

## Setting the Stage for a Non-Supersymmetric UV-Complete String Phenomenology

---

### Steven Abel\*

*IPPP, Durham University, Durham, DH1 3LE, UK*  
*E-mail: s.a.abel@durham.ac.uk*

### Keith R. Dienes

*Department of Physics, University of Arizona, Tucson, AZ 85721 USA*  
*Department of Physics, University of Maryland, College Park, MD 20742 USA*  
*E-mail: dienes@email.arizona.edu*

### Eirini Mavroudi

*IPPP, Durham University, Durham, DH1 3LE, UK*  
*E-mail: irene.mavroudi@durham.ac.uk*

In this talk, I discuss our recent work concerning the construction of non-supersymmetric heterotic string models which have exponentially suppressed dilaton tadpoles and thus greatly enhanced stability properties. The existence of such models opens the door to realistic non-supersymmetric string model-building, and I discuss how semi-realistic string models resembling the Standard Model or any of its unified variants may be constructed. These models maintain modular invariance and exhibit a misaligned supersymmetry which ensures UV finiteness, even without spacetime supersymmetry. I also discuss the potential implications for phenomenology.

*18th International Conference From the Planck Scale to the Electroweak Scale*  
*25-29 May 2015*  
*Ioannina, Greece*

---

\*Speaker.

## 1. Introduction

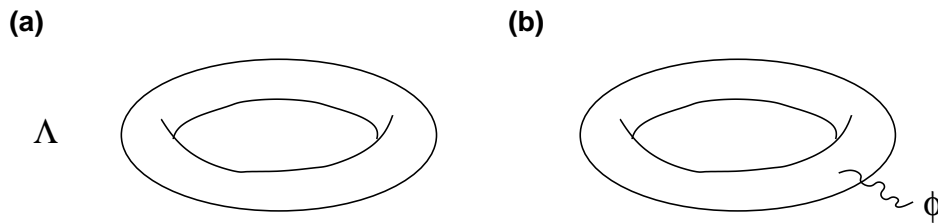
Recent data taking from the LHC seems to hint against the most minimal version of supersymmetry, and there has been much recent interest in alternatives. It seems a good moment to ask what the implications are for string phenomenology, given that supersymmetry is such an apparently integral part of string theory. In this talk I report on recent work done in Ref.[1], that establishes a starting point for performing entirely non-supersymmetric string phenomenology directly in string theory (focussing on the heterotic theory, although we think many of the principles could be applied to other configurations). The work demonstrates the existence of a class of models that have parametrically exponentially suppressed dilaton tadpoles, and hence virtually no stability problems.

Why might one be motivated to try such an approach? First, the major stumbling block for phenomenology is undoubtedly the hierarchy problem, namely how to protect the electro-weak scale against quantum corrections from the UV completion of the theory. There are many ideas based on symmetry that have been explored within field theory. However it is known that string theory provides additional symmetries, modular invariance for example, that cannot be seen within the effective field theory, except insofar as they might lead to approximate symmetries (such as non-compact shift symmetries). Moreover almost all field theory explorations of this subject (barring perhaps ones based on asymptotic safety) lack the very UV completion that one is trying to protect against. Therefore any successful explanation within effective field theory must be able to shield against *any* UV completion, however brutal. Supersymmetry is remarkable in that it *does* protect against UV completions of any kind, but it seems important to ask if UV complete but non-supersymmetric theories might provide subtler answers to the hierarchy problem.

Ref.[1] shows that hierarchically separated scales can be natural within the context of non-supersymmetric string theory, with supersymmetry being spontaneously broken by a stringy and generalised version of Scherk-Schwarz compactification, that respects modular invariance. It is the latter property in particular that ensures that the theory remains UV finite with or without supersymmetry. Indeed it is modular invariance that ensures the fundamental domain for the one loop integral simply does not contain the region corresponding to the UV divergences of field theory. The question we would like to answer is whether scales can be hierarchically less than the generic scale of supersymmetry breaking within such a theory.

In Ref.[1] we begin the hunt for this kind of structure by first tackling the issue of stability, in the guise of the cosmological constant. In any non-supersymmetric string theory this is the first object that one would like to make hierarchically smaller than its generic expectation, because a typical cosmological constant (i.e. one of order the scale of supersymmetry breaking) leads to a disastrously large dilaton tadpole as in Fig.1. Only those special theories in which the cosmological constant vanishes to leading order have a chance of being consistently stabilised. In order to find them we turn to the Scherk-Schwarz mechanism.

Scherk-Schwarz theories are typically said to have "spontaneously broken supersymmetry": they have for example an identifiable order parameter for the breaking, namely the compactification scale, which we shall refer to generically as  $1/R$  (although typically it will have an interesting dependence on all the moduli that describe the compactification manifold). The cosmological constant (which is essentially the Casimir energy) *generically* goes like  $1/R^4$ . Likewise



**Figure 1:** (a) The one-loop Casimir energy (cosmological constant)  $\Lambda$ . (b) The one-loop one-point dilaton “tadpole” diagram. In general, the value of the dilaton tadpole is always proportional to  $\Lambda$ . As a result, a non-zero cosmological constant implies a non-vanishing one-loop dilaton tadpole diagram, in turn indicating a linear term in  $\phi$  in the effective potential.

the Kaluza-Klein (KK) modes are typically split non-supersymmetrically in the Scherk-Schwarz mechanism at the scale  $1/2R$ . This for example is the mass of the gravitino in any of the theories we construct. Nevertheless it is important to emphasise from the outset that in string theories the subsequent spectrum (and hence the entire theory) is non-supersymmetric at all scales. The winding modes of the theory have masses proportional to  $R$  and so experience gross shifts in their masses due to the Scherk-Schwarz compactification, that only increase with  $R$ . At small radius, winding modes and KK modes are interchanged, and more often than not the theories becomes entirely non-supersymmetric non-compact ones in the  $R \rightarrow 0$  limit. Therefore, the distinction between supersymmetric and non-supersymmetric is *not* merely a question of the *energy scale* at which supersymmetry is broken. It would be wrong to view non-supersymmetric string models as having been supersymmetric at high energy scales but subsequently subjected to some sort of SUSY-breaking mechanism at lower energies. That the supersymmetric theory reached at large  $R$  is an extra dimensional one is another indication of this fact: the gravitino and gaugino masses are the same order as the KK masses, so there is no scale at which 4D broken supersymmetry provides a good description of the phenomenology.

Despite that it *does* make sense, at least partially, to speak of an effective spontaneously broken supersymmetric field theory, at the lowest orders of perturbation theory. The heavy string modes provide a threshold contribution to the effective field theory, which, thanks to the miracle of UV completion, is indeed finite and well behaved. The nett result is a theory where supersymmetry breaking terms can be dialled to any value, even to the string scale itself, with the non-supersymmetric threshold effects, for example violations of the non-renormalisation theorem and hard supersymmetry breaking operators, becoming more pronounced as the supersymmetry breaking approaches the string scale. In this way the Scherk-Schwarz mechanism allows us parametrically to deform the theory away from one with a supersymmetric content towards an entirely non-supersymmetric one. This property of interpolation is an integral and we believe important feature of our construction that I will discuss in more detail below.

