PROCEEDINGS OF SCIENCE

Implications of the top (and Higgs) mass for vacuum stability

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The discovery of the Higgs boson by the LHC and the measurement of its mass at around 125 GeV, taken together with the absence of signals of physics beyond the standard model, make it possible that we might live in a metastable electroweak vacuum. Intriguingly, we seem to be very close to the boundary of stability and this near-criticality makes our vacuum extremely long-lived. In this talk I describe the state-of-the-art calculation leading to these results, explaining what are the ingredients and assumptions that enter in it, with special emphasis on the role of the top mass. I also discuss possible implications of this metastability for physics beyond the standard model.

8th International Workshop on Top Quark Physics, TOP2015
14-18 September, 2015
Ischia, Italy

PoS(TOP2015)043

1. The Metastability of the Electroweak Vacuum and Near-criticality

So far, LHC has taught us that a light Higgs boson, with $M_h \simeq 125\,\mathrm{GeV}$ [1] exists and has SM-like couplings (with room for deviations). No BSM physics has showed up, with bounds on the scale of new physics, $\Lambda > \mathscr{O}(\mathrm{TeV})$. For those willing to hold on to the naturalness paradigm, the EW hierarchy problem implies new physics around the corner. However, if naturalness misled us, the SM might be valid up to very high energy scales, possibly up to $\Lambda \sim M_P$. Fig. 1-left shows the SM couplings extrapolated to high scales [2]. The three gauge couplings almost unify at $\mu \sim 10^{14}$ GeV. The top Yukawa coupling decreases due to α_s effects. The zoomed-in right plot in Fig. 1 shows λ becoming negative at $\mu \sim 10^{10}$ GeV.

The steep slope of $\lambda(\mu)$ is due to one-loop top corrections to $\beta_\lambda = d\lambda/d\log\mu = -6y_t^4/(16\pi^2) + \ldots$ where y_t is the top Yukawa coupling. The fourth power of y_t explains the sensitivity of the running of λ to the top quark mass M_t (see 3σ gray band in Fig. 1-right). The smaller sensitivity to α_s (through its indirect effect on the running $y_t(\mu)$) is shown by the thinner 3σ red band in the same plot. The thinnest blue band corresponds to 3σ changes of M_h. Note how λ flattens out after becoming negative: in that range of large scales gauge couplings are comparable in size to y_t (Fig. 1-left) and their positive contribution to β_λ balances the top one, leading to $\beta_\lambda \simeq 0$.

The trouble with $\lambda < 0$ is the following: at high field values the quartic term dominates the potential $V(h)$. A good approximation to $V(h)$ requires λ to be evaluated at a renormalization scale $\mu \sim h$: $V(h \gg M_t) \simeq (1/4)\lambda(\mu = h)h^4$. For $\lambda(h) < 0$ the potential is deeper than the EW vacuum, which is no longer the true vacuum. We should then worry about the lifetime of our vacuum against decay through quantum tunneling. The unstable EW vacuum can decay by nucleation of

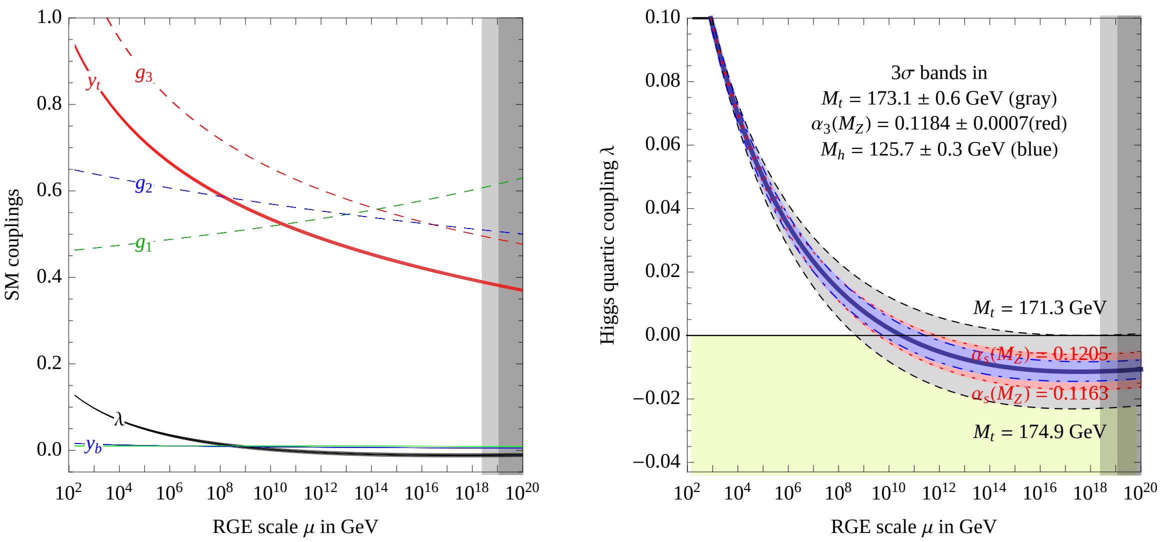

Figure 1: *Left: UV extrapolation of SM couplings. Right: Evolution of the Higgs quartic coupling with 3σ uncertainties in M_t, α_s and M_h shown by the colored intervals as indicated [2].*

