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D-Meson Mixing in 2+1-Flavor Lattice QCD

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We present results for neutral D-meson mixing in 2+1-flavor lattice QCD. We compute the matrix elements for all five operators that contribute to D mixing at short distances, including those that only arise beyond the Standard Model. Our results have an uncertainty similar to those of the ETM collaboration (with 2 and with 2+1+1 flavors). This work shares many features with a recent publication on B mixing and with ongoing work on heavy-light decay constants from the Fermilab Lattice and MILC Collaborations.

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1. Introduction

These proceedings contain a status update of an ongoing calculation of D^0-$\bar{D}^0$ mixing matrix elements [1], similar to our published work on B^0-$\bar{B}^0$ mixing [2]. We present nearly final results for all five matrix elements, sufficient to describe D^0-$\bar{D}^0$ mixing not only in the Standard Model, but also in any high-energy extension that modifies only the local $\Delta C = 2$ interaction.

In the Standard Model, neutral-meson mixing is mediated by one-loop, GIM-suppressed processes, shown in Fig. 1. In extensions of the Standard Model, other particles could appear in the boxes; there could even be tree-level flavor-changing neutral currents. Mixing has been observed in all four neutral-meson systems—K^0, D^0, B^0, and B_s^0—but the pattern of internal quark masses and CKM factors explains why the phenomenology differs so greatly from one system to another.

Because the W bosons and b quarks have masses well above the QCD scale, mixing can be re-expressed as stemming both from a local $\Delta C = 2$ interaction and two $\Delta C = 1$ interactions separated by a distance of order $1/\Lambda_{\mathrm{QCD}}$. From degenerate perturbation theory, the off-diagonal term in the mass-width matrix is [3]

$$M_{12} - \frac{i}{2}\Gamma_{12} \propto \langle D^0|\mathscr{L}^{\Delta C=2}|\bar{D}^0\rangle + \sum_n \frac{\langle D^0|\mathscr{L}^{\Delta C=1}|n\rangle\langle n|\mathscr{L}^{\Delta C=1}|\bar{D}^0\rangle}{M_D - E_n + i0^+}. \tag{1.1}$$

The second term is very difficult to estimate. For D^0 mesons it is also not negligible, unlike for B^0 and B_s^0, where t, c, and u quarks appear in the box. (For kaons, the second term is important but not dominant.) One can relate the measured mass and width differences, ΔM and $\Delta\Gamma$, to $|M_{12}|$, $|\Gamma_{12}|$, and the relative phase $\arg(\Gamma_{12}/M_{12})$ [4]. In some extensions of the Standard Model, only the first term and, thus, M_{12} is altered [5].

The effective Lagrangian $\mathscr{L}^{\Delta C=2}$ (at energies below the b-quark mass) is built out of the following operators (and their Wilson coefficients) [6, 7, 8]:

$$\mathscr{O}_1 = \bar{c}\gamma^\mu Lu\,\bar{c}\gamma_\mu Lu, \qquad \tilde{\mathscr{O}}_1 = \bar{c}\gamma^\mu Ru\,\bar{c}\gamma_\mu Ru, \tag{1.2}$$

$$\mathscr{O}_2 = \bar{c}Lu\,\bar{c}Lu, \qquad \tilde{\mathscr{O}}_2 = \bar{c}Ru\,\bar{c}Ru, \tag{1.3}$$

$$\mathscr{O}_3 = \bar{c}^\alpha Lu^\beta\,\bar{c}^\beta Lu^\alpha, \qquad \tilde{\mathscr{O}}_3 = \bar{c}^\alpha Ru^\beta\,\bar{c}^\beta Ru^\alpha, \tag{1.4}$$

