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# $D$ meson semileptonic decays in lattice QCD with Möbius domain-wall quarks

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We report on our study of the $D$ meson semileptonic decays in 2+1 flavor lattice QCD. Gauge ensembles are generated at three lattice cutoffs up to 4.5 GeV and with pion masses as low as 300 MeV. We employ the Möbius domain-wall fermion action for both light and charm quarks. We report our preliminary results for the vector and scalar form factors and discuss their dependence on the momentum transfer, quark masses and lattice spacing.

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## 1. Introduction

The $D \to \pi l \nu$ and $D \to K l \nu$ semileptonic decays provide a precise determination of the Cabibbo-Kobayashi-Maskawa matrix elements $|V_{cd}|$ and $|V_{cs}|$, and play an important role in the search for new physics in the charm sector [1]. The vector and scalar form factors $f_{\{+,0\}}^{DP}$ describe non-perturbative QCD effects, and are defined from the relevant hadronic matrix elements as

$$\langle P(p')|V_\mu|D(p)\rangle = (p+p')_\mu f_+^{DP}(t) + (p-p')_\mu f_-^{DP}(t), \tag{1.1}$$

$$f_0^{DP}(t) = f_+^{DP}(t) + \frac{t}{M_D^2 - M_P^2} f_-^{DP}(t), \tag{1.2}$$

where $P$ specifies the light meson ($P = \pi, K$) and $t = (p-p')^2$ is the momentum transfer. The current accuracy of $|V_{cd}|$ and $|V_{cs}|$ is limited by the theoretical uncertainty of $f_{\{+,0\}}^{DP}$ [2]. Lattice QCD is the only known method to calculate $f_{\{+,0\}}^{DP}$ with controlled and systematically-improvable uncertainties.

In this article, we report on our calculation of these form factors in $N_f = 2+1$ QCD with the tree-level improved Symanzik gauge action and the Möbius domain wall quark action [3]. Numerical simulations are carried out at three lattice cutoffs $a^{-1} \sim 2.5$, 3.6 and 4.5 GeV. On such fine lattices, we employ the domain-wall action also for charm quarks. The simulated values of $m_{ud}$, that is the mass of the degenerate up and down quarks, cover a range of the pion mass 300 MeV $\lesssim M_\pi \lesssim$ 500 MeV. We take a strange quark mass $m_s$ close to its physical value. An additional value of $m_s$ is simulated at certain choices of $(a, m_{ud})$ in order to study the $m_s$ dependence of the form factors. The charm quark mass is set to its physical value determined from the $D$ meson spectrum [4]. The physical charm quark mass extracted from the same set of simulations is $m_c(3\ \text{GeV}) = 1.003(10)$ GeV, which is consistent with the present world average [5]. Our simulation parameters are summarized in Table 1. After the previous report [6], we extend our simulation to the two finer lattices and improve our measurement method to reduce the statistical uncertainty.

At each simulation point, our lattice size satisfies a condition $M_\pi L \gtrsim 4$ to control finite volume effects, and we accumulate 5,000 Molecular Dynamics time. Chiral symmetry is preserved to good accuracy by choosing the sign function approximation and the kernel operator in the 4-dimensional

**Table 1:** Simulation parameters.

| lattice parameters | $m_{ud}$ | $m_s$ | $M_\pi$[MeV] | $M_K$[MeV] | $N_{x_{4,\rm src}}$ |
|---|---|---|---|---|---|
| $\beta = 4.17,\ a^{-1} = 2.453(4),\ 32^3 \times 64 \times 12$ | 0.0190 | 0.0400 | 499(1) | 618(1) | 2 |
| | 0.0120 | 0.0400 | 399(1) | 577(1) | 2 |
| | 0.0070 | 0.0400 | 309(1) | 547(1) | 4 |
| | 0.0190 | 0.0300 | 498(1) | 563(1) | 2 |
| $\beta = 4.35,\ a^{-1} = 3.610(9),\ 48^3 \times 96 \times 8$ | 0.0120 | 0.0250 | 501(2) | 620(2) | 2 |
| | 0.0080 | 0.0250 | 408(2) | 582(2) | 2 |
| | 0.0042 | 0.0250 | 300(1) | 547(2) | 4 |
| | 0.0120 | 0.0180 | 499(1) | 557(2) | 2 |
| $\beta = 4.47,\ a^{-1} = 4.496(9),\ 64^3 \times 128 \times 8$ | 0.0030 | 0.0150 | 284(1) | 486(1) | 1 |