bubbles that probe the instability region and are large enough to grow, eating the whole of space. The probability p of such decay is given by the decay-rate per unit time and unit volume [7] $\sim h_t^4 \exp(-S_4)$, with h_t the Higgs field value beyond the region of instability to which the tunneling occurs (the only relevant mass/energy scale), and with $S_4 \simeq -8\pi^2/(3|\lambda(h_t)|)$ the action of the 4D Euclidean bounce solution for tunneling that interpolates between the EW phase and h_t. Tunneling takes place through bubbles at the scale h_t at which $\lambda(h)$ reaches its minimum. The decay rate is then $dp/(dV\ dt) \sim h_t^4 \exp[-2600/(|\lambda|/0.01)]$. This tiny number has to be multiplied by the huge 4D volume inside our past lightcone $\sim \tau_U^4 \sim (e^{140}/M_{Pl})^4$, with τ_U the age of the universe. For the typical $\lambda(h_t) \sim -0.01$ the decay is extremely suppressed or, in terms of the EW vacuum lifetime, $\tau_{EW} \gg \tau_U$. We conclude that the potential instability does not require stabilizing new physics. This would have been different for smaller M_h with $\lambda(\mu)$ entering the dangerous region $\lambda(\mu) < -0.05$ which corresponds to a vacuum lifetime $\tau_{EW} < \tau_U$ (the really dangerous instability region).

Fig. 2 shows different regions of the $\{M_h, M_t\}$ plane according to the structure of the Higgs potential: stable (green) with $\lambda(\mu) > 0$ for all $\mu < M_P$; unstable (yellow and red) with $\lambda(\mu) < 0$ below M_P. The lifetime of the metastable EW vacuum is shorter (larger) than τ_U in the red (yellow) region. With the current precision of the measurements of M_h and M_t and of the theoretical calculation of the stability bound, one concludes that the EW vacuum is most likely metastable (given the assumptions about the absence of BSM physics). More precisely, Tevatron plus LHC give [10], $M_t = 173.34 \pm 0.76\ (0.36_{stat} \pm 0.67_{syst})$ GeV,[1] while stability needs [6]:

$$M_t < (171.36 \pm 0.15 \pm 0.25_{\alpha_s} \pm 0.17_{M_h})\,\mathrm{GeV} = (171.36 \pm 0.46)\,\mathrm{GeV}\ . \tag{1.1}$$

In the last formula, the small theory error (due to higher orders and achieved only recently [5, 2, 6]) is combined in quadrature with the experimental uncertainties from $\alpha_s(M_z) = 0.1184 \pm 0.0007$ and M_h. An EW vacuum stable up to M_P requires M_t in $\sim 2-3\sigma$ tension with its central value. There is controversy on the relation between the top mass measured at the Tevatron and LHC and the top pole-mass. Although naively the difference between the two is of order Λ_{QCD} (or even smaller according to some educated guesses) a better understanding of the theoretical errors in the top mass determination would be most welcome [12].

Fig. 2-right, a zoomed-out version of the left plot, emphasizes that we might live in a very special place, really close to the critical stability boundary. This intriguing fact has motivated many speculations on its possible deep meaning. Is $\lambda(M_P) \simeq 0$ related to our living very close to the phase boundary that separates the EW broken and unbroken phases? This second near-criticality is associated to the smallness of the mass parameter in the Higgs potential, m^2: $m^2/M_P^2 \ll 1$. In fact, the Higgs potential has a very special form at M_P, with both λ and m^2 being very small in natural units. Moreover, also $\beta_\lambda \simeq 0$ not far from M_P. Why do Higgs potential parameters take these intriguing values at the scale relevant for gravitational physics, which is unrelated to the breaking of the EW symmetry? So far there is no compelling theoretical explanation for this.

2. Vacuum Instability and Physics Beyond the Standard Model

Near-criticality would be an accident if BSM below M_P modifies significantly $\lambda(\mu)$. In fact,

[1]This world combination is already superseded by the CMS one: $M_t = 172.44 \pm 0.48\ (0.13_{stat} \pm 0.47_{syst})$ GeV, [11] but, in absence of a more up-to-date world combination, we still resort to the last one.

