PROCEEDINGS OF SCIENCE

Further Study of BRST-Symmetry Breaking on the Lattice

Attilio Cucchieri[*†]
Instituto de Física de São Carlos, Universidade de São Paulo, Caixa Postal 369, 13560-970 São Carlos, SP, Brazil
E-mail: attilio@ifsc.usp.br

Tereza Mendes
Instituto de Física de São Carlos, Universidade de São Paulo, Caixa Postal 369, 13560-970 São Carlos, SP, Brazil
E-mail: mendes@ifsc.usp.br

We evaluate the so-called Bose-ghost propagator $Q(p^2)$ for SU(2) gauge theory in minimal Landau gauge, considering lattice volumes up to 120^4 and physical lattice extents up to $13.5\,fm$. In particular, we investigate discretization effects, as well as the infinite-volume and continuum limits. We recall that a nonzero value for this quantity provides direct evidence of BRST-symmetry breaking, related to the restriction of the functional measure to the first Gribov region. Our results show that the prediction (from cluster decomposition) for $Q(p^2)$ in terms of gluon and ghost propagators is better satisfied as the continuum limit is approached.

34th annual International Symposium on Lattice Field Theory
24-30 July 2016
University of Southampton, UK

PoS(LATTICE2016)394

*Speaker.

†We acknowledge partial support from CNPq. We would like to acknowledge computing time provided on the Blue Gene/P supercomputer supported by the Research Computing Support Group (Rice University) and Laboratório de Computação Científica Avançada (Universidade de São Paulo).

1. BRST-Symmetry Breaking

The minimal Landau gauge in Yang-Mills theories [1] is obtained by restricting the functional integral to the set of transverse gauge configurations for which the Faddeev-Popov (FP) matrix $\mathcal{M}$ is non-negative, the so-called first Gribov region Ω. On the lattice, this gauge condition is imposed by considering a minimization procedure. On the contrary, in the Gribov-Zwanziger (GZ) approach in the continuum [2], this restriction is forced by adding a nonlocal horizon-function term $\gamma^4 S_{\rm h}$ to the usual (Landau-gauge) action. The resulting (nonlocal) GZ action may be localized by introducing the auxiliary fields $\phi_\mu^{ab}(x)$ and $\omega_\nu^{cd}(y)$, yielding $S_{\rm GZ} = S_{\rm YM} + S_{\rm gf} + S_{\rm aux} + S_\gamma$. Here, $S_{\rm YM}$ is the usual four-dimensional Yang-Mills action, $S_{\rm gf}$ is the covariant-gauge-fixing term, $S_{\rm aux}$ is defined as

$$S_{\rm aux} = \int {\rm d}^4 x \left[\overline{\phi}_\mu^{ac} \, \partial_\nu \left(D_\nu^{ab} \phi_\mu^{bc} \right) - \overline{\omega}_\mu^{ac} \, \partial_\nu \left(D_\nu^{ab} \omega_\mu^{bc} \right) - g_0 \left(\partial_\nu \overline{\omega}_\mu^{ac} \right) f^{abd} \, D_\nu^{be} \eta^e \phi_\mu^{dc} \right]$$

and is necessary to localize the horizon function, and S_γ, given by

$$S_\gamma = \int {\rm d}^4 x \left[\gamma^2 D_\mu^{ab} \left(\phi_\mu^{ab} + \overline{\phi}_\mu^{ab} \right) - 4 \left(N_c^2 - 1 \right) \gamma^4 \right] ,$$

allows one to fix the γ parameter through the so-called horizon condition. Also, one can define [3] for these fields a nilpotent BRST transformation s that is a simple extension of the usual (perturbative) BRST transformation leaving $S_{\rm YM} + S_{\rm gf}$ invariant. However, in the GZ case, the BRST symmetry s is broken by terms proportional to a power of the Gribov parameter γ. Since a nonzero value of γ is related to the restriction of the functional integration to Ω, it is somewhat natural to expect a breaking of the (extended) BRST symmetry s, as a direct consequence of the nonperturbative gauge-fixing.[1] More precisely —as nicely explained in Ref. [7]— an infinitesimal gauge transformation is formally equivalent to a (perturbative) BRST transformation. Since the region Ω is free of infinitesimal gauge copies, applying s to a configuration in Ω should result in a configuration outside Ω. The breaking of the BRST symmetry in minimal Landau gauge is then inevitable, since the functional integration is limited to the region Ω. This interpretation is supported by the introduction [6] of a nilpotent nonperturbative BRST transformation s_γ that leaves the local GZ action invariant. The new symmetry is a simple modification of the extended BRST transformation s, by adding (for some of the fields) a nonlocal term proportional to a power of the Gribov parameter γ.