I should mention the many other works that have considered non-supersymmetric string theories, including of course the ones that originally adapted the Scherk-Schwarz mechanism to string theory. These include the original studies of the ten-dimensional  $SO(16) \times SO(16)$  heterotic string [2], studies of the one-loop cosmological constants of non-supersymmetric strings [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17], their their finiteness properties [10, 11, 18], and

their strong/weak coupling duality symmetries [19, 20, 21, 22]. There have even been studies of the landscapes of such strings [23, 24]. All studies of strings at finite temperature are also implicitly studies of such non-supersymmetric strings (for early work in this area, see, *e.g.*, Refs. [25, 26, 27, 28, 29]). In general, the non-supersymmetric string models which were studied were either non-supersymmetric by construction or exhibited a form of spontaneous supersymmetry breaking [3, 30, 31, 32, 33, 34, 35, 36, 37], achieved through a stringy version of the Scherk-Schwarz mechanism [38] — indeed, potentially viable models within this class were constructed in Refs. [39, 40, 41, 4, 42, 43, 22, 44, 45]. Non-supersymmetric string models have also been explored in a wide variety of other configurations [46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59], including studies of the relations between scales in various schemes [60, 61, 62, 63, 64, 65, 66].

## 2. The importance of interpolation: Proto-gravitons

I begin by discussing the importance of interpolation. The issue rests on how to control the various contributions to radiative corrections, in particular a critical component coming from "unphysical" (by which I mean not level matched) states. In general there can be many different kinds of physical and unphysical states which contribute to the one-loop partition function,

$$Z(\tau) = \tau_2^{1-D/2} \sum_{m,n} a_{mn} \bar{q}^m q^n, \quad (2.1)$$

where  $q = e^{2\pi i\tau}$  in the usual nome,  $\tau_1 \equiv \text{Re } \tau$ ,  $\tau_2 \equiv \text{Im } \tau$ , and  $a_{mn}$  counts the bose–fermi non-degeneracy at level  $(m, n)$ . However, *every non-supersymmetric string model necessarily contains off-shell tachyonic states with  $(m, n) = (0, -1)$* . This is a theorem [8] which holds regardless of the specific class of non-supersymmetric string model under study, and regardless of the particular GSO projections that might be imposed.

It is easy to understand the origin of these states and their effect on the partition function. We know that every string model contains a completely NS/NS sector from which the gravity multiplet arises:

$$\text{graviton} \subset \tilde{\psi}_{-1/2}^\mu |0\rangle_R \otimes \alpha_{-1}^V |0\rangle_L. \quad (2.2)$$

Here  $|0\rangle_{R,L}$  are the right- and left-moving vacua of the heterotic string,  $\tilde{\psi}_{-1/2}^\mu$  represents the excitation of the right-moving world-sheet Neveu-Schwarz fermion  $\tilde{\psi}^\mu$ , and  $\alpha_{-1}^V$  represents the excitation of the left-moving coordinate boson  $X^V$ . Indeed, no self-consistent GSO projection can possibly eliminate this gravity multiplet from the string spectrum. However, given that the graviton is always in the string spectrum, then there must also exist in the string spectrum a corresponding state for which the left-moving coordinate oscillator is *not* excited:

$$\text{proto-graviton:} \quad \tilde{\psi}_{-1/2}^\mu |0\rangle_R \otimes |0\rangle_L. \quad (2.3)$$

This “proto-graviton” state has world-sheet energies  $(E_R, E_L) = (m, n) = (0, -1)$ , and is thus off-shell and tachyonic. Nevertheless, it is always there in the string spectrum along with the graviton.

Normally one ignores such things in phenomenology, firstly because they cannot appear as asymptotic states in any scattering (hence they are referred to as “unphysical” which we consider to be something of a misnomer), and secondly because, in a supersymmetric theory, any contribution

to the partition function from the proto-graviton is automatically cancelled by an equal and opposite one from its superpartner, the proto-gravitino. In the context of non-supersymmetric strings however, the latter is absent (or lifted to the SUSY breaking scale). Thus we can quite generally write the first term in the  $q$ -expansion of *any* non-SUSY string theory. As evident from Eq. (2.3), the proto-graviton states transform as vectors under the transverse spacetime Lorentz symmetry  $SO(D-2)$ . Thus, any non-supersymmetric string theory in  $D$  spacetime dimensions must have a partition function which begins with the contribution

$$Z(\tau) = \frac{D-2}{q} + \dots \quad (2.4)$$

One may easily evaluate the contributions that the various states make to the cosmological constant,

$$\Lambda^{(D)} \equiv -\frac{1}{2} \mathcal{M}^D \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z(\tau) \quad (2.5)$$

where  $D$  is the number of uncompactified spacetime dimensions,  $\mathcal{M}$  is the reduced string scale, and

$$\mathcal{F} \equiv \{\tau: |\operatorname{Re} \tau| \leq \frac{1}{2}, \operatorname{Im} \tau > 0, |\tau| \geq 1\} \quad (2.6)$$

is the fundamental domain of the modular group.

As a toy example, and also to illustrate the general structure of interpolating models, consider a  $D = 10$  theory compactified on a twisted circle. Any such  $(D-1)$ -dimensional model has a partition function that takes the general form [3, 25, 26, 27, 28]

$$Z_{\text{string}}(\tau, R) = Z^{(1)}(\tau) \mathcal{E}_0(\tau, R) + Z^{(2)}(\tau) \mathcal{E}_{1/2}(\tau, R) \\ + Z^{(3)}(\tau) \mathcal{O}_0(\tau, R) + Z^{(4)}(\tau) \mathcal{O}_{1/2}(\tau, R) \quad (2.7)$$

where  $Z^{(1)} + Z^{(2)}$  reproduces the partition function of a supersymmetric ten-dimensional model  $M_1$  and where  $Z^{(1)} + Z^{(3)}$  reproduces the partition function of a different non-supersymmetric ten-dimensional model  $M_2$ .

The bose–fermi non-degeneracy (i.e. the  $a_{mn}$ ) is shown in fig.2 for this theory. It clearly illustrates the non-softness of the supersymmetry breaking, namely the fact that no matter how much one “restores” supersymmetry by increasing the radius  $\sqrt{\alpha'}/R = a \rightarrow 0$ , there is always a scale (the mass of the lowest lying winding modes) above which the spectrum is entirely non-supersymmetric. In this sense the supersymmetric and interpolating non-supersymmetric theories are always entirely distinct as I stressed earlier. Despite that, the low lying spectrum adopts a characteristic supersymmetric (albeit extra-dimensional) form in this limit, while the heavy spectrum yields non-supersymmetric threshold effects.