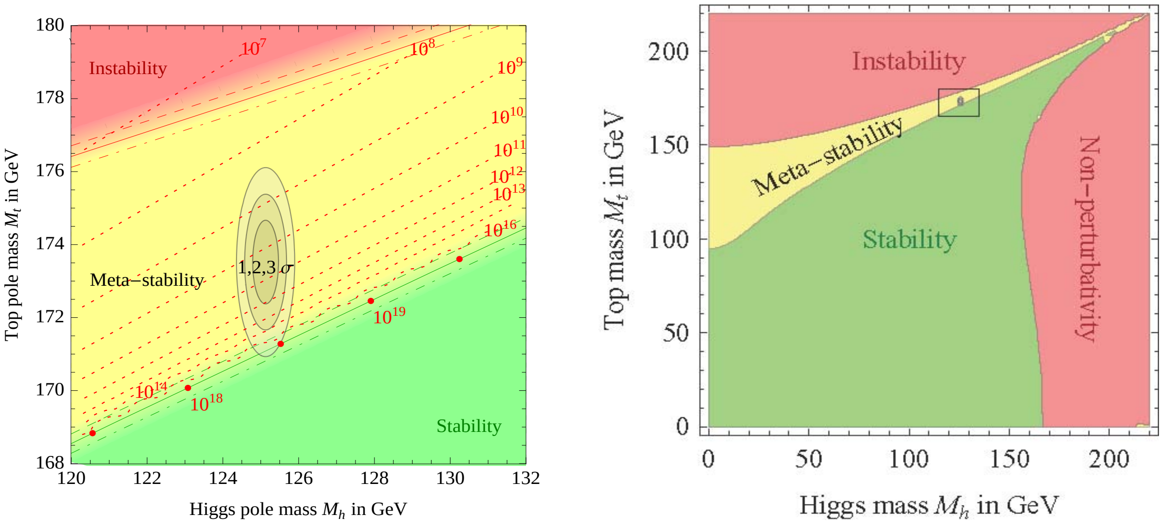

Figure 2: *Regions in the $\{M_h, M_t\}$ plane corresponding to: stability (green), metastability with $\tau_{EW} > \tau_U$ (yellow), and instability with $\tau_{EW} < \tau_U$ (red), calculated at NNLO precision [6]. Red-dashed lines in the left plot give the instability scale in GeV. The right plot [2] includes the region with the Higgs quartic coupling becoming non-perturbative below M_P.*

BSM physics is needed to explain dark matter, dark energy, neutrino masses, inflation or the baryon asymmetry. How does new physics affect the near-criticality of the Higgs potential?

BSM states can *a)* make the stability worse; *b)* be irrelevant; or *c)* cure it. Examples of the three options are easy to find e.g. in the simple case of type I seesaw neutrinos. Neutrinos affect $\beta_\lambda(\mu)$ through their Yukawa couplings, which scale as $y_\nu^2 \sim M_N m_\nu / v^2$, where m_ν is the light neutrino mass, M_N is the mass of the heavy right handed neutrinos and $v = 246\,\text{GeV}$ is the Fermi scale. The three cases are as follows: a) If $M_N \geq 10^{13-14}$ GeV, y_ν must be large to give $m_\nu \simeq 0$–1 eV and give a sizable negative contribution to β_λ. The quartic becomes too negative, $\lambda(\mu) < -0.05$, and $\tau_{EW} < \tau_U$. This conflicts with our survival and can be used to put an upper bound on M_N [13, 3]. b) For M_N smaller than in *a)* y_ν's are too small and do not change β_λ significantly. c) A powerful tree-level stabilization mechanism [14] with a heavy singlet field S, with nonzero $\langle S \rangle$, coupled to the Higgs boson as $\lambda_{HS} S^2 |H|^2$ can be used in a seesaw scenario with $M_N = \langle S \rangle$ smaller than the SM instability scale $\sim 10^{10}$ GeV, and satisfying also the lower bounds on M_N from leptogenesis [14].

Alternative stabilization mechanisms exist, and most extensions of the SM at the TeV scale modify the behavior (or existence) of the Higgs at high energies. Potential stability arguments can be used to constrain extra BSM sources of instability. For the possible impact of Planckian physics on the previous discussions see the ArXiv's version of these proceedings [15].

3. Conclusions

Already from the example of seesaw neutrinos we learned that it is easier to destroy near-criticality than to explain it. However, the interest of the near-criticality of the Higgs potential hinted at by LHC is that it might be trying to tell us something deep about nature. In this respect one can compare it with gauge coupling unification. LEP-II gave us a tantalizing hint for gauge coupling unification (with a supersymmetric spectrum). LHC has given us a tantalizing hint about the possible near-criticality of the Higgs potential. Although admitedly grand unification rested on a more respectable theoretical foundation, it is worth considering seriously the possible theoretical reasons that might be lying behind the LHC hints of a special nature of the Higgs potential. From this point of view, the stability of the Higgs potential is certainly a good motivation to improve (both in the experimental and theoretical fronts) the determination of the top mass, which is the main parameter that controls how close we are to the stability line.

Acknowledgments

This work has been partly supported by the Spanish Ministry MINECO under grants FPA2011-25948, FPA2013-44773-P and FPA2014-55613-P; by the Severo Ochoa excellence program of MINECO (grant SO-2012-0234); and by the Generalitat de Catalunya grant 2014-SGR-1450.

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