Theory Summary

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I review few selected topics on recent theoretical progress in top physics. In particular I will discuss recent progress in the computation of the relationship between the $\overline{\text{MS}}$ and pole top mass, in the NNLO calculation of top differential distributions, and in the simulation of top production and decays. Implications for top mass measurements will be discussed.

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1. Introduction

In the current year there has been considerable progress in theoretical studies regarding top quark physics. In the present workshop, summaries of recent developments in several areas have been presented. As reminded in the introductory talk [1], in several extensions of the Standard Model, the large mass of the top quark plays an important role in driving the Electro-Weak symmetry breaking. It is thus natural to expect new physics effects to show up in processes involving the top. It was also reminded [2] that, having the largest Yukawa coupling, the top can destabilize the Higgs potential. It is thus important to explore top physics in great detail, both with regard to searches for New Physics signals, and with regard to precision measurements of its properties, in particular of its mass.

At the LHC, top production is a very abundant process, thus offering new opportunities for the search of new physics effects in flavour violating top decays [3] and in anomalous couplings [3–5]. Signals of new physics are expected to become stronger for highly boosted tops. The corresponding reduction in rates can be compensated by the use of boosted top techniques, that allow for tagging hadronic top decays [6]. In general, all these studies require a quite precise understanding of the production and decay properties of the top in the Standard model. Several recent results have appeared on the computation of higher order QCD corrections to top production and decays, including their matching to parton shower generators [7–10], and of EW corrections to various top processes [11–13]. Theoretical issues in the measurement of the top mass have been summarized in [14].

It is impossible to summarize all the material presented in this workshop, especially in view of the fact that several talks were already summaries on their own. Here I will focus in detail upon three topics where very recent and important progress was made: the relationship between the $\overline{\text{MS}}$ and pole top mass, and its implications for top mass measurements; the first presentation of NNLO distributions for $t\bar{t}$ production, that was given by D. Heymes at this conference [7]; and finally I will discuss the first implementations of NLO calculation matched with shower generators (NLO+PS) that can handle properly finite width effects in the cases when resonances can decay into coloured particles, and its application to top production processes.

In the conclusions, I will discuss in particular the relevance of this recent progress for the top mass measurement at the LHC.

2. The $\overline{\text{MS}}$ and pole mass relation for the top

The top mass term in the QCD lagrangian arises from the Yukawa coupling of the top to the Higgs field. Mass renormalization can be naturally carried out with the same methods adopted for all other couplings, i.e. the $\overline{\text{MS}}$ scheme. Alternatively we can define the mass parameter as the particle physical mass, i.e. its rest energy. This parameter is also called the “pole mass”, since it really corresponds to the pole position in the particle propagator. This definition is sometimes more convenient, in view of the fact that the physical mass is often a parameter that can be directly measured, and that it remains fixed as we perform higher order calculations. If we insist in using the $\overline{\text{MS}}$ mass, on the other hand, we would find that, as we raise the perturbative order, the expression of the physical mass receives higher order corrections.
In case of stable coloured massive particles, there are practical and theoretical obstacles in defining the pole mass. Such particles, because of colour confinement, cannot be isolated. We thus expect to find them in bound states. Intuitively, we expect that the mass of these bound states will differ from the pole mass by corrections of order $\Lambda_{\text{QCD}}$. It is remarkable that this argument, relying upon color confinement, is also supported by perturbation theory alone, that suggests that a similar ambiguity in the pole mass should also arise due to self-energy corrections. The argument can be summarized as follows. It can be shown that infrared gluons in a heavy quark self-energy yield mass corrections of the form

$$\delta m \propto \int_{l_0}^{m} \, dl \, \alpha_s,$$  \hspace{1cm} (2.1)

where $l$ is the gluon virtuality. This expression is infrared finite, but one should not forget that the inclusion of higher order corrections leads to the running of the strong coupling constant

$$\alpha_s \rightarrow \alpha_s(l^2) = \frac{1}{b_0 \log(l^2/\Lambda_{\text{QCD}}^2)} = \frac{\alpha_s}{1 - \alpha_s b_0 \log(m^2/l^2)},$$  \hspace{1cm} (2.2)

where $\alpha_s = \alpha_s(m^2)$, $b_0 = (33 - 2n_l)/(12\pi)$, and $n_l$ is the number of light flavours. This leads to a divergence in (2.1) when $l = \Lambda_{\text{QCD}}$.