The contributions to the cosmological constant in the  $\sqrt{\alpha'}/R = a \rightarrow 0$  limit from a given state with world-sheet energies  $(m, n)$  in the different  $\mathcal{E}/\mathcal{O}$ -sectors are found to be as follows:

sector	state	contribution to $\Lambda$
$\mathcal{E}_0 - \mathcal{E}_{1/2}$	$m = n = 0$	$-[4(D/2 - 1)!/\pi^{D/2}] a^{D-1}$
$\mathcal{E}_0 - \mathcal{E}_{1/2}$	$m = n \neq 0$	$4(2\sqrt{ma})^{(D-1)/2} e^{-4\pi\sqrt{m}/a}$
$\mathcal{E}_0 - \mathcal{E}_{1/2}$	$m \neq n$	$-[4\sqrt{2}/\pi] e^{-2\pi(m+n)} a^2 e^{-\pi/a^2}$
$\mathcal{O}_{0,1/2}$	any $(m, n)$	$[2\sqrt{2}/\pi] e^{-2\pi(m+n)} a^2 e^{-\pi/a^2}$

(2.8)

In this table,  $D$  represents the dimensionality of the theory in question *prior* to the compactification on the twisted circle. These expressions are in fact general for compactification on a twisted circle from any  $D$ . At large radii, the leading contribution to the cosmological constant is given by the nett contribution coming from the massless  $m = n = 0$  physical states, of the form,  $\sim (N_b^{(0)} - N_f^{(0)})a^{D-1}$ . In more general compactifications from  $D$  down to  $d$  dimensions one would find

$$\Lambda_d \sim (N_b^{(0)} - N_f^{(0)})a^d + \dots, \quad (2.9)$$

recovering the same contribution to the Casimir energy that one would infer in extra-dimensional field theory. Our ultimate task then is to find models where states at the massless level have degenerate numbers of bosons and fermions, despite having no supersymmetry. In such models, at large radius the leading contribution comes from the massive physical  $m = n \neq 0$  states which is exponentially suppressed.

Returning to the proto-graviton, we conclude that the one-loop cosmological constant is exponentially suppressed *provided* the unphysical states do not contribute significantly. Their contribution is independent of the number of dimensions. One can understand this from the fact that dimension dependence requires states to be able to propagate long distances, which unphysical states are not able to do. Nevertheless they still contribute to the cosmological constant because the bottom of the fundamental domain (i.e. the UV end of the one-loop integral) is curved, and in particular the proto-graviton term from  $(m, n) = (-1, 0)$  exceeds that of even the massless  $(0, 0)$  physical contributions for  $a \lesssim 0.54$ . Thus Scherk-Schwarz compactified theories with  $R \lesssim 2$  have little chance of being stable, at least on the grounds of any arguments based on the physical spectrum.

All of the above makes the cleanest assumption about the compactification scale, that  $1/R = M_c \sim M_{\text{string}}$ , problematic. Indeed, in such models it is not always clear how to separate oscillator states from KK states and/or winding states; there even exist examples of such models which transcend the notion of having a compactification geometry altogether and in which no compactification geometry can even be identified. In such models we typically obtain a cosmological constant of order  $\Lambda \sim M_{\text{string}}$ . Of course, even within such string models, there remains the possibility that  $\Lambda$  might still vanish through some other mechanism. For example, the proposals in Refs. [7, 8, 12] all rely on different kinds of symmetry arguments for cancelling  $\Lambda$  within closed string models for which  $M_c \sim M_{\text{string}}$ . Unfortunately, no string models have ever been constructed exhibiting the symmetries proposed in Refs. [7, 8], and the mechanism proposed in Ref. [12] may actually fail at higher loops [14, 15].

The alternative possibility that we are proposing is to consider models in which  $M_{\text{string}}$  is fixed but  $M_c$  is taken to be a free, adjustable variable. Indeed, we can go even further and imagine that our compactification volume is characterized by many different compactification scales  $M_c^{(i)}$ , each of which we might consider a free parameter; such a scenario would emerge, for example, if our  $d$ -dimensional compactification manifold is a  $d$ -torus with different radii of compactification  $R_i$ ,  $i = 1, \dots, d$ . In general, as the volume of compactification  $V_d$  is taken to infinity, we effectively produce a string model in  $d$  additional spacetime dimensions. This higher-dimensional model is the general equivalent of our  $M_1$  above. For closed strings, T-duality then ensures that we also produce a model in  $d$  additional spacetime dimensions as  $V_d \rightarrow 0$ , which is the equivalent of  $M_2$ . The model with variable compactification volumes can be said to *interpolate* between the two

higher-dimensional endpoint models,  $M_1$  and  $M_2$ , in the exact same manner as the interpolation between the two 10 dimensional models above.

Such interpolating models offer a number of distinct advantages when it comes to suppressing the cosmological constant. If the model  $M_1$  is supersymmetric, we are assured that  $\Lambda = 0$  when  $V_d \rightarrow \infty$ . Moreover, if  $M_2$  is non-supersymmetric, then spacetime supersymmetry is likely to be broken for all finite  $V_d$ . It is therefore reasonable to assume that we can dial  $V_d$  to a sufficiently large value in order to obtain a cosmological constant of whatever size we wish. Even more compellingly, there is a widespread belief that spacetime supersymmetry, if it exists at all in nature, is broken at the TeV-scale, with superpartners having masses  $\sim \mathcal{O}(\text{TeV})$ . Indeed, as first suggested in Refs. [3, 61], these sorts of scenarios with large compactification volumes are relatively easy to incorporate with the interpolating-model framework with  $M_c \sim \mathcal{O}(\text{TeV})$ .