$$\mathscr{O}_4 = \bar{c}Lu\,\bar{c}Ru, \tag{1.5}$$

$$\mathscr{O}_5 = \bar{c}^\alpha Lu^\beta\,\bar{c}^\beta Ru^\alpha, \tag{1.6}$$

where L (R) denotes a left-(right-)handed projector on the Dirac indices, and α and β are color indices. By parity conservation in QCD, $\langle D^0|\tilde{\mathscr{O}}_i|\bar{D}^0\rangle = \langle D^0|\mathscr{O}_i|\bar{D}^0\rangle$, $i = 1, 2, 3$. Thus, the five matrix elements $\langle D^0|\mathscr{O}_i|\bar{D}^0\rangle$, $i = 1,\ldots 5$, suffice to describe the short-distance part of all $\Delta C = 2$ processes, whether their origin is W-b box or something else. In these proceedings, we report on a calculation of all five matrix elements using lattice QCD with 2+1 flavors of sea quarks.

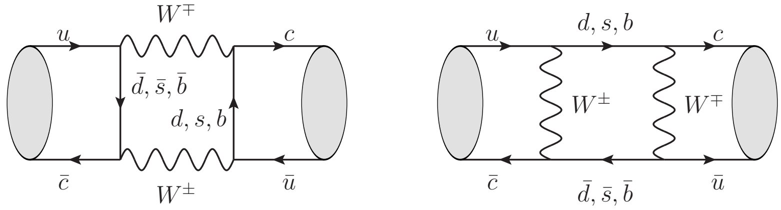

Figure 1: Box diagrams mediating D^0-$\bar{D}^0$ mixing in the Standard Model.

2. Lattice-QCD calculation

Our *D*-meson calculations have much in common with our published *B*-meson work [2]. We use the same ensembles (generated by the MILC collaboration) with 2+1 flavors of sea quark [9]. The light quarks (valence and sea) are based on the staggered asqtad action; the heavy c (or b) quark on the Fermilab interpretation of the clover action. The lattice spacings for the ensembles satisfy $a \approx 0.045$ fm, ≈ 0.06 fm, ≈ 0.09 fm, and ≈ 0.12 fm. The sea-quark masses yield pions with

$$177\ \text{MeV} \lesssim M_\pi \lesssim 555\ \text{MeV}, \tag{2.1}$$

$$257\ \text{MeV} \lesssim M_\pi^{\text{rms}} \lesssim 670\ \text{MeV}, \tag{2.2}$$

The ensembles contain 600–2200 gauge-field configurations, and we use 4 or 8 sources/config.

To carry out the chiral-continuum extrapolation, we take into account the subtle way in which spin emerges for staggered fermions with staggered-Wilson four-fermion lattice operators. The three-point correlation function, it turns out, contains contributions not only from the continuum-limit operator of desired spin, but also some of the wrong spin [10]. Because only the five operators in Eqs. (1.2)–(1.6) can arise, we automatically have the information needed to disentangle this effect. We use the one-loop chiral-perturbation-theory formulas of Ref. [10] to remove the wrong-spin contribution in the course of our chiral-continuum fit.

The operators in Eqs. (1.2)–(1.6) require renormalization for any ultraviolet regulator. We carry out the renormalization of the lattice operators corresponding to Eqs. (1.2)–(1.6) together with matching to $\overline{\text{MS}}$ schemes in continuum QCD. We use a mostly nonperturbative method to handle the largest lattice-to-continuum matching corrections [11, 12], supplemented with a one-loop calculation of the remaining, small renormalization parts [13, 2]. We choose the renormalization scale for *D*-meson matrix elements to be 3 GeV, while we chose m_b for $B_{(s)}$ mesons.