If we expand eq. (2.2) in powers of $\alpha_s$ we get

$$\alpha_s \rightarrow \alpha_s(l^2) = \alpha_s \sum_{k=0}^{\infty} \left( \alpha_s b_0 \log(m^2/l^2) \right)^k.$$  \hspace{1cm} (2.3)

Replacing this expansion in eq. (2.1) we get

$$\delta m \propto \int_{l_0}^{m} \, dl \, \alpha_s(l^2) = \alpha_s \sum_{k=0}^{\infty} \left( \alpha_s b_0 \log(m^2/l^2) \right)^k \int_{l_0}^{m} \, dl \, \log^k \frac{m}{l} = m \alpha_s \sum_{k=0}^{\infty} (2\alpha_s b_0)^k k!,$$  \hspace{1cm} (2.4)

so that, even if each term of the expansion is finite, the sum is divergent because of the factorially growing coefficients. This sums are known as infrared renormalons (see ref. [15] for a review). The name reminds us of the infrared nature of the problem (i.e. the small values of momenta involved) and from its relation to the running coupling.

From eq. (2.2) we are led to conclude that there is an intrinsic ambiguity of order $\Lambda_{\text{QCD}}$ in the heavy quark pole mass. In fact, when $l \approx \Lambda_{\text{QCD}}$, the coupling constant becomes of order one, and we are no longer allowed to use the perturbative expansion, so that we are unable to compute the integral (2.1) in the region $l \lesssim \Lambda_{\text{QCD}}$. A similar conclusion follows if we use the expansion of eq. (2.4) as an asymptotic expansion, stopping the summation when the terms of the series stop decreasing. This happen when

$$\frac{(2\alpha_s b_0)^n n!}{(2\alpha_s b_0)^n (n - 1)!} = 2b_0 \alpha_s n \approx 1.$$  \hspace{1cm} (2.5)

that is to say when $n \approx 1/(2b_0 \alpha_s)$. The size of the last term at this value of $n$ is

$$(2\alpha_s b_0)^n n! = n^{-n} n! \approx n^{-n} (n^ne^{-n}) \approx \exp \left( -\frac{1}{2b_0 \alpha_s} \right) \approx \frac{\Lambda_{\text{QCD}}}{m},$$  \hspace{1cm} (2.6)

yielding again a mass ambiguity of order $\Lambda_{\text{QCD}}$. 

The presence of infrared renormalons in the relation between the $\overline{\text{MS}}$ and pole mass of heavy quarks was first pointed out in ref. [16]. In particular, in ref. [17] it was found that the structure of the renormalon has the form

$$m_p = m_{\overline{\text{MS}}} \left( 1 + \sum_{n=0}^{\infty} r_n \alpha_s^{n+1} \right),$$

where $b$ and the $s_i$ coefficients can all be computed in terms of the coefficients of the $\beta$ function. The normalization $N$ cannot be computed with present techniques.

It is often stated that, if the top pole mass is measured, an intrinsic error of order of few hundred MeV is to be expected, and should thus be ascribed to all LHC mass measurement that rely on observables strongly sensitive to the mass of the system comprising the top decay products.