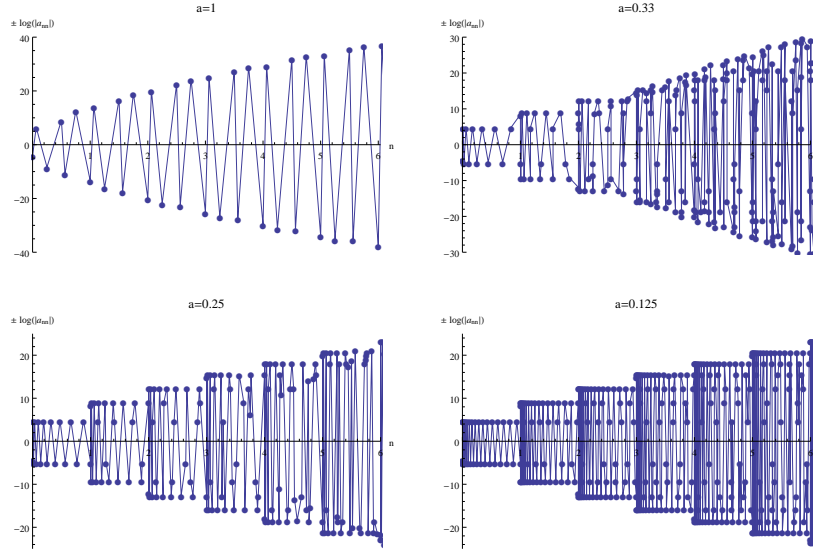
But second, and equally importantly, we also find that the *scale* of the cosmological constant need not necessarily be tied to the effective scale of the supersymmetry breaking. In particular, although we can consider the scale of supersymmetry breaking in these models to be given by  $M_c = 1/R$ , there are circumstances under which the contribution of massless physical states and their KK modes to the Casimir energy vanish, because they have bose-fermi degeneracy,  $N_b^{(0)} = N_f^{(0)}$ . The cosmological constant is then very generally expected to be *exponentially suppressed*, with  $\Lambda \sim \mathcal{O}(e^{-M_c/M_{\text{string}}})$ , due to the string sized masses of the states that are able to make a nett contribution propagating in the loop. This additional stability is a pre-cursor of what one might eventually hope to achieve for scalar masses.

### 3. Interpolating string models with exponentially small cosmological constant

I will now outline the construction for non-supersymmetric theories that interpolate to supersymmetric ones. For technical reasons it is advantageous to interpolate between  $M_1$  and  $M_2$  models in *six* dimensions rather than five. We therefore begin with six-dimensional  $M_1$ 's that have  $\mathcal{N} = 1$  supersymmetry. Such models are most conveniently obtained by lifting to six dimensions semi-realistic four-dimensional  $\mathcal{N} = 1$  string models, for example those in Refs.[73, 74, 75, 76, 77, 78, 79] which are already on the market. The objective is to retain as far as possible their desirable phenomenological features.

Once we have constructed  $M_1$ , the next step is to compactify back down to four dimensions. The four-dimensional  $\mathcal{N} = 1$  model that results from compactifying back to four dimensions on a  $T_2/\mathbb{Z}_2$  orbifold can be compared with the four-dimensional  $\mathcal{N} = 0$  model that results from a CDC compactification on the same orbifold using the techniques of Refs.[30, 31, 32].

Our final step is to take the resulting  $\mathcal{N} = 0$  model and introduce modifications to render  $N_b^{(0)} = N_f^{(0)}$ , yielding an exponentially suppressed cosmological constant. There are several different ways in which this can be done. One way is to alter the final Scherk-Schwarz (CDC) twist but retain the prior GSO symmetry breaking: this can produce an SM-like model. By contrast, altering the final twist and also removing prior GSO projections can lead to a variety of additional models: a Pati-Salam-like model, a flipped- $SU(5)$  ‘‘unified’’ model, and an  $SO(10)$  ‘‘unified’’ model, each also with  $N_b^{(0)} = N_f^{(0)}$ . The procedure is outlined in fig.3. Undoubtedly these models are only several within an entire new terrain which deserves exploration.



**Figure 2:** Degeneracies of physical states for the 9D toy model with  $a = 1$  (upper left),  $a = 0.33$  (upper right),  $a = 0.25$  (lower left),  $a = 0.125$  (lower right). Within each plot, points are connected in order of increasing world-sheet energy  $n$ . In all cases we see that surpluses of bosonic states alternate with surpluses of fermionic states as we proceed upwards in  $n$ ; this behavior is the signal of an underlying “misaligned supersymmetry” which exists within all modular-invariant non-supersymmetric tachyon-free string theories and which is ultimately responsible for the finiteness of closed strings — even in the absence of spacetime supersymmetry. For  $R = \sqrt{\alpha'}$  (or  $a = 1$ ), we see that this oscillation between bosonic and fermionic surpluses occurs within the exponentially growing envelope function  $|a_{nn}| \sim e^{c\sqrt{n}}$  associated with a Hagedorn transition. However, as the compactification radius increases (or equivalently as  $a \rightarrow 0$ ), we see that a hierarchy begins to emerge between the oscillator states and their KK excitations; the oscillator states continue to experience densities of states which are exponentially growing as functions of  $n$ , but their corresponding KK excitations are densely packed within each interval  $(n, n + 1)$  and, as expected, exhibit constant state degeneracies.

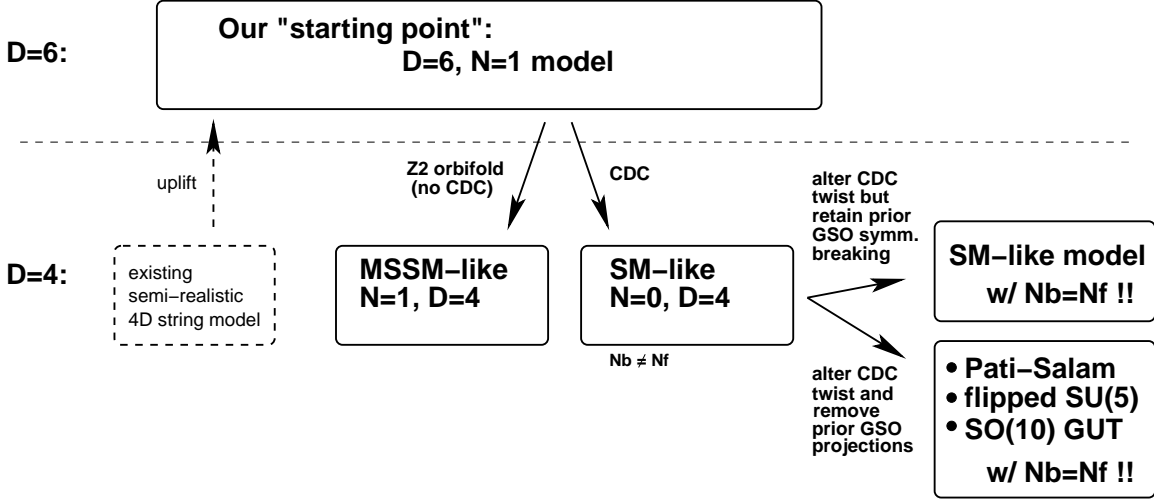
### 3.1 Pati-Salam model

As an illustration I will present the bose-fermi degenerate Pati-Salam model. It is defined by the following generalised GSO vectors (where the notation is standard in fermionic string constructions – and is summarized in the Appendix of Ref.[1]):

$$\begin{aligned}
 V_0 &= -\frac{1}{2} [ 11 111 111 | 1111 11111 111 11111111 ] \\
 V_1 &= -\frac{1}{2} [ 00 011 011 | 1111 11111 111 11111111 ] \\
 V_2 &= -\frac{1}{2} [ 00 101 101 | 0101 00000 011 11111111 ] \\
 b_3 &= -\frac{1}{2} [ 10 100 001 | 0001 11111 001 10000111 ] \\
 V_4 &= -\frac{1}{2} [ 00 101 101 | 0101 00000 011 00000000 ] \\
 V_5 &= -\frac{1}{2} [ 00 000 011 | 0100 11100 000 11100111 ] \\
 \mathbf{e} &= \frac{1}{2} [ 00 101 101 | 1011 00000 000 00011111 ] .
 \end{aligned} \tag{3.1}$$