2. The Bose-Ghost Propagator

As implied above, the Gribov parameter γ is not introduced explicitly on the lattice, since in this case the restriction of gauge-configuration space to the region Ω is achieved by numerical minimization. Nevertheless, the breaking of the BRST symmetry s induced by the GZ action may be investigated by the lattice computation of suitable observables, such as the so-called Bose-ghost propagator

$$Q_{\mu\nu}^{abcd}(x,y) = \langle \, s(\, \phi_\mu^{ab}(x) \, \overline{\omega}_\nu^{cd}(y)) \, \rangle = \langle \, \omega_\mu^{ab}(x) \, \overline{\omega}_\nu^{cd}(y) + \phi_\mu^{ab}(x) \, \overline{\phi}_\nu^{cd}(y) \, \rangle \, .$$

[1] This issue has been investigated in several works (see e.g. [4, 5, 6, 7] and references therein).

PoS(LATTICE2016)394

Since this quantity is BRST-exact with respect to the (extended) BRST transformation s, it should be zero for a BRST-invariant theory, but it does not necessarily vanish if the symmetry s is broken. On the lattice, however, one does not have direct access to the auxiliary fields $(\bar{\phi}_\mu^{ac}, \phi_\mu^{ac})$ and $(\bar{\omega}_\mu^{ac}, \omega_\mu^{ac})$. Nevertheless, these fields enter the continuum action at most quadratically and they can be integrated out exactly, giving for the Bose-ghost propagator an expression that is suitable for lattice simulations. This yields

$$Q_{\mu\nu}^{abcd}(x-y) = \gamma^4 \left\langle R_\mu^{ab}(x) R_\nu^{cd}(y) \right\rangle , \tag{2.1}$$

where

$$R_\mu^{ac}(x) = \int \mathrm{d}^4 z \, (\mathcal{M}^{-1})^{ae}(x,z) \, B_\mu^{ec}(z) \tag{2.2}$$

and $B_\mu^{ec}(z)$ is given by the covariant derivative $D_\mu^{ec}(z)$. One can also note that, at the classical level, the total derivatives $\partial_\mu(\phi_\mu^{aa} + \bar{\phi}_\mu^{aa})$ in the action S_γ can be neglected [3, 4]. In this case the expression for $B_\mu^{ec}(z)$ simplifies to

$$B_\mu^{ec}(z) = g_0 \, f^{ebc} A_\mu^b(z) , \tag{2.3}$$

as in Ref. [4]. Let us stress that, in both cases, the expression for $Q_{\mu\nu}^{abcd}(x-y)$ in Eq. (2.1) depends only on the gauge field $A_\mu^b(z)$ and can be evaluated on the lattice.

3. Numerical Simulations and Results

The first numerical evaluation of the Bose-ghost propagator in minimal Landau gauge was presented —for the SU(2) case in four space-time dimensions— in Refs. [8, 9]. In particular, we evaluated the scalar function $Q(k^2)$ defined [for the SU(N_c) gauge group] through the relation

$$Q^{ac}(k) \equiv Q_{\mu\mu}^{abcb}(k) \equiv \delta^{ac} N_c \, P_{\mu\mu}(k) \, Q(k^2) ,$$

where $P_{\mu\nu}(k)$ is the usual transverse projector and k is the wave vector with components $k_\mu = 0, 1, \ldots, N-1$, for a lattice of N points per directions. The lattice momentum $p^2(k)$ is obtained using the improved definition (see Ref. [8]). This calculation has been recently extended in Ref. [10], where we have investigated the approach to the infinite-volume and continuum limits by considering four different values of the lattice coupling β and different lattice volumes $V = N^4$, with physical volumes ranging from about $(3.366\,fm)^4$ to $(13.462\,fm)^4$. We find no significant finite-volume effects in the data. As for discretization effects, we observe small such effects for the coarser lattices, especially in the IR region. We also tested three different discretizations[2] for the sources $B_\mu^{bc}(x)$, used in the inversion of the FP matrix $\mathcal{M}$, and find that the data are fairly independent of the chosen lattice discretization of these sources.