The main difference between our work on D vs. $B_{(s)}$ mesons is the analysis of the correlation functions. The signal-to-noise ratio is much better for *D*-meson correlators. For the two-point correlators, the optimal time range $t_{\text{min}} \lesssim t \lesssim t_{\text{max}}$ differs: $t_{\text{min}} \approx 0.7\,(0.2)$ fm, $t_{\text{max}} \approx 3.0\,(2.4)$ fm for D ($B_{(s)}$) mesons. The difference for the three-point correlators is more striking. We fix the four-quark operators at $t = 0$ and the meson creation (annihilation) operator at time $t_x < 0$ ($t_y > 0$). As

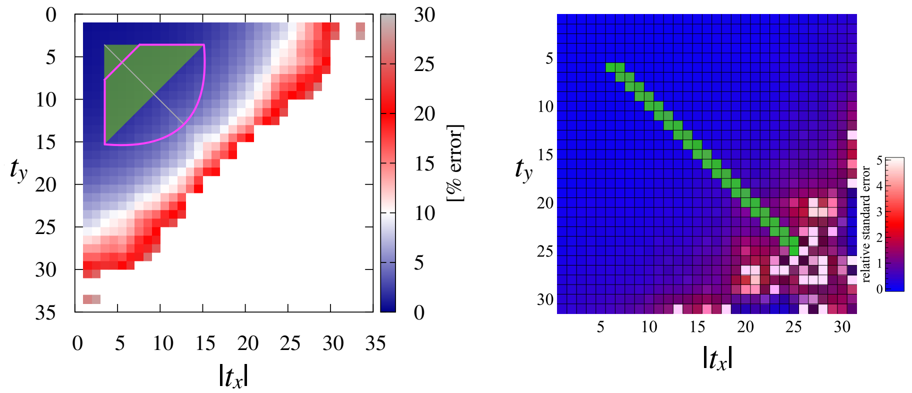

Figure 2: Fitting ranges for three-point correlators: triangular (green) and-or fan-shaped (magenta) regions for B mixing (left); two-strip diagonal region (green) for D mixing (right). Background color shows the signal-to-noise ratio from good (blue) to bad (red).

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shown in Fig. 2, we use a triangular and-or fan-shaped region in the $|t_x|$-t_y plane for $B_{(s)}$ mesons [2], while we use a long diagonal of width 2 for D mixing, $\{|t_x| = t_y\} \cup \{|t_x| = t_y + 1\}$. The long diagonal makes it easier to disentangle the lowest-lying state, if the signal persists that far. A simultaneous fit to two- and three-point functions is used to extract the matrix elements $\langle O_i \rangle \equiv \langle D^0 | O_i | \bar{D}^0 \rangle$.

3. Chiral-continuum extrapolation

To carry out the chiral-continuum extrapolation, we develop a fit function based on chiral perturbation theory (χPT), Symanzik effective field theory, and heavy-quark effective theory (HQET). It takes the form

$$F_i = F_i^{\text{logs}} + F_i^{\text{analytic}} + F_i^{\text{HQ disc}} + F_i^{\alpha_s a^2 \text{ gen}} + F_i^{\text{renorm}} + F_i^{\kappa}, \tag{3.1}$$

where F_i^{logs} denotes the next-to-leading order description from heavy-meson rooted staggered χPT, with nonanalytic terms including those that disentangle the wrong-spin contributions [10]; F_i^{analytic} is a polynomial of various terms that arise in χPT at next-to-leading or higher order; $F_i^{\text{HQ disc}}$ describes heavy-quark discretization effects using HQET as a theory of cutoff effects [12]; $F_i^{\alpha_s a^2 \text{ gen}}$ parametrizes generic cutoff effects of light quarks and gluons, à la Symanzik; and F_i^{renorm} allows the fit to be sensitive to higher orders in α_s for matching and renormalization. Finally, F_i^{κ} incorporates a correction for tuning the charm-quark hopping parameter κ, based on extra runs at $a \approx 0.12$ fm.