In the current year, in ref. [18] (from now on MSSS for Marquard, Smirnov, Smirnov and Steinhauser) remarkable progress was made in the calculation of the relation between the $\overline{\text{MS}}$ and pole mass of a heavy quark, reaching now 4-loop accuracy. For the top, this reads

$$m_p = m_{\overline{\text{MS}}} \left( 1 + 0.4244 \alpha_s + 0.8345 \alpha_s^2 + 2.375 \alpha_s^3 + (8.49 \pm 0.25) \alpha_s^4 \right)$$

(2.8)

where $m_{\overline{\text{MS}}}$ is the top $\overline{\text{MS}}$ mass evaluated at a scale equals to itself. The strong coupling $\alpha_s$ is the 6-flavour strong coupling, also evaluated at $m_{\overline{\text{MS}}}$. The corresponding corrections to the top mass (assuming $m_{\overline{\text{MS}}} = 163.643$ GeV) are given by

$$m_p = 163.643 + 7.557 + 1.617 + 0.501 + 0.195 \text{ GeV},$$

(2.9)

showing that the perturbative expansion is still well behaved, the last term being less than $1/2$ of the previous term. We could just take this as an indication that the intrinsic error on the pole mass is less than 200 MeV. It is possible, however, to do better than this if we recognize that the behaviour of the perturbative expansion up to the fourth order is well-fitted by formula (2.7). This is shown in fig. 1. As we can see, by fitting the normalization $N$ in eq. (2.7) using the exact value of the 4th order coefficient from the MSSS calculation, the 3rd and 2nd order coefficients turn out to fit the exact perturbative calculation quite well.\footnote{This fact was already studied in $b$ physics contexts in [19, 20].} Assuming thus that the perturbative expansion is dominated by the renormalon for higher order terms, we can guess the rest of the expansion. For example, the term of order 5 in eq. (2.8) is $45.4 \alpha_s^5$, corresponding roughly to a 110 MeV correction in eq. (2.9). The approximate perturbative series reaches its minimum for $n \approx 8$, and the size of the last term corresponds to a 66 MeV mass correction.\footnote{A more detail analysis of this issue is in preparation [21].}

Summarizing, there is a strong suggestion that the renormalon problem in the pole mass is not very severe, at least for the precision that the LHC is aiming to. This is good news for the standard LHC top mass determinations [22], that make use of observables that are intimately related to the pole mass.
3. NNLO differential distributions for $t\bar{t}$ pair production

D. Heymes has presented in this workshop the first NNLO differential distributions for top pair production. Needless to say, this is an outstanding result. A relative publication has appeared [23]. Figure 2, taken from [23], reports NLO and NNLO results for the top transverse momentum distribution, compared to CMS data. As also shown by Hindrichs at this conference [24], this particular distribution is not very well described by available LO+PS and NLO+PS generators, that also include shower and finite width effects (not present in the fixed order results of fig. 2). Discrepancies
show up at very low and very high top transverse momentum. In particular, the measured cross section is roughly 10% higher in the first transverse momentum bin, and it is lower in the high transverse momentum tail. Keeping this mind, we may use fig. 2 as a guidance for what variation we may expect relative to an NLO calculations when NNLO corrections are included. The slope of the NNLO/NLO red band on the right plot of the figure seems to indicate that the inclusion of higher order effects should alleviate both the low and the high \( p_T \) discrepancies. It should be said, however, that the use of a fixed renormalization and factorization scale is bound to overestimate the cross section in the high transverse momentum region, where a scale of the order of the top transverse mass (i.e. \( \sqrt{p_T^2 + m^2} \)) should be more appropriate, and is in fact used in LO+PS and NLO+PS generators. In the last bin of the plot, the transverse mass is roughly 400 GeV, not far from the high value of the scale variation, that is twice the top mass. On the other hand, the right plot seems to indicate that the NNLO result would not be lower than the NLO one if the transverse mass was used as central scale, and that the NNLO/NLO slope may disappear, thus leading to no improvement in the comparison with data. Further numerical studies may clarify this issue.