effective action [7]. The residual mass is suppressed to $O(1\text{ MeV})$ at the coarsest lattice, and even smaller $\lesssim 0.2$ MeV at finer lattices with moderate sizes in the fifth dimension $\sim 10$.

## 2. Calculation of form factors

We calculate the three-point function

$$C_{V_\mu}^{DP}(\mathbf{p},\mathbf{p}';\Delta x_4,\Delta x_4') = \frac{1}{N_s^3 N_{x_{4,\rm src}}} \sum_{x_{4,\rm src}} \sum_{\mathbf{x},\mathbf{x}',\mathbf{x}''} \langle \mathcal{O}_P(\mathbf{x}'',x_{4,\rm src}+\Delta x_4+\Delta x_4') \times V_\mu(\mathbf{x}',x_{4,\rm src}+\Delta x_4)\mathcal{O}_D^\dagger(\mathbf{x},x_{4,\rm src})\rangle e^{-i\mathbf{p}'(\mathbf{x}''-\mathbf{x}')} e^{-i\mathbf{p}(\mathbf{x}'-\mathbf{x})}, \tag{2.1}$$

where $N_s$ is the spatial lattice size and $\mathbf{p}^{(\prime)}$ represents the momentum of the initial (final) meson. We apply a Gaussian smearing to the meson interpolating operators $\mathcal{O}_{\{\pi,K,D\}}$. The temporal coordinate of the source operator is denoted by $x_{4,\rm src}$, and $\Delta x_4^{(\prime)}$ represents the temporal separation between the source (sink) operator and the vector current $V_\mu$.

In this study, we calculate $C_{V_\mu}^{DP}(\mathbf{p},\mathbf{p}';\Delta x_4,\Delta x_4')$ by varying $\Delta x_4$ with $\Delta x_4+\Delta x_4'$ kept fixed. Its physical length is the same for the three cutoffs and is chosen as $\Delta x_4+\Delta x_4'=28a$ at $\beta=4.17$ [6]. The $D$ meson is at rest ($\mathbf{p}=\mathbf{0}$), and we simulate four different values of the momentum transfer $t$ with light meson momenta $|\mathbf{p}'|^2=0,1,2,3$ in units of $(2\pi/L)^2$. For $P=\pi(K)$, the minimum value of the momentum transfer is typically $t_{\rm min}\approx 0.3(0.2)\,\text{GeV}^2$, while the maximum is $t_{\rm max}\approx 2.6(1.8)\,\text{GeV}^2$.

We also calculate two-point functions of $\pi$, $K$ and $D$ mesons. The amplitudes of the correlators are extracted by the following exponential fits in terms of $\Delta x_4$

$$C_{V_\mu}^{DP}(\mathbf{p},\mathbf{p}';\Delta x_4,\Delta x_4') = A_{V_\mu}^{DP}(\mathbf{p},\mathbf{p}')e^{-E_D(\mathbf{p})\Delta x_4}e^{-E_P(\mathbf{p}')\Delta x_4'} \quad (P=\pi,K), \tag{2.2}$$

$$C^Q(\mathbf{p};\Delta x_4) = B^Q(\mathbf{p})e^{-E_Q(\mathbf{p})\Delta x_4} \quad (Q=\pi,K,D). \tag{2.3}$$