The vector  $\mathbf{e}$  shows the action of the CDC on the right-moving space-time world-sheet degrees of freedom (on the left) and the left-moving internal degrees of freedom (on the right). The vector dot





**Figure 3:** Roadmap illustrating the procedure for constructing semi-realistic non-supersymmetric string models with  $N_b^{(0)} = N_f^{(0)}$ , as discussed in the text.

products and  $k_{ij}$  structure constants for this model are given by

$$V_i \cdot V_j = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & 1 & 0 & \frac{3}{2} \\ 0 & \frac{1}{2} & 1 & 0 & 0 & \frac{3}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & \frac{3}{2} & 0 & 0 \end{pmatrix} \text{ mod } (2), \quad k_{ij} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The gauge-group structure is

$$G = SO(4) \otimes U(1) \otimes U(1) \otimes \underbrace{SO(6) \otimes SO(4)}_{\text{contains SM}} \otimes U(1) \otimes U(1) \otimes U(1) \otimes U(1) \otimes SO(4) \otimes SO(4) \otimes SO(6), \tag{3.2}$$

where the Pati-Salam group corresponding to the visible sector is indicated. This model, which has four quasi-supersymmetric chiral generations of massless untwisted matter but no twisted matter, has  $N_b^{(0)} = N_f^{(0)} = 416$  complex massless degrees of freedom in the untwisted sector. The spectrum is shown in tables 1 and 2. Many similar examples can be found.

## 4. Phenomenological properties

### 4.1 Spectrum

The phenomenological structure of models such as those discussed above, are very general. First the spectrum itself consists of a largely unaffected and still supersymmetric (at tree-level) twisted set of sectors, together with untwisted sectors that have relatively shifted KK towers. When  $N_b^{(0)} = N_f^{(0)}$  the spectrum has to take the characteristic form shown in Fig.4. As the Standard-Model does not have bose-fermi degeneracy in a realistic model one would have to insist on a hidden sector

Sector	States remaining after CDC	Spin	$SU(4) \otimes SU(2)_L \otimes SU(2)_R$	Particle
$V_0 + V_2$	$ \alpha\rangle_R \otimes \bar{\psi}_0^i \bar{\psi}_0^a  \hat{\alpha}\rangle_L$	$\frac{1}{2}$	$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	$\mathbb{F}_L$
	$ \alpha\rangle_R \otimes \bar{\psi}_0^1 \bar{\psi}_0^2 \bar{\psi}_0^3 \bar{\psi}_0^a  \hat{\alpha}\rangle_L$			
	$ \alpha\rangle_R \otimes  \hat{\alpha}\rangle_L$			
	$ \alpha\rangle_R \otimes \bar{\psi}_0^4 \bar{\psi}_0^5  \hat{\alpha}\rangle_L$	$\frac{1}{2}$	$(\mathbf{4}, \mathbf{1}, \mathbf{2})$	$\mathbb{F}_R$
	$ \alpha\rangle_R \otimes \bar{\psi}_0^i \bar{\psi}_0^j  \hat{\alpha}\rangle_L$			
	$ \alpha\rangle_R \otimes \bar{\psi}_0^i \bar{\psi}_0^j \bar{\psi}_0^4 \bar{\psi}_0^5  \hat{\alpha}\rangle_L$			
$\overline{V_1 + V_2}$	$ \alpha\rangle_R \otimes  \beta\rangle_L$	0	$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	Exotic spinor $\mathbb{E}$
	$ \alpha\rangle_R \otimes  \beta\rangle_L$	0	$(\mathbf{4}, \mathbf{1}, \mathbf{2})$	Complex scalar $\mathbb{K}$

**Table 1:** Chiral ( $\mathbb{Z}_2$ -untwisted) multiplets of the  $\mathcal{N} = 1, D = 4$  Pati-Salam model that remain massless after the CDC. Here  $i, j \in SU(4)$  and  $a \in SU(2)_L \otimes SU(2)_R$ . The  $|\alpha\rangle_R$  represent right-moving Ramond ground states (space-time spinors), while  $|\hat{\alpha}\rangle_L$  (respectively  $|\beta\rangle_L$ ) represent the left-moving Ramond excitations that do not (respectively do) overlap with the Pati-Salam gauge group. Again the multiplets are essentially the decomposition of the  $\mathbf{16}$  of  $SO(10)$ . The same decomposition applies for the two massless generations of the  $b_3$ - and  $b_4$ - twisted-sector matter fields.