The discrepancy in the low transverse momentum bin also deserves some comment. It yields a useful example of the interplay of Monte Carlo studies, higher order calculations and comparison with data. First of all, we notice that also the invariant mass of the \( t\bar{t} \) pair, shown in Fig. 3, displays a discrepancy in the threshold region. Within the framework of the CERN top working group, it was pointed out in several circumstances that the NLO+PS generator that was yielding the best description of the top transverse momentum spectrum was POWHEG interfaced with HERWIG. As far as the threshold region is concerned, this fact was eventually traced back to the way in which momentum reshuffling is handled in HERWIG [25]. Since reshuffling in NLO+PS generators yields corrections that are formally of NNLO order, it was argued that until a fully differential NNLO result was available, the reshuffling ambiguity should have been considered as a source of a theoretical error. An NNLO calculation is now available, and we can proceed to investigate whether it justifies the HERWIG reshuffling strategy. From fig. 2 it appears that an enhancement at very low transverse momentum actually arises in the NNLO result. However, before attributing it to reshuffling effects, we should consider other sources of enhancements that may appear at the NNLO level. New Sudakov logarithms arise at the NNLO level, but they should also be reasonably modeled at the leading logarithmic level by shower Monte Carlo. On the other hand, NNLO \( 1/\nu \) singularities (where \( \nu \) is the top velocity in the \( t\bar{t} \) frame) are not modeled at all in Monte Carlo generators. The coefficient of the \( 1/\nu^2 \) term at NNLO is quite large, having relative size equal to \( (\alpha_s/4\pi)^2 \times 68.5/\nu^2 \) [26]. It is easy to implement such correction in the POWHEG-BOX-V2/hvq package. In fig. 3 I display the effect of such modification for the invariant mass of the \( t\bar{t} \) pair and for the top transverse momentum. As one can see, the effect in the low \( p_T \) and low invariant mass region is quite compatible with what is observed in the NNLO result. This hints to the fact that Coulomb effects, rather than recoil, are responsible for the mismodeling of the low invariant mass region. A detailed study of this problem is in preparation [27].

4. NLO+PS simulation of top pair production and decay

This section is about current progress in the simulation of heavy quark pair production. NLO+PS

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This point was raised by M. L. Mangano at this conference.
generators for this process have been available for a long time [28,29], and are today available also for associated production with jets [10]. Top decay is only treated approximately in these generators, with no inclusion of NLO corrections. In view of the interest in an accurate determination of the top mass, the inclusion of radiation in decay, and also of interference effects, has been addressed in the literature.

In ref. [30], an NLO-PS generator including full spin correlation and radiation in decay was presented. This generator uses zero-width, factorized matrix elements for heavy flavour production and decay. Interference effects in radiation are thus not included. Corrections are however applied in order to account for finite width, non-resonant and interference effects at least at the leading order. This generator has been illustrated in the talk by E. Re in these proceedings [10].

In ref. [31], an NLO+PS generator for the full process $W^+W^-b\bar{b}$ was built in the so called POWHEL framework (consisting essentially of the POWHEG-BOX-V2 interfaced to the HELAC matrix elements). The POWHEG-BOX-V2 framework does not treat resonances in any special way, i.e. it generates radiation from the top or anti-top decay product without accounting for the fact that near the resonance peak this radiation should preserve the mass of the resonances. It is not clear at the moment whether this is causing any mismodeling in practice. It is important, however, to develop a correct treatment of resonances in the NLO+PS framework, since it can be proven that standard NLO+PS generators, in the narrow width limit, will develop a distortion of the jet-mass.