Here the meson energies $E_{\{\pi,K,D\}}$ are estimated from their rest masses [4] and the dispersion relation in the continuum limit. The matrix elements are given as

$$\langle P(\mathbf{p}')|V_\mu|D(\mathbf{p})\rangle = 2Z_V\sqrt{\frac{E_D(\mathbf{p})\,E_P(\mathbf{p}')\,|A_{V_\mu}^{DP}(\mathbf{p},\mathbf{p}')|^2}{B^D(\mathbf{p})\,B^P(\mathbf{p}')}}, \tag{2.4}$$

where we use the renormalization factor $Z_V$ non-perturbatively calculated in Ref. [8]. The relevant semileptonic form factors are then extracted via Eqs. (1.1) and (1.2).

In order to improve the statistical accuracy, we average the three- and two-point functions over the locations of the source operator. As for the temporal location, we repeat our measurement over $N_{x_{4,\rm src}}$ different values of $x_{4,\rm src}$. Our choice of $N_{x_{4,\rm src}}$ is summarized in Table 1. An important improvement from Ref. [6] is to average over the spatial coordinates $\mathbf{x}$ as well by putting the Gaussian source operator associated with a $Z_2$ noise at each lattice site at a given time-slice $x_{4,\rm src}$.

Figure 1 compares our results for the amplitude $A_{V_4}^{D\pi}$ with different measurement setups on our coarsest lattice at the heaviest sea quark masses. We observe about 30 % reduction of the statistical error by averaging over 2 $t_{\rm src}$'s and an additional 30 % reduction by averaging over $\mathbf{x}$: about a factor of two improvement in total. Averaging over $\mathbf{p}$ further improves the statistical accuracy: at $|\mathbf{p}|^2=2$, for instance, about factor of two improvement by averaging over 12 $\mathbf{p}$'s. The typical statistical accuracy is $1-2$ % at $t_{\rm max}$ and $M_\pi\sim 500$ MeV, and $6-9$ % at $t_{\rm min}$ and $M_\pi\sim 300$ MeV.



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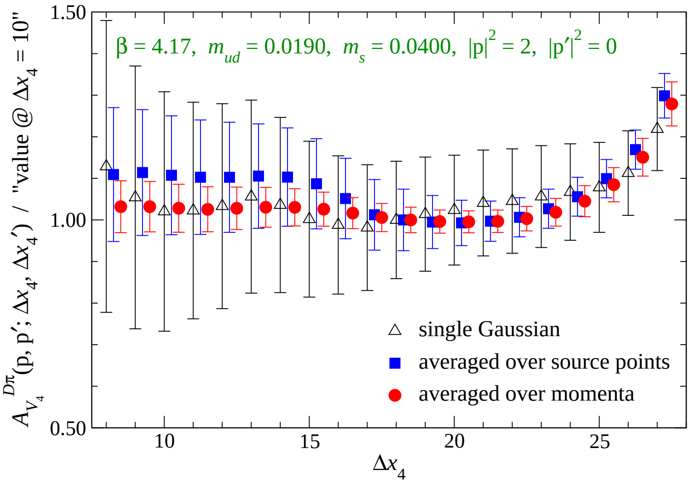


**Figure 1:** Effective value of amplitude $A_{V_4}^{D\pi}(\mathbf{p},\mathbf{p}')$ as a function of $\Delta x_4$. We plot data with $|\mathbf{p}|=2$ and $|\mathbf{p}'|^2=0$ at $\beta=4.17$ and $(m_{ud},m_s)=(0.0190,0.0400)$. The open triangles show data with a "local" Gaussian source and a single choice of $\mathbf{p}$. The blue squares and red circles are obtained by averaging over the location of the source operator and then over 12 $\mathbf{p}$'s. All data are normalized by their value at $\Delta x_4=18$.

