]. Preliminary results from those fits, except for the ensemble with $a \approx 0.042$ fm that has not yet been analyzed, are shown on the right hand side of Fig. 2. The (preliminary) errors in the data points are statistical only, obtained from 500 bootstraps. They range

| $\approx a$(fm) | $m_l/m_s$ | $m_\pi L$ | $N_{conf} \times N_{src}$ (2014) | $N_{conf} \times N_{src}$ (2016) | $am_s^{sea}$ | $am_s^{val}$ |
|---|---|---|---|---|---|---|
| 0.15 | 0.035 | 3.30 | $1000 \times 4$ | $1000 \times 4$ | 0.0647 | 0.0691 |
| 0.12 | 0.2 | 4.54 | $1053 \times 8$ | $1053 \times 8$ | 0.0509 | 0.0535 |
| | 0.1 | 3.22 | - | $1020 \times 8$ | 0.0507 | 0.053 |
| | 0.1 | 4.29 | $993 \times 4$ | $993 \times 4$ | 0.0507 | 0.053 |
| | 0.1 | 5.36 | $391 \times 4$ | $1029 \times 8$ | 0.0507 | 0.053 |
| | 0.035 | 3.88 | $945 \times 8$ | $945 \times 8$ | 0.0507 | 0.0531 |
| 0.09 | 0.2 | 4.50 | $773 \times 4$ | $773 \times 4$ | 0.037 | 0.038 |
| | 0.1 | 4.71 | $853 \times 4$ | $853 \times 4$ | 0.0363 | 0.038 |
| | 0.035 | 3.66 | $621 \times 4$ | $950 \times 8$ | 0.0363 | 0.0363 |
| 0.06 | 0.2 | 4.51 | $362 \times 4$ | $1000 \times 8$ | 0.024 | 0.024 |
| | 0.035 | 3.95 | - | $692 \times 6$ | 0.022 | 0.022 |
| 0.042 | 0.2 | 3.23 | - | $432 \times 12$ | 0.0158 | 0.0158 |

**Table 2:** Parameters of the $N_f = 2+1+1$ gauge-field ensembles used in this work and details of the correlation functions generated. $N_{\rm conf}$ is the number of configurations included in our analysis, $N_{\rm src}$ the number of time sources used on each configuration, and $L$ the spatial size of the lattice. The ensemble with $a \approx 0.12$ fm, $m_l/m_s = 0.1$ and $m_\pi L = 3.22$ is not included in the main analysis, but used to analyze finite-volume effects.

from 0.16% to 0.38%.

In Fig 2, for comparison, we show both the results in our 2014 analysis (left plot) and the preliminary results we are reporting here (right plot). In the right plot we also show our previous chiral interpolation curve and the physical result (after performing the chiral-continuum interpolation/extrapolation) together with the corresponding errors, in order to make the comparison easier. Solids symbols on that plot correspond to the ensembles for which our data set has not changed since 2014, while open symbols correspond to ensembles where we have increased the statistics or ensembles for which we did not have data previously. The dashed magenta line is the preliminary chiral interpolation including all the new data except for the ensemble with $a \approx 0.12$ fm and $m_\pi L = 3.22$, and that with $a \approx 0.042$ fm. We use the same fit function as in our 2014 analysis, a NLO partially quenched staggered ChPT (PQSChPT) expression [15], plus regular NNLO continuum ChPT terms [16], plus extra analytic terms which parametrize higher order discretization and chiral effects:

$$f_+(0) = 1 \, + \, f_2^{\rm PQSChPT}(a) + K_1 \sqrt{r_1^2 a^2 \bar\Delta \left(\frac{a}{r_1}\right)^2} + K_3 \left(\frac{a}{r_1}\right)^4 + f_4^{\rm cont.}$$
$$+ \, r_1^4 (m_\pi^2 - m_K^2)^2 \left[ C_6 + K_2 \sqrt{r_1^2 a^2 \bar\Delta \left(\frac{a}{r_1}\right)^2} + K_2' r_1 a^2 \bar\Delta + C_8 \, m_\pi^2 + C_{10} \, m_\pi^4 \right] \, , \quad (2.2)$$

where the constants $K_i$ and $C_i$ are fit parameters to be determined by the chiral fits using Bayesian techniques, $\bar\Delta$ is the average taste splitting $\bar\Delta = \frac{1}{16}(\Delta_P + 4\Delta_A + 6\Delta_T + 4\Delta_V + \Delta_I)$, and $r_1^2 a^2 \bar\Delta$ is a proxy for $\alpha_s^2 a^2$. More details about this fit function, priors used in the Bayesian approach and tests performed can be found in Refs. [1, 2]. The only change we have made to the fit function for this preliminary analysis is updating the values of the taste splitting, the relative scale $r_1/a$ and the pion decay constant, but the updates do not vary very much from the old values, and those changes have very little effect on the fit results.

As explained in Refs. [1, 2], the error in the physical values of $f_+(0)$ depicted in Fig. 2 includes statistical (bootstrap), chiral extrapolation and discretization errors, as well as the uncertainty associated to those inputs treated as constrained fit parameters ($O(p^4)$ LEC's and taste-violating hairpin parameters). The HISQ taste splittings are known precisely enough that their error has no impact on the final uncertainty.