Figure 3: Left: results for the invariant mass of the $t\bar{t}$ system. Right: Invariant mass of the $t\bar{t}$ pair and transverse momentum of the top obtained with the POWHEG-BOX-V2/hvq generator, modified with the inclusion of the $1/v^2$ NLO term, compared to the plain result.
spectrum when \( m_{\text{jet}} \approx \Gamma E_{\text{jet}} \), where \( \Gamma \) is the resonance width, and \( m_{\text{jet}}, E_{\text{jet}} \) are respectively the jet mass and its energy [32]. A formalism for such treatment has been put forward in ref. [32], where it was applied to the process of \( t \)-channel single top production. In this conference, an analogous effort in the \text{MadGraph5	extunderscore aMC@NLO} framework was presented [8]. An NLO+PS generator for \( W^+W^-b\bar{b} \) production where resonances are treated according to the formalism of ref. [32] is also in preparation [33]. In the following I will present preliminary results obtained with this last generator in comparison with the generator of ref. [30]. In fig. 4, I compare the two generators results for the

![Figure 4: Comparison of the ECNR (for Ellis-Campbell-Nason-Re) generator with respect to the JLNOP (for Jezo-Lindert-Oleari-Nason-Pozzorini) one for \( W^+W^-b\bar{b} \) production. The left plot represents the invariant mass distribution of the \( W^+b_{\text{jet}} \) system, the central plot represents the fragmentation function of the \( B \) meson defined in the \( W^+b_{\text{jet}} \) centre-of-mass, and the right plot represents the transverse momentum of the \( B \) meson with respect to the recoiling \( W^+ \) boson in the \( W^+b_{\text{jet}} \) centre-of-mass.](image)

invariant mass of the \( W^+b_{\text{jet}} \) system, and for the \( B \) meson fragmentation function and transverse momentum distribution (relative to the recoiling \( W \) boson) in the top rest frame. We see that the description of the \( W^+b_{\text{jet}} \) mass distribution is very similar in the two generators. We notice instead sensible differences in the \( B \) fragmentation function. The transverse momentum plot on the right suggests that these differences may have to do with the description of gluon emission for gluon energies near a GeV. They could thus be due to genuine interference effects between emission in production and in decay, or to the way in which radiation is assigned to either production or decay in the formalism of ref. [32]. This assignment becomes in fact ambiguous for gluon energies of the order of the top width. Further studies are needed to clarify this issue.

5. Conclusions and observation on the top mass measurement

In this brief review I have focused on recent progress in theoretical calculations relevant to the physics of top production at the LHC. This progress will be vital for the top physics program at the LHC, both for search of new physics effects and for testing our understanding of collider physics in general. There is however one specific topic for which this year’s theoretical progress is particularly relevant, i.e. the top mass measurement.

It has been argued that, since the top mass is extracted by fitting Monte Carlo computed distributions to experimental data, what is really measured is a Monte Carlo parameter, a fact that was also reminded in the experimental talk on top mass measurements at this conference [22]. On the theory side, several arguments supporting this view are given. It is often claimed, for example, that
unless the generators being used have NLO accuracy in both top production and decays, one cannot claim that the measurement is extracting a theoretically well-defined top mass parameter \cite{14}. It has also been argued that, since soft radiation from top quark is simulated by general purpose shower Monte Carlo, that typically lack a rigorous treatment of the boundary between perturbative and non-perturbative effects, its mass definition does not correspond to a well defined field theoretical one \cite{34}. While (on the positive side) these claims may lead to a more critical view on what is actually measured, they are in fact unnecessary once the theoretical error in the simulation of the top production and decay is properly considered. As a simple example, we can consider two kinds of determination of the top mass: one obtained from its total production cross section, and one obtained by fitting observables related to the mass of $W^{-} b_{\text{jet}}$ system. In the first case, assuming that we didn’t have any better calculation, we could compute the cross section using a general purpose Monte Carlo, and fit its normalization to the data by adjusting its top mass parameter. Once we include the associated theoretical uncertainty (obtained for example by scale variation), we would find a large variation of order $\alpha_s m_t$. It is certainly true that, since the generator we are using is only accurate at leading order, we would not be able to distinguish between the $\overline{\text{MS}}$ and pole mass, since their difference is of order $\alpha_s m_t$. However, the estimated error itself would lead to the same conclusion. Rather than concluding that we are measuring a “Monte Carlo” mass, we would conclude that, because of the large error, we cannot distinguish among the two mass definitions.