## 3. Momentum transfer dependence

We parametrize the momentum transfer dependence of the form factors in terms of the so-called $z$ parameter [9]

$$z(t,t_0) \;=\; \frac{\sqrt{t_+-t}-\sqrt{t_+-t_0}}{\sqrt{t_+-t}+\sqrt{t_+-t_0}}, \tag{3.1}$$

where $t_+=(M_D+M_P)^2$ represents the $DP$ threshold ($P=\pi,\,K$). The free parameter $t_0$ is chosen so that our simulated region $t\in[t_{\rm min},t_{\rm max}]$ is mapped into a shortest segment $z\in[-|z|_{\rm max},+|z|_{\rm max}]$ centered at the origin. Typical size of the $z$ parameter is $|z|_{\rm max}\lesssim 0.2$.

The momentum transfer dependence of the form factors are then parametrized by using this small parameter as

$$f_{\{+,0\}}^{DP}(t) \;=\; \frac{1}{B(t)}\sum_{k=0}^{N_{\{+,0\}}} a_{\{+,0\},k}\,z^k. \tag{3.2}$$

In this preliminary analysis, we test two choices of the factor $B(t)$. In the so-called Bourrely-Caprini-Lellouch (BCL) parametrization [10] with

$$B(t)=1-\frac{t}{M_{\rm pole}^2}, \tag{3.3}$$

possibly small deviation from the lowest pole contribution $1/B(t)$ is expanded in terms of $z$. We also test a naive polynomial expansion of the form factors themselves with

$$B(t)=1. \tag{3.4}$$

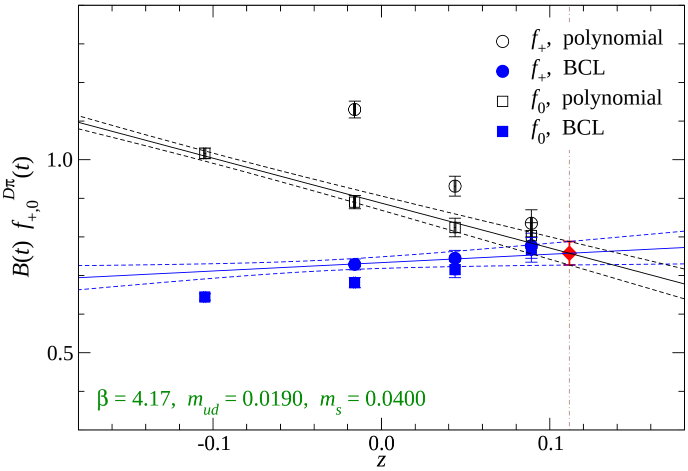


**Figure 2:** Plot of $B(t)f^{D\pi}_{\{+,0\}}(t)$ as a function of $z$ at $\beta = 4.17$ and $(m_{ud}, m_s) = (0.0190, 0.0400)$. Circles and squares are data for $f_+$ and $f_0$, respectively. Filled symbols show $(1 - t/M^2_{D^*_{(0)}})f^{D\pi}_{+(0)}(t)$ for the BCL parametrization, whereas open symbols for the polynomial expansion are just the form factors themselves. A simultaneous fit to $f_+$ (filled symbols) and $f_0$ (open symbols) is shown by solid and dashed lines. The vertical dot-dashed line represents $z$ corresponding $t = 0$, and the diamond is the value extrapolated to $t = 0$.

Figure 2 shows $z$-dependence of a quantity $B(t)f^{D\pi}_{\{+,0\}}(t)$ to be expanded in terms of $z$. Namely, $B(t)f^{D\pi}_{+(0)}(t) = (1 - t/M^2_{D^*_{(0)}})f^{D\pi}_{+(0)}(t)$ for the BCL parametrization with Eq. (3.3), whereas it is just the form factor for the polynomial expansion with Eq. (3.4).

For $f_+^{DP}$, we use the vector meson masses $M_{D^*_{(s)}}$ calculated at the simulation points, which are well below the threshold $t_+$. We observe that the $z$ dependence of $B(t)f_+^{DP}(t)$ is significantly reduced by switching from the polynomial expansion (3.4) to the BCL parametrization (3.3). This suggests that the vector meson dominance (VMD) hypothesis is a reasonably good approximation of $f_+^{DP}$, and we can expand the small deviation from the VMD in terms of small $z$. In this study, we test two BCL parametrization including the linear ($N_+ = 1$) and quadratic terms ($N_+ = 2$).