Our results concerning the BRST symmetry-breaking and the form of the Bose-ghost propagator are similar to the previous analysis [8, 9], i.e. we find a $1/p^6$ behavior at large momenta and a double-pole singularity at small momenta,[3] in agreement with the one-loop analysis carried out

[2] See Eqs. (30), (31) and (32) in Ref. [10].

[3] As proven in Ref. [10], the Fourier transform of the quantity $R_\mu^{ac}(x)$, defined in Eq. (2.2) above, is trivially equal to 0 at zero momentum, i.e. $\sum_x R_\mu^{ac}(x) = 0$. Thus, one needs to consider sufficiently large lattice volumes, in order to have the IR behavior of the Bose-ghost propagator under control.

PoS(LATTICE2016)394

in Ref. [11]. These behaviors can be clearly seen in Fig. 1, where we fit the data for the Bose-ghost propagator $Q(p^2)$ using the fitting function

$$f(p^2) = \frac{c}{p^4} \frac{p^2 + s}{p^4 + u^2 p^2 + t^2} , \tag{3.1}$$

which can be related [see Eq. (3.2) below] to an IR-free FP ghost propagator $G(p^2) \sim 1/p^2$ in combination with a massive gluon propagator $D(p^2)$.

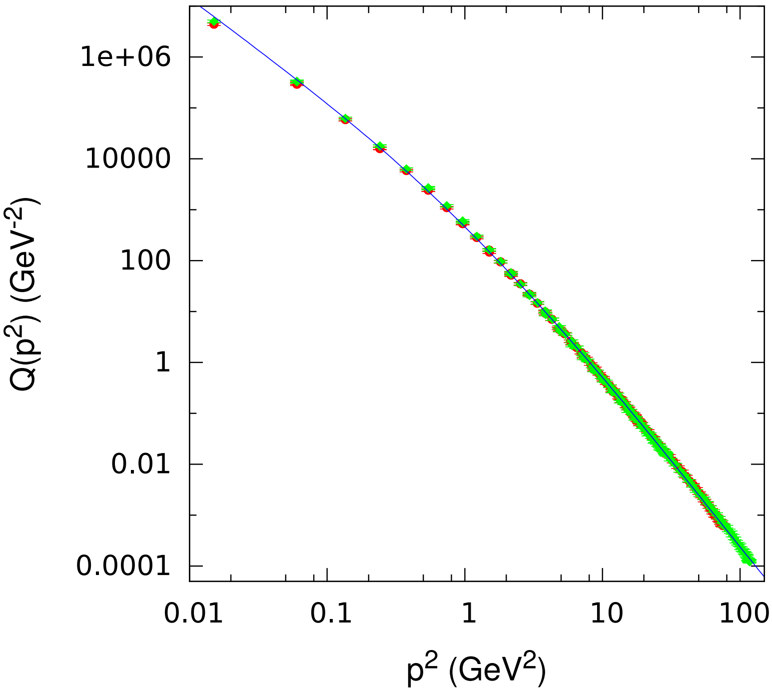

Figure 1: The Bose-ghost propagator $Q(p^2)$ as a function of the (improved) lattice momentum squared p^2. Here we used as sources $B^{ec}_{\mu}(z)$ the formula reported in Eq. (32) of Ref. [10]. We plot data for $\beta_2 \approx 2.44$, $V = 96^4$ (●) and $\beta_3 \approx 2.51$, $V = 120^4$ (♦), after applying a matching procedure [12] to the former set of data. We also plot, for $V = 120^4$, a fit using the fitting function (3.1). Note the logarithmic scale on both axes.

In Figs. 2 and 3 we compare the Bose-ghost propagator $Q(p^2)$ to the product $g_0^2 G^2(p^2) D(p^2)$, where g_0 is the bare coupling constant. To this end, the data of the Bose-ghost propagator have been rescaled in order to agree with the data of the product $g_0^2 G^2(p^2) D(p^2)$ at the largest momentum.[4] This comparison is based on the result

$$Q(p^2) \sim g_0^2 G^2(p^2) D(p^2) , \tag{3.2}$$

obtained in Ref. [4] using a cluster decomposition. Even though there is a clear discrepancy between these two quantities we find that this discordance seems to decrease when the continuum limit is considered.