As Fig. 2 shows, the addition of the new data only slightly changes the central value of $f_+(0)$, while the error is significantly reduced. These preliminary results seem to indicate that this error will be under 0.2%, which was our goal.

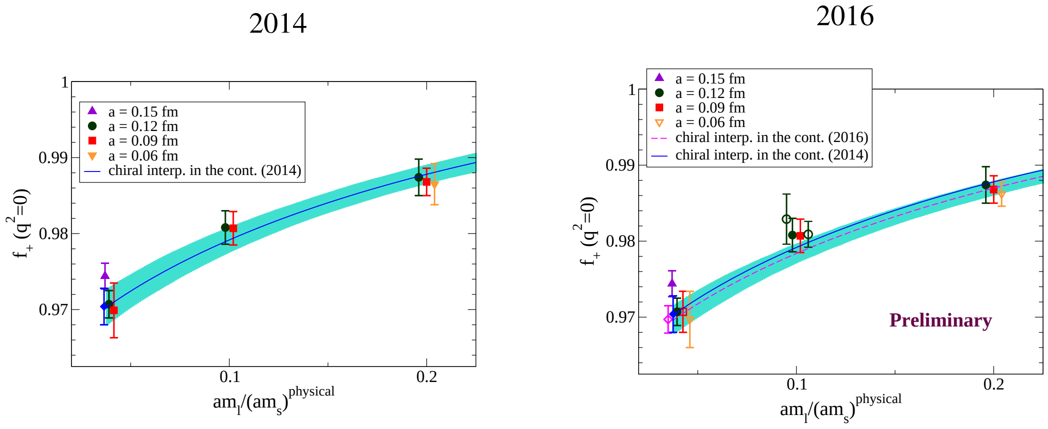


**Figure 2:** Form factor $f_+(0)$ vs. light-quark mass. Errors shown are statistical only, obtained from 500 bootstraps. Different symbols and colors denote different lattice spacings, and open symbols correspond to either new ensembles or ensembles on which we have increased the statistics since our previous calculation in Ref. [1]. The solid blue line is the interpolation in the light-quark mass, keeping $m_s$ equal to its physical value, and turning off all discretization effects, from Ref. [1]. The dashed magenta line shows preliminary results for the same type of interpolation but including the new data generated since 2014. The blue and magenta diamonds are the corresponding interpolations at the physical point. The turquoise error band includes statistical, discretization and higher order chiral errors, as well as the uncertainty from some of the input parameters (see the text). Data at the same light-quark mass but different lattice spacing are off-set horizontally.

## 3. Finite-Volume Effects

The other dominant source of error in our 2014 calculation [1], as shown in Table 1, was the uncertainty due to finite-volume effects. For an estimate of the finite-volume error we compared data obtained on two ensembles with the same parameters but different spatial volumes, those with $a \approx 0.12$ fm and $m_l = 0.1 m_s$ and $m_\pi L = 4.29, 5.36$ (third and fourth lines with $a \approx 0.12$ fm in Table 2). The difference was about half of the statistical error, so we took the finite volume error to be the full size of the statistical error, 0.2%.

For our new analysis, we have increased the statistics on the largest $a \approx 0.12$ fm, $m_l = 0.1 m_s$ volume and generated data on a third, smaller, volume, so we have results for three different spatial volumes (with all other parameters fixed). The preliminary values we obtain for $f_+(0)$ on these three volumes are shown in Table 3. With the improved statistics the two largest volume results are basically the same, and the smaller volume gives a larger result but well within the statistical error. Disregarding the result on the $m_\pi L = 3.22$ ensemble (which is smaller than all of the ensembles used in our main analysis), we conclude that finite-volume effects are smaller than our statistical errors, or 0.17%.

To further reduce the finite-volume errors, we can study finite-volume effects systematically within the framework of ChPT, and then use the computed corrections to extrapolate our results to infinite volume. The use of partially twisted boundary conditions in the generation of the relevant correlation functions complicates the analysis in several ways. For example, at finite volume we need an extra form factors, $h_\mu$, to parametrize the weak current

| $m_\pi L$ | 3.22 | 4.29 | 5.36 |
|---|---|---|---|
| $f_+(0)$ | 0.9829(33) | 0.9808(22) | 0.9809(17) |
| variation | +0.2% | $\sim 0\%$ | 0% |

**Table 3:** $f_+(0)$ on three different spatial volumes with $a \approx 0.12$ fm and $m_l = 0.1 m_s$, and variation respect to the largest volume result.

$$\langle \pi^-(p')|V_\mu|K^0(p)\rangle = f_+(p_\mu + p'_\mu) + f_- q_\mu + h_\mu\,, \tag{3.1}$$

where the three form factors depend on the exact choice of twisting angles, not only on the value of $q^2$. At finite volume, then, the Ward-Takahashi identity that relates the matrix element of a vector current with that of a scalar current at zero momentum transfer leads to the following relation

$$f_+^V(0) = \frac{m_s - m_l}{\tilde{m}_K^2 - \tilde{m}_\pi^2}\langle \pi(p_\pi)|S|K(p_K)\rangle - q_\mu h_\mu\,, \tag{3.2}$$

where the $\tilde{m}_P^2$ include one-loop finite-volume corrections [17].