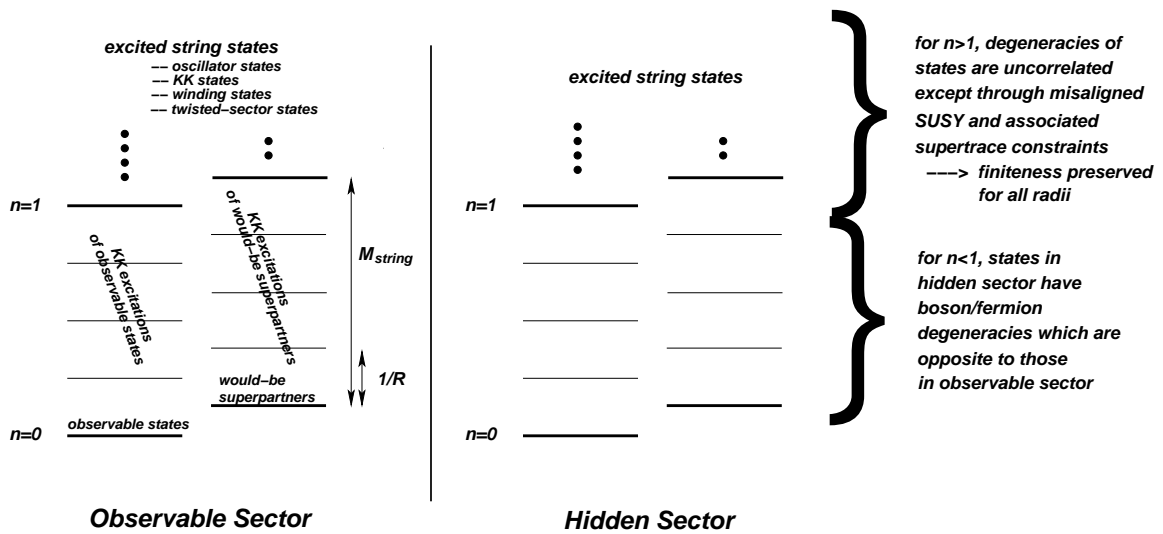
Sector	States removed by CDC	Spin	$SU(4) \otimes SU(2)_L \otimes SU(2)_R$	Particle
$V_1 + V_2$	$ \alpha\rangle'_R \otimes  \beta\rangle_L$	$\frac{1}{2}$	$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	Spinor $\tilde{\mathbb{E}}$
	$ \alpha\rangle'_R \otimes  \beta\rangle_L$	$\frac{1}{2}$	$(\mathbf{4}, \mathbf{1}, \mathbf{2})$	Spinor $\tilde{\mathbb{K}}$
$V_0 + V_2$	$ \alpha\rangle'_R \otimes \bar{\psi}_0^i \bar{\psi}_0^a  \hat{\alpha}\rangle_L$	0	$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	$\tilde{\mathbb{F}}_L$
	$ \alpha\rangle'_R \otimes \bar{\psi}_0^1 \bar{\psi}_0^2 \bar{\psi}_0^3 \bar{\psi}_0^a  \hat{\alpha}\rangle_L$			
	$ \alpha\rangle'_R \otimes  \hat{\alpha}\rangle_L$			
	$ \alpha\rangle'_R \otimes \bar{\psi}_0^4 \bar{\psi}_0^5  \hat{\alpha}\rangle_L$	0	$(\mathbf{4}, \mathbf{1}, \mathbf{2})$	$\tilde{\mathbb{F}}_R$
	$ \alpha\rangle'_R \otimes \bar{\psi}_0^i \bar{\psi}_0^j  \hat{\alpha}\rangle_L$			
	$ \alpha\rangle'_R \otimes \bar{\psi}_0^i \bar{\psi}_0^j \bar{\psi}_0^4 \bar{\psi}_0^5  \hat{\alpha}\rangle_L$			

**Table 2:** Chiral ( $\mathbb{Z}_2$ -untwisted) multiplets of the  $\mathcal{N} = 1, D = 4$  Pati-Salam model which are given masses  $\frac{1}{2}\sqrt{R_1^{-2} + R_2^{-2}}$  by the CDC. Here  $i, j \in SU(4)$  while  $a \in SU(2)_L \otimes SU(2)_R$ . The  $|\alpha\rangle'_R$  represent right-moving Ramond ground states that are not space-time spinors.

with the exact equal and opposite nett bose-fermi number in order to cancel overall. This structure is also apparent in the (misaligned supersymmetry) plots of nett bose-fermi number in Fig.5. Here one can see that as the radius is increased the entire KK spectrum has bose-fermi degeneracy below the string scale despite the fact the theory is completely non-supersymmetric.

### 4.2 Cosmological constant

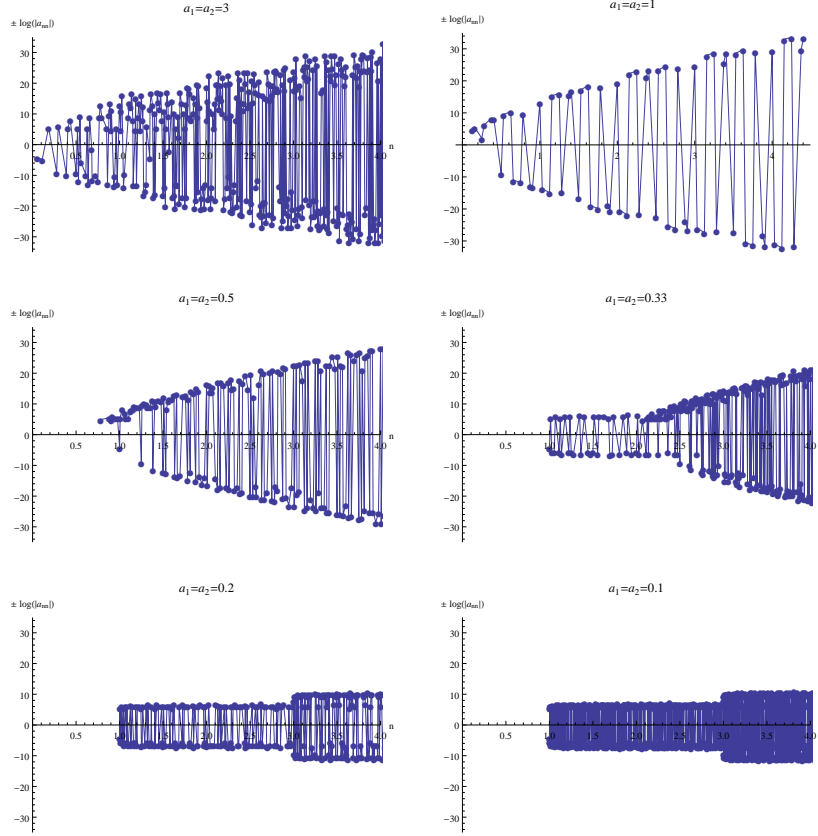
It is interesting also to examine the interpolating cosmological constant  $\Lambda(a)$  of the Pati-Salam model as a function of the inverse radius  $a = \sqrt{\alpha'}/R$ . Our results are shown in Fig. 6, and are consistent with the gross features that one would expect from the above discussion, namely that the cosmological constant is finite for all radii, exponentially suppressed in the large-radius limit, and radius-independent in the small-radius limit. This last observation suggests the existence of a zero-radius endpoint model (a.k.a.  $M_2$ ) with an entirely non-supersymmetric but tachyon-free spectrum — one which most likely corresponds to a 6D fermionic string constructed with discrete torsion. More surprisingly, however, just above (but not at) the self-dual radius, we find a stable anti-de Sitter minimum. This turn-over could indicate a restoration of gauge symmetry and/or supersymmetry, and is similar to the situation encountered in the Type II models of Ref. [37].



**Figure 4:** The structure of the spectrum of a generic interpolating model with suppressed cosmological constant in the limit of large interpolating radius. States with masses below  $M_{string}$  (or below  $n = 1$ ) consist of massless observable states, massless hidden-sector states, their would-be superpartners, and their lightest KK excitations. For these lightest states, the net (bosonic minus fermionic) numbers of degrees of freedom from the hidden sector are exactly equal and opposite to those from the observable sector for all large radii. Note that this cancellation of net physical-state degeneracies between the observable and hidden sectors bears no connection with any supersymmetry, either exact or approximate, in the string spectrum. Nevertheless, it is this conspiracy between the observable and hidden sectors which suppresses the overall cosmological constant and enhances the stability of these strings. For the heavier states, by contrast, the observable and hidden sectors need no longer supply equal and opposite numbers of degrees of freedom. Nevertheless the entire theory remains finite at one-loop.