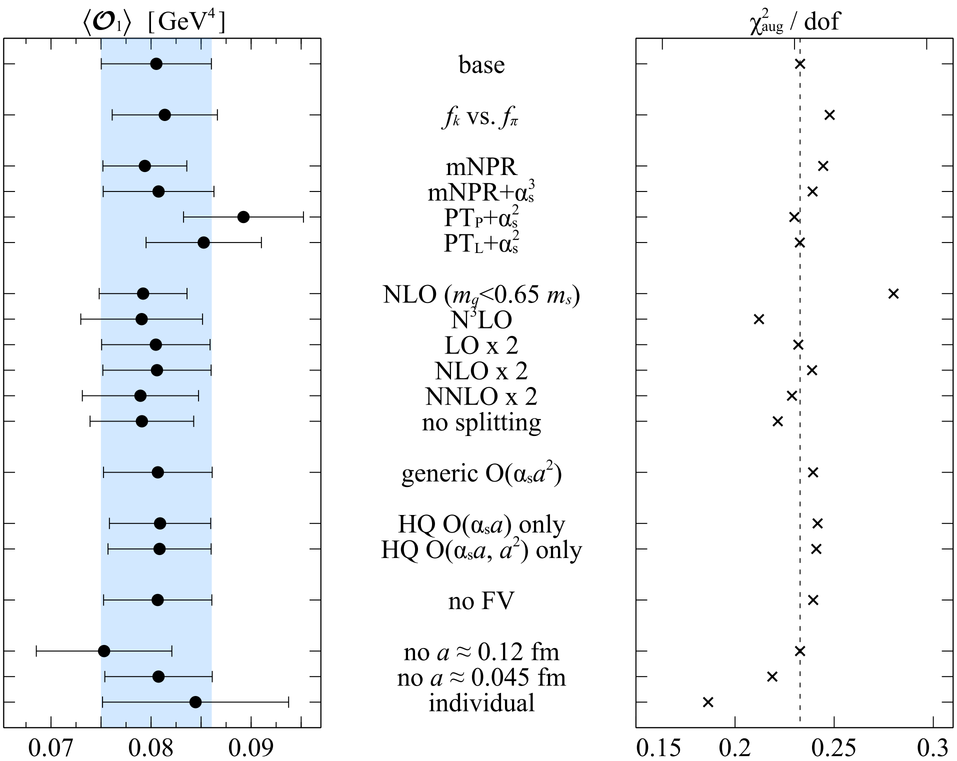

Figure 3: Stability of the chiral-continuum extrapolation for several variants of the fit function F_i: $\langle \mathscr{O}_1 \rangle$ (left), minimized χ^2_{aug}/dof (right). Stability plots for the other $\langle \mathscr{O}_i \rangle$ look similar.

BBGLN [16]	$\langle \mathscr{O}_i \rangle / M_D$ (GeV^3)	$f^2_{B_q} B_{B_q}$ (GeV^2)	
		$q = d$	$q = s$
$\mathscr{O}_1$	0.0432(29)(9)	0.0342(29)(7)	0.0498(30)(10)
$\mathscr{O}_2$	−0.0833(38)(17)	0.0303(27)(6)	0.0449(29)(9)
$\mathscr{O}_3$	0.0248(16)(5)	0.0399(77)(8)	0.0571(77)(11)
$\mathscr{O}_4$	0.1469(69)(30)	0.0390(28)(8)	0.0534(30)(11)
$\mathscr{O}_5$	0.0554(38)(11)	0.0361(35)(7)	0.0493(36)(10)
μ	3 GeV	m_b	m_b

Table 1: Results for D [this work] and B [2] mixing in the renormalization scheme of Ref. [16].