The authors of Ref. [17] have performed the one-loop ChPT calculation of finite-volume effects for all the form factors involved in $K \to \pi \ell \nu$ decays, $f_+$, $f_-$ and $h_\mu$, including all choices of partially (and fully) twisted boundary conditions, both in the continuum and including the leading staggered effects. The main results are also reported at this conference [18]. Both the partially quenched and full-QCD cases are studied in Ref. [17]. When applying the one-loop formulae in Ref. [17] to all ensembles included in our calculation, [1] we conclude that the dominant finite-volume effects in our data are under 0.05% and higher order effects can be safely neglected. The one-loop corrections are added before the chiral+continuum extrapolation and any remaining finite-volume corrections are negligible.

## 4. Conclusions

We have described here how we address the main two sources of uncertainty in our previous calculation of the vector form factor $f_+(0)$: statistics+discretization+chiral interpolation and finite volume effects. We summarize in the third column of Table 1 our preliminary error budget for this quantity. We expect the final error to be reduced from 0.33% to $\sim 0.2\%$, matching the level of

[1] We exclude here the ensemble with 0.12 fm, $m_l = 0.1 m_s$ and $m_\pi L = 3.22$ because it is not used in the main analysis.



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precision of the average of experimental measurements, which includes the uncertainty from strong and electromagnetic isospin corrections.

With this reduction in the error of $f_+(0)$, using the same experimental input and assuming that the central value does not vary, the unitarity test in Eq. (1.1) would become

$$\Delta_u \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00122(27)_{V_{us}}(41)_{V_{ud}}\,, \tag{4.1}$$

which is $\sim 2.5\sigma$ away from the unitarity prediction with an error dominated by the uncertainty on $|V_{ud}|$. This will make revisiting the determination of $|V_{ud}|$ a priority for CKM tests. Again assuming the central value of the form factor does not change in our final analysis, the reduction of the error in $f_+(0)$ would increase the tension between the leptonic and semileptonic determinations of $|V_{us}|$ to the $2.7\sigma$ level. Further, the semileptonic determination of $|V_{us}|$ would become more precise than the leptonic determination from $f_K$, and as precise as the leptonic determination using the ratio $f_K/f_\pi$ but without the need of the $|V_{ud}|$ input.

## References


[1] A. Bazavov *et al.*, Phys. Rev. Lett. **112** (2014) no.11, 112001 [arXiv:1312.1228 [hep-ph]].

[2] E. Gámiz *et al.* [Fermilab Lattice and MILC Collaborations], PoS LATTICE **2013** (2014) 395 [arXiv:1311.7264 [hep-lat]].

[3] M. Moulson, talk at CKM2014, arXiv:1411.5252 [hep-ex].

[4] J. C. Hardy and I. S. Towner, Phys. Rev. C **91** (2015) no.2, 025501 [arXiv:1411.5987 [nucl-ex]].

[5] N. Carrasco, P. Lami, V. Lubicz, L. Riggio, S. Simula and C. Tarantino, Phys. Rev. D **93** (2016) no.11, 114512 [arXiv:1602.04113 [hep-lat]].

[6] P. A. Boyle *et al.* [RBC/UKQCD Collaboration], JHEP **1506** (2015) 164 [arXiv:1504.01692 [hep-lat]].

[7] J. L. Rosner, S. Stone and R. S. Van de Water, [arXiv:1509.02220 [hep-ph]].

[8] K. Maltman, these proceedings; H. Ohki, these proceedings.

[9] M. González-Alonso and J. Martin Camalich, arXiv:1606.06037 [hep-ph].

[10] J. Bijnens and G. Ecker, Ann. Rev. Nucl. Part. Sci. **64** (2014) 149 [arXiv:1405.6488 [hep-ph]].

[11] A. Lusiani, talk at CKM2014 arXiv:1411.4526 [hep-ex]; E. Passemar, talk at CKM2014.

[12] A. Bazavov *et al.* [MILC Collaboration], Phys. Rev. D **87**, 054505 (2013) [arXiv:1212.4768 [hep-lat]].

[13] A. Bazavov *et al.* [Fermilab Lattice and MILC Collaborations], Phys. Rev. D **87**, 073012 (2013) [arXiv:1212.4993 [hep-lat]].

[14] E. Gámiz *et al.* [Fermilab Lattice and MILC Collaborations], PoS LATTICE **2012**, 113 (2012) [arXiv:1211.0751 [hep-lat]].

[15] C. Bernard, J. Bijnens and E. Gámiz, Phys. Rev. D **89** (2014) no.5, 054510 [arXiv:1311.7511 [hep-lat]].

[16] J. Bijnens, P. Talavera, Nucl. Phys. **B669** (2003) 341-362. [hep-ph/0303103].

[17] C. Bernard, J. Bijnens, E. Gámiz and J. Relefors, work in progress.

[18] J. Bijnens, these proceedings.