In the second example, since we fit observables related to the mass of the $W^{-} b_{\text{jet}}$ system, we are clearly sensitive the top pole mass, that has no NLO corrections by definition. Of course, the relation of the reconstructed mass to the underlying parameter in the Lagrangian still has perturbative and non-perturbative errors that should be estimated, but this does not mean that we are not measuring the pole mass. The error estimate in our model of production and decay would clarify how close we are, theoretically, to the pole mass.

The issue of the interplay of soft radiation from top and non-perturbative effect is certainly more subtle. It should not be forgotten, however, that Shower Monte Carlo’s do cut off gluon radiation from top at scales of order of its width, and this suppression also emerges naturally in rigorous perturbative calculation that include finite width effects. The boundary between perturbative and non-perturbative effects is thus more likely to be approached by soft radiation from the top decay products, rather than from the top itself, and it is implausible to ascribe the associated error to the pole mass definition in the Monte Carlo.

The presence of the infrared renormalon in the top pole mass, and the fact that (since the top is coloured) there is no unambiguous definition of a reconstructed top, have lead many authors to pessimistic conclusions regarding the top mass measurement, sometimes summarized by the statement that a further error of the order of 1 GeV should be always ascribed to the so called “Monte Carlo mass”. As we have seen, in reality the error due to the renormalon is much smaller than 1 GeV. The inability to define a reconstructed top even at the theoretical level is instead a serious problem, that is certainly bound to yield an irreducible theoretical error of the order of few hundred MeV on the top mass. The real question is whether “few” is one or ten.

The standard methods for measuring the top mass (reviewed in \cite{22} at this conference) make use of a reconstructed top definition comprising in essence the $W^{-} b_{\text{jet}}$ system. Assuming that we have a perfect detector, and can measure exactly the mass distribution of $W^{-} b_{\text{jet}}$ system, the question is how to relate this distribution to the fundamental parameter of the Standard Model.
Lagrangian. As we have seen in this conference, we do have generators that include radiative corrections in top production and decays. Furthermore, generators that also include interference effects arising from radiation in production and decays are around the corner. These generators can and should be used to estimate the perturbative error on the extracted top pole mass, by considering, as usual, different scale choices and variations of other parameters in the generators. Comparison among different generators of the same accuracy can also be used to better assess the error.

Estimating the error due to hadronization and non-perturbative effects is a more difficult task, since we do not have a rigorous theoretical framework for modeling these effects. This problem, however, is not new, and not specific to top production. In the early times of jet physics at LEP, studies on the determination of the strong coupling constant where performed by correcting shape variables for hadronization effects, carried out with general purpose Shower Monte Carlo. These corrections were leading to more consistent values in the determination of the strong coupling constant obtained from different shape variables. At that time, comparison of the hadronization corrections obtained with Herwig and Pythia were used to assess the hadronization error. A similar attitude can be adopted for the top mass measurement, although it is certainly recommendable to work harder on the error estimate. As a general indication, different physically motivated hadronization schemes that yield reasonable fits to hadronization data should be tried and used for this purpose. Studies like the ones presented by Corcella in this conference [14], where the Monte Carlo performance in case of a fictitious long-lived top is compared to the realistic case, and like those presented in refs. [35, 36], where alternative colour reconnection models are considered, should be critically considered and extended. More ideas and research along these lines would be welcome.

There are several proposals of alternative observables to be used for the determination of the top mass. Some of them have been already used by the experimental collaborations, and have been reviewed at this conference [37]. These proposals are all valuable, and it is likely that in the future they may reach competitive precisions. It is likely that, at the end, consistency between different determinations will yield a convincing assessment of the overall error. It should not be forgotten, however, that, as the precision increases, more and more effort should be made in trying to estimate and reduce the theoretical error, and it is unlikely that this task will be easier for these alternative methods than with the current main-stream approach.

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