[4]This is equivalent to imposing a given renormalization condition for the propagators at the largest momentum.

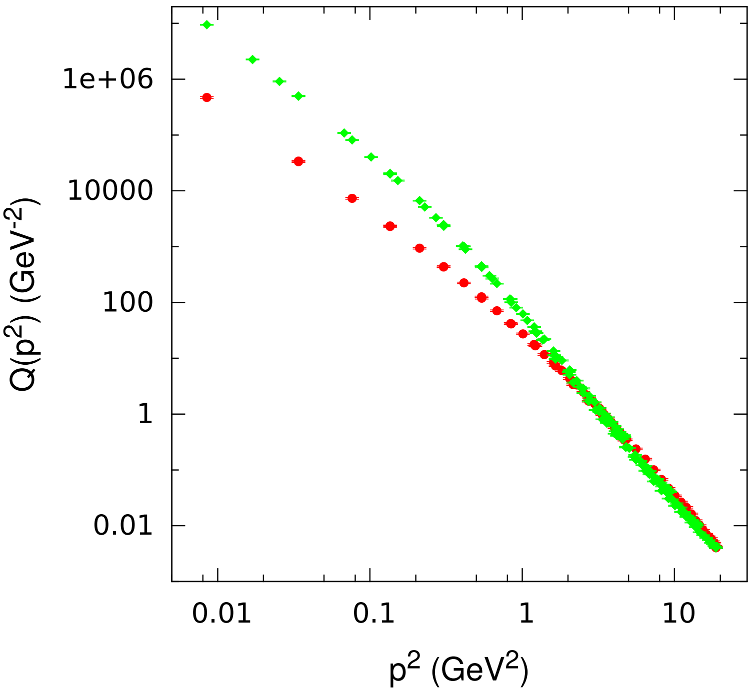

Figure 2: The Bose-ghost propagator $Q(p^2)$ (●) and the product $g_0^2\, G^2(p^2)\, D(p^2)$ (♦) as a function of the (improved) lattice momentum squared p^2 for the lattice volume $V = 64^4$ at β_0. Here we used as sources $B_\mu^{ec}(z)$ the formula reported in Eq. (32) of Ref. [10]. Note the logarithmic scale on both axes.

References

[1] V. N. Gribov, Nucl. Phys. B **139**, 1 (1978).

[2] D. Zwanziger, Nucl. Phys. **B378**, 525 (1992).

[3] N. Vandersickel and D. Zwanziger, Phys. Rept. **520**, 175 (2012).

[4] D. Zwanziger, arXiv:0904.2380 [hep-th];

[5] D. Zwanziger, Nucl. Phys. **B412**, 657 (1994); N. Maggiore and M. Schaden, Phys. Rev. **D50**, 6616 (1994); M. Ghiotti, A. C. Kalloniatis and A. G. Williams, Phys. Lett. **B628**, 176 (2005); L. von Smekal, M. Ghiotti and A. G. Williams, Phys. Rev. **D78**, 085016 (2008); D. Dudal, J. A. Gracey, S. P. Sorella, N. Vandersickel and H. Verschelde, Phys. Rev. **D78**, 125012 (2008); L. Baulieu and S. P. Sorella, Phys. Lett. **B671**, 481 (2009); D. Dudal, S. P. Sorella, N. Vandersickel and H. Verschelde, Phys. Rev. **D79**, 121701 (2009); S. P. Sorella, Phys. Rev. **D80**, 025013 (2009); K. I. Kondo, arXiv:0905.1899 [hep-th]; S. P. Sorella *et al.*, AIP Conf. Proc. **1361**, 272 (2011); M. A. L. Capri *et al.*, Phys. Rev. **D82**, 105019 (2010); Phys. Rev. **D83**, 105001 (2011); D. Dudal and N. Vandersickel, Phys. Lett. **B700**, 369 (2011); P. Lavrov, O. Lechtenfeld and A. Reshetnyak, JHEP **1110**, 043 (2011); P. M. Lavrov, O. V. Radchenko and A. A. Reshetnyak, Mod. Phys. Lett. **A27**, 1250067 (2012). A. Weber, Phys. Rev. **D85**, 125005 (2012); J. Phys. Conf. Ser. **378**, 012042 (2012); A. Maas, Mod. Phys. Lett. **A27**, 1250222 (2012); D. Dudal and S. P. Sorella, Phys. Rev. **D86**, 045005 (2012); M. A. L. Capri *et al.*, Annals Phys. **339**, 344 (2013); V. Mader, M. Schaden, D. Zwanziger and