We have not yet calculated the scalar meson masses $M_{D^*_{(s)0}}$, and hence it is not clear whether there exist corresponding isolated poles below $t_+$ at simulated $M_\pi$'s. In this analysis, we employ the simple linear expansion (3.4) for $f_0^{DP}$. We also test the BCL parametrization ($N_0 = 1$) with the experimental value of $M_{D^*_{(s)0}}$ by assuming its mild dependence on $m_{ud}$.

We estimate the normalization $f_+^{DP}(0) = f_0^{DP}(0)$ from the simultaneous fit using the BCL parametrization with $N_+ = 1$ for $f_+^{DP}$ and the linear parametrization ($N_0 = 1$) for $f_0^{DP}$. We also test above-mentioned alternative forms to estimate the systematic uncertainty of the extrapolation to $t = 0$. As shown in Fig. 2, however, the $z$ dependence of our results is mild except the polynomial parametrization for $f_+^{DP}$. The systematic uncertainty is not large compared to the statistical accuracy.

## 4. Continuum and chiral extrapolation

In Fig. 3, we compare $f^{D\pi}_{\{+,0\}}$ at different pion masses (left panel) and at different lattice spac-

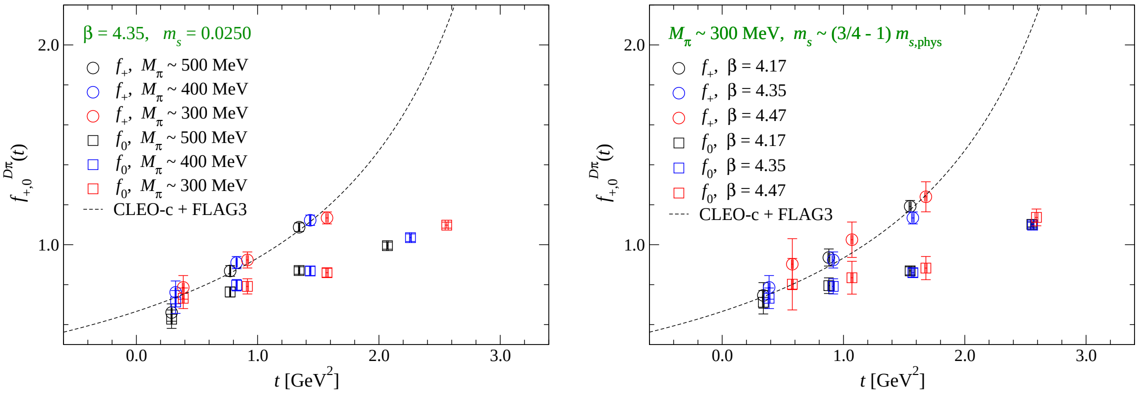


**Figure 3:** Comparison of $f^{D\pi}_{\{+,0\}}$ among different $M_\pi$'s (left panel) and different $a$'s (right panel). We plot data at $\beta = 4.35$ and $m_s = 0.0250$ in the left panel, whereas the right panel shows data at $M_\pi \sim 300$ MeV and larger $m_s$. We also plot the Becirevic-Kaidalov parametrization [11] of the CLEO-c data of ${f_+}^{D\pi}(t)$ [12] combined with an average of recent lattice estimates of ${f_+}^{D\pi}(0)$ [13].

ings (right panel). The reasonable consistency in both panels suggests a mild dependence on $M_\pi$ and $a$. We note that the decay constants $f_{D_{(s)}}$ also have small discretization errors with our choice of the lattice action and cutoffs [14].