Both the renormalization and wrong-spin effects mix operators 1, 2, and 3 with each other, and also 4 and 5 with each other. It is thus natural to fit the matrix elements in each sector simultaneously. Some ingredients in F_i^{logs} are common for all i, such as masses, f_π, light-meson χPT constants [14], and the D^*-D-π coupling. We introduce these external inputs with Gaussian priors, for example $g_{D^*D\pi} = 0.53 \pm 0.8$. Because of these common ingredients, we choose to fit all five matrix elements simultaneously. We form a χ^2 function from $F_i - \langle \bar{O}_i \rangle$ and the sample covariance matrix of the $\langle \bar{O}_i \rangle$, where $\bar{O}_i$ denotes the renormalized lattice operators (which differ from the continuum $\mathscr{O}_i$ by discretization effects and higher-order matching effects). We then augment this χ^2 with Gaussian priors for the fit parameters implied in Eq. (3.1), choosing a central value of 0 and width of ± 1 in natural units for χPT and HQET [15] and minimize the resulting χ^2_{aug}. We reconstitute the fit function at zero lattice spacing and physical quark masses to obtain our estimate of the $\langle \mathscr{O}_i \rangle$ and their uncertainty.

We have 510 data points for $\langle \bar{O}_i \rangle$, ranging over the ensembles, valence-quark masses, and five operators. In our base version of F_i, there are 127 parameters. To check whether the final results are robust, we repeat the procedure with several variants of F_i, as illustrated in Fig. 3. We express the χPT with f_K instead of f_π; we choose different orders of α_s in F_i^{renorm} and even replace the mostly nonperturbative (mNPR) matching with a fully perturbative (PT) one; we check various alternatives for the polynomial F_i^{analytic} (NLO, NNLO, N^{3}LO); we check what happens when the χPT prior widths in F_i^{analytic} are doubled; we check alternatives for the heavy-quark discretization errors; we substitute infinite-volume one-loop integrals for the finite-volume sums in one-loop χPT; we omit the data from the coarsest or finest lattice spacing; we fit each matrix element separately, thereby ignoring data constraints on wrong-spin contributions. As one can see from Fig. 3, the results for the $\langle \mathscr{O}_1 \rangle$ are very stable, so we take these variations in the fit as cross checks. The same applies to the other $\langle \mathscr{O}_i \rangle$. The largest deviations are $\sim 1\sigma$ and come from fits that omit important information. Our nearly final results for D mixing are given in Table 1, together with published results for $B_{(s)}$ mixing from Ref. [2]. These matrix elements (as noted above) depend on the renormalization scheme; the tabulated results are in the $\overline{\text{MS}}$ scheme with naive (fully commuting) γ^5 and the evanescent-operator basis used by Beneke, Buchalla, Greub, Lenz, and Nierste (BBGLN) [16].

The MILC asqtad ensembles omit the charmed-quark sea. As in Ref. [2], we assign an additional 2% uncertainty to account for this omission. This uncertainty is given separately, in the second set of parentheses, in Table 1.

4. Outlook

Our results agree well with and have similar uncertainty as previous lattice-QCD results from the ETM collaboration, with 2 [17] or 2+1+1 [18] flavors in the sea. The comparison of these results tests not only the flavor-dependence of the matrix elements but also the sensitivity to lattice fermion formulation: ETM employs twisted-mass Wilson fermions, while we employ staggered fermions. All these calculations use several lattice spacings and take the continuum limit. References [17, 18] report the so-called "bag factors" often used in phenomenology [7]; a detailed comparison would require choices of quark masses and decay constants (and their uncertainties) that would obscure the error budget of one or the other set of results. We have a set of calculations underway [19] to compute the D- and $B_{(s)}$-meson decay constants on the same ensembles and will report the bag factors then.

Estimates of the contribution to M_{12} of the second term in Eq. (1.1) range over $(10^{-3}$–$10^{-2})\Gamma$ [20], where Γ is the total width of the neutral D meson. It turns out, however, that all Standard-Model phases appearing in Eq. (1.1) are small. Thus, in a TeV-scale model that might produce a large phase in M_{12}, the results for the $\langle \mathscr{O}_i \rangle$ can be used to constrain the model's parameters. Furthermore, until a method is developed to tame the second term in Eq. (1.1), the accuracy achieved in this work and Refs. [17, 18] should suffice for this purpose.

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