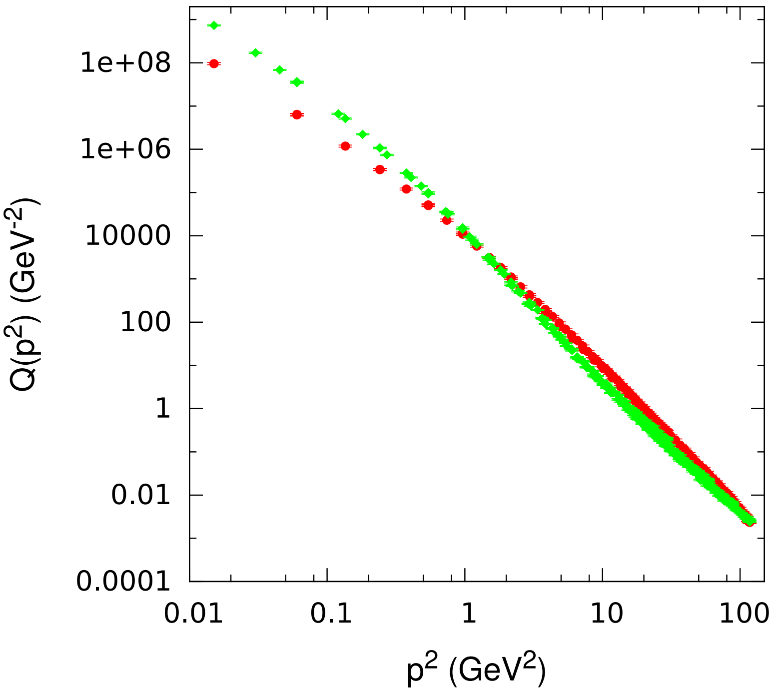

Figure 3: The Bose-ghost propagator $Q(p^2)$ (●) and the product $g_0^2 G^2(p^2) D(p^2)$ (♦) as a function of the (improved) lattice momentum squared p^2 for the lattice volume $V = 120^4$ at $\beta_3 \approx 2.51$. Here we used as sources $B_\mu^{ec}(z)$ the formula reported in Eq. (32) of Ref. [10]. Note the logarithmic scale on both axes.

R. Alkofer, Eur. Phys. J. **C74**, 2881 (2014); A. Reshetnyak, Int. J. Mod. Phys. **A29**, 1450184 (2014); arXiv:1412.8428 [hep-th]; N. Brambilla *et al.*, Eur. Phys. J. **C74**, no. 10, 2981 (2014); P. Y. Moshin and A. A. Reshetnyak, Nucl. Phys. **B888**, 92 (2014); arXiv:1607.07253 [hep-th]; M. A. L. Capri, M. S. Guimaraes, I. F. Justo, L. F. Palhares and S. P. Sorella, Phys. Rev. **D90**, 085010 (2014); M. Schaden and D. Zwanziger, Phys. Rev. **D92**, no. 2, 025001 (2015); M. A. L. Capri *et al.*, Phys. Rev. **D93**, no. 6, 065019 (2016); Phys. Rev. D **94**, no. 2, 025035 (2016).

[6] M. A. L. Capri *et al.*, Phys. Rev. **D92**, no. 4, 045039 (2015).

[7] A. D. Pereira, arXiv:1607.00365 [hep-th].

[8] A. Cucchieri, D. Dudal, T. Mendes and N. Vandersickel, Phys. Rev. **D90**, 051501 (2014).

[9] PoS **LATTICE2014**, 347 (2014).

[10] A. Cucchieri and T. Mendes, Phys. Rev. **D94**, no. 1, 014505 (2016).

[11] J. A. Gracey, JHEP **1002**, 009 (2010).

[12] D. B. Leinweber *et al.* [UKQCD Collaboration], Phys. Rev. **D60**, 094507 (1999) [Erratum-ibid. **D61**, 079901 (2000)]; A. Cucchieri, T. Mendes and A. R. Taurines, Phys. Rev. **D67**, 091502 (2003).