In this preliminary analysis, therefore, we parametrize the $a$, $m_{ud}$ and $m_s$ dependences of $f_+^{DP}(0)$ by the following simple linear form

$$f_+^{DP}(0) \;=\; c^{DP} + c_a^{DP} a^2 + c_\pi^{DP} M_\pi^2 + c_{\eta_s}^{DP} M_{\eta_s}^2, \tag{4.1}$$

where $M_{\eta_s}^2 = 2M_K^2 - M_\pi^2$. This continuum and chiral extrapolation is plotted in Fig 4. We obtain $\chi^2/\mathrm{d.o.f} \sim 1.6-1.8$. All the coefficients $c^{DP}_{\{a,\pi,\eta_s\}}$ have $\gtrsim 75\,\%$ statistical error: namely, consistent with zero as expected from the good consistency in Fig. 3. This fit is therefore not sensitive to higher order corrections, and we estimate the systematic uncertainty from three fits in which one of the three coefficients $c^{DP}_{\{a,\pi,\eta_s\}}$ is set to zero. Our preliminary estimates

$$f_+^{D\pi}(0) = 0.644(49)(27), \qquad f_+^{DK}(0) = 0.701(46)(33). \tag{4.2}$$

are consistent with recent lattice averages $f_+^{D\pi}(0) = 0.666(29)$ and $f_+^{DK}(0) = 0.747(19)$ [13].

## 5. Summary

In this article, we report on our lattice calculation of the $D \to \pi$ and $D \to K$ semileptonic form factors. We employ the Möbius domain-wall quark action both for light and charm quarks, and simulate lattice cutoffs up to 4.5 GeV.

Our preliminary results for ${f_+}^{D\pi(K)}(0)$ have uncertainty of 8 (9) %. We expect significant improvement in the near future by increasing statistics ($N_{x_{4,\mathrm{src}}}$) on the finest lattice and extending our measurements to smaller $M_\pi \sim 230$ MeV.

We observe small discretization errors of the $D$ meson form factors with our simulation setup. It is therefore interesting to extend our study to the $B$ meson semileptonic decays, which are being precisely measured at SuperKEKB/Belle II and LHCb experiments.

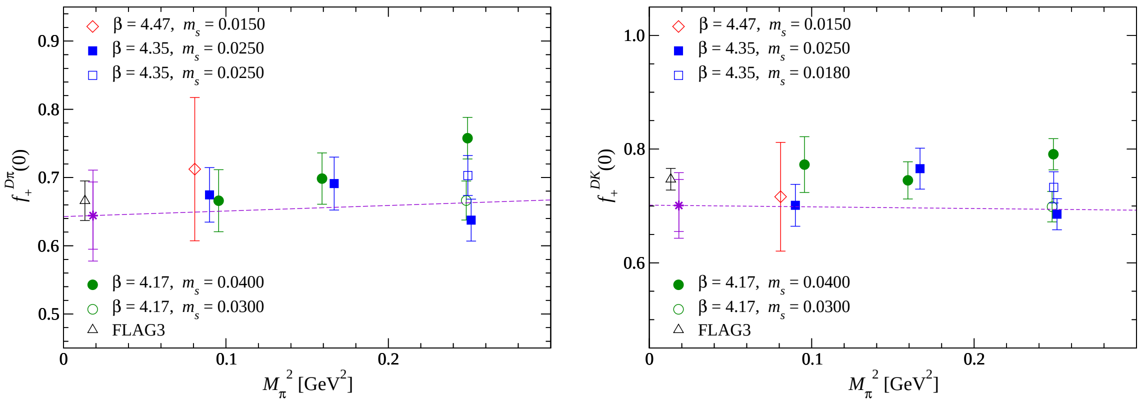


**Figure 4:** Continuum and chiral extrapolation of $f_+{}^{D\pi}(0)$ (left panel) and $f_+{}^{DK}(0)$ (right panel). Data at different $a$'s and $m_s$'s are plotted by different symbols as a function of $M_\pi^2$. The dashed lines show the fit line in the continuum limit and at the physical strange quark mass. The value extrapolated to the physical point is plotted by the stars. We also plot averages of recent lattice estimates [13] by the triangles.

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