### 4.3 Scalar masses

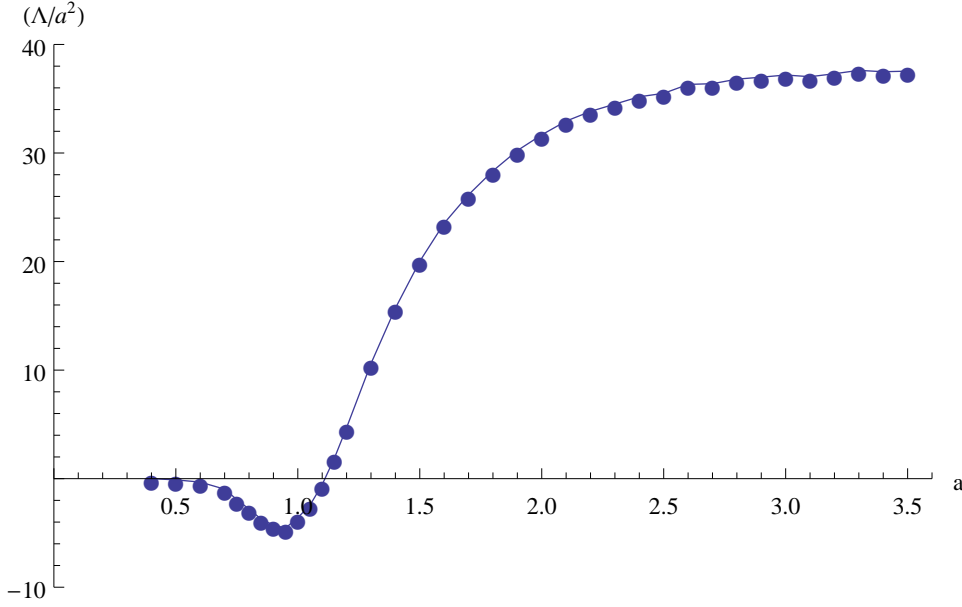
Finally let me discuss the stringy threshold corrections that generate scalar masses, *etc.*. These are all in principle calculable. Indeed at one loop their chief contribution at large radius can be understood by a field theoretical calculation as it is dominated by the physical modes propagating



**Figure 5:** Degeneracies of physical states for the Pati-Salam model with exponentially suppressed cosmological constant. The inverse radius  $a = \sqrt{\alpha'}/R$  varies from  $a = 3$  (upper left) to  $a = 0.1$  (lower right). Comparing with Fig. 2, we see that all of the general features associated with interpolating models survive, including a smoothly growing exponential envelope function for  $a \sim \mathcal{O}(1)$  which slowly deforms into a discretely step-wise growing exponential function as  $a \rightarrow 0$ . This reflects the emerging hierarchy between KK states and oscillator states. However, we also observe a critical new feature which reflects the fact that this model has an exponentially suppressed cosmological constant: the removal or “evacuation” of all non-zero nett state degeneracies  $a_{nm}$  for  $n \leq 1$  for sufficiently small  $a$ . Thus, for sufficiently large radius, the spectrum of such models develops an exact boson/fermion degeneracy for all relevant mass levels  $n < 1$ , even though there is no supersymmetry anywhere in the spectrum. Indeed, as illustrated in Fig. 4, this degeneracy does *not* occur through a pairing of states with their would-be superpartners, but rather as the result of the balancing of non-zero nett degeneracies associated with a non-supersymmetric *observable* sector against the degeneracies associated with a non-supersymmetric *hidden* sector.

in the loops. The string theoretical calculation of these effects can be carried out in an analogous fashion to the usual gauge beta function calculation - namely by directly determining the two point function for the scalar, but with the appropriately Scherk-Schwarz modified partition function. As one might expect the result no longer cancels, and the amplitude can be written as

$$A(k, -k) = -(2\pi)^4 \frac{g_{\text{YM}}^2}{16\pi^2} \int_{\mathcal{F}} \frac{d^2\tau}{4\tau_2} \sum_{\alpha, \beta, \ell} \left( \frac{Y^2}{g_{\text{YM}}^2} - \frac{1}{4\pi\tau_2} \right) \frac{|\vec{\ell}|^2}{\tau_2^2} Z_{\ell, \mathbf{0}Z} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}. \quad (4.1)$$



**Figure 6:** The rescaled cosmological constant  $\Lambda/a^2$  for the Pati-Salam model versus  $a \equiv \sqrt{\alpha'}/R$ . For large  $a$ , we find that  $\Lambda/a^2$  tends to a constant indicating that the  $a \rightarrow \infty$  limit of this model is non-supersymmetric and tachyon-free. We also see that the entire curve is finite, which indicates that no tachyons emerge at any intermediate radii. Thus this model lacks Hagedorn-like instabilities. However we observe that the small- $a$  behaviour of this curve is radically different from the generic case. First, we see that  $\Lambda$  does not have the usual Casimir  $a^4$  behaviour, but rather is exponentially suppressed. Second, and somewhat surprisingly, we observe that  $\Lambda$  changes sign as  $a \rightarrow 0$  increases past unity. Indeed, we see that the cosmological constant appears to have a stable minimum near (but not precisely at) the self-dual radius, and moreover that the cosmological constant crosses zero at yet another (slightly higher) radius. It is not clear whether there might exist enhanced symmetry at either of these specific radii.

We can split the contributions into those from massless physical states and those from massive ones. The term  $(4\pi\tau_2)^{-1}$  will be proportional to the overall cosmological constant and therefore exponentially suppressed. The contribution from the massless-sector terms to the canonically normalised 4D Higgs squared-masses are then

$$\begin{aligned}
 M_{H_1}^2 &= \frac{1}{16\pi^2} \int_{\frac{1}{a^2} \approx 1}^{\infty} \frac{d\tau_2}{4\tau_2^5} \sum_{\ell=\text{odd}, i} Y^2 (N_{fH}^i - N_{bH}^i) |\vec{\ell}|^2 e^{-\frac{\pi}{2}|\vec{\ell}|^2} e^{-\pi\tau_2\alpha' m_i^2} \\
 &\approx \frac{2}{\alpha'} \frac{Y^2}{16\pi^2} (N_{fH}^0 - N_{bH}^0) \frac{\pi^2 a^6}{320}, \tag{4.2}
 \end{aligned}$$

where the sum is divided into mass-levels  $m_i$ , while the contributions from the massive states are

$$M_{H_1}^2 = \frac{2}{\alpha'} \frac{Y^2}{16\pi^2} (N_{fH}^i - N_{bH}^i) \sum_{\ell=\text{odd}} |\vec{\ell}|^{-5/2} (\sqrt{\alpha'} m_i)^{7/2} e^{-2\pi\sqrt{\alpha'} m_i |\vec{\ell}|}. \tag{4.3}$$

The first of these expressions does not necessarily vanish even if its analogue does for the cosmological constant, because the Higgs couples differently to the states that are projected out by the CDC. Note however the interesting possibility of exponentially suppressed Higgs masses as well.

To finish, let me mention several interesting developments recently concerning the question of large volume “decompactification” in Refs.[90, 91, 92, 93, 94]. The decompactification problem (i.e. reaching large volumes while avoiding large gauge couplings) has also been discussed in the past literature in a somewhat different guise [95]. In the context of Scherk-Schwarz breaking of supersymmetry it is to a certain extent a dimensional transmutation of the hierarchy problem; namely the fact that achieving order one couplings in a generic theory appears to require a fine-tuning of one-loop corrections against tree-level ones. There are schemes to overcome this fine-tuning problem that will be presented in forthcoming work [96]. I should add though that the entire benefit of being able to generate exponentially suppressed scales is that extremely large volumes may not be required. Clearly a primary objective would be then to extend exponential suppression to the Higgs mass itself. This will be explored in Ref.[97].

The work of KRD was supported in part by the US Department of Energy under Grant DE-FG02-13ER-41976, and by the US National Science Foundation through its employee IR/D program. The opinions and conclusions expressed herein are those of the authors and do not represent any funding agency. EM is in receipt of an STFC fellowship.

## References

- [1] S. Abel, K. R. Dienes and E. Mavroudi, “Towards a nonsupersymmetric string phenomenology,” *Phys. Rev. D* **91**, no. 12, 126014 (2015) doi:10.1103/PhysRevD.91.126014 [arXiv:1502.03087 [hep-th]].
- [2] L. Alvarez-Gaume, P. H. Ginsparg, G. W. Moore and C. Vafa, “An  $O(16) \times O(16)$  Heterotic String,” *Phys. Lett. B* **171**, 155 (1986);  
L. J. Dixon and J. A. Harvey, “String Theories In Ten-Dimensions Without Space-Time Supersymmetry,” *Nucl. Phys. B* **274**, 93 (1986).
- [3] R. Rohm, “Spontaneous Supersymmetry Breaking in Supersymmetric String Theories,” *Nucl. Phys. B* **237**, 553 (1984).
- [4] V. P. Nair, A. D. Shapere, A. Strominger and F. Wilczek, “Compactification of the Twisted Heterotic String,” *Nucl. Phys. B* **287**, 402 (1987);  
P. H. Ginsparg and C. Vafa, “Toroidal Compactification of Nonsupersymmetric Heterotic Strings,” *Nucl. Phys. B* **289**, 414 (1987).
- [5] H. Itoyama and T. R. Taylor, “Supersymmetry Restoration in the Compactified  $O(16) \times O(16)$ -prime Heterotic String Theory,” *Phys. Lett. B* **186**, 129 (1987).
- [6] H. Itoyama and T. R. Taylor, “Small Cosmological Constant in String Models,” FERMILAB-CONF-87-129-T, C87-06-25.
- [7] G. W. Moore, “Atkin-Lehner Symmetry,” *Nucl. Phys. B* **293**, 139 (1987) [Erratum-ibid. B **299**, 847 (1988)];  
J. Balog and M. P. Tuite, “The Failure Of Atkin-Lehner Symmetry For Lattice Compactified Strings,” *Nucl. Phys. B* **319**, 387 (1989);  
K. R. Dienes, “Generalized Atkin-Lehner Symmetry,” *Phys. Rev. D* **42**, 2004 (1990).
- [8] K. R. Dienes, “New string partition functions with vanishing cosmological constant,” *Phys. Rev. Lett.* **65**, 1979 (1990).
- [9] D. Kutasov and N. Seiberg, “Number of degrees of freedom, density of states and tachyons in string theory and CFT,” *Nucl. Phys. B* **358**, 600 (1991).

- [10] K. R. Dienes, “Modular invariance, finiteness, and misaligned supersymmetry: New constraints on the numbers of physical string states,” Nucl. Phys. B **429**, 533 (1994) [hep-th/9402006]; “How strings make do without supersymmetry: An Introduction to misaligned supersymmetry,” In \*Syracuse 1994, Proceedings, PASCOS '94\* 234-243 [hep-th/9409114]; “Space-time properties of (1,0) string vacua,” In \*Los Angeles 1995, Future perspectives in string theory\* 173-177 [hep-th/9505194].
- [11] K. R. Dienes, M. Moshe and R. C. Myers, “String theory, misaligned supersymmetry, and the supertrace constraints,” Phys. Rev. Lett. **74**, 4767 (1995) [hep-th/9503055]; “Supertraces in string theory,” In \*Los Angeles 1995, Future perspectives in string theory\* 178-180 [hep-th/9506001].
- [12] S. Kachru, J. Kumar and E. Silverstein, “Vacuum energy cancellation in a nonsupersymmetric string,” Phys. Rev. D **59**, 106004 (1999); [hep-th/9807076];  
S. Kachru and E. Silverstein, “On vanishing two loop cosmological constants in nonsupersymmetric strings,” JHEP **9901**, 004 (1999) [hep-th/9810129].
- [13] J. A. Harvey, “String duality and nonsupersymmetric strings,” Phys. Rev. D **59**, 026002 (1999) [hep-th/9807213];  
S. Kachru and E. Silverstein, “Selfdual nonsupersymmetric type II string compactifications,” JHEP **9811**, 001 (1998) [hep-th/9808056];  
R. Blumenhagen and L. Gorlich, “Orientifolds of nonsupersymmetric asymmetric orbifolds,” Nucl. Phys. B **551**, 601 (1999) [hep-th/9812158];  
C. Angelantonj, I. Antoniadis and K. Forger, “Nonsupersymmetric type I strings with zero vacuum energy,” Nucl. Phys. B **555** (1999) 116 [hep-th/9904092];  
M. R. Gaberdiel and A. Sen, “Nonsupersymmetric D-brane configurations with Bose-Fermi degenerate open string spectrum,” JHEP **9911**, 008 (1999) [hep-th/9908060].
- [14] G. Shiu and S. H. H. Tye, “Bose-Fermi degeneracy and duality in nonsupersymmetric strings,” Nucl. Phys. B **542**, 45 (1999) [hep-th/9808095].
- [15] R. Iengo and C. J. Zhu, “Evidence for nonvanishing cosmological constant in nonSUSY superstring models,” JHEP **0004**, 028 (2000) [hep-th/9912074].
- [16] E. D'Hoker and D. H. Phong, “Two loop superstrings 4: The Cosmological constant and modular forms,” Nucl. Phys. B **639**, 129 (2002) [hep-th/0111040]; “Lectures on two loop superstrings,” Conf. Proc. C **0208124**, 85 (2002) [hep-th/0211111].
- [17] A. E. Faraggi and M. Tsulaia, “Interpolations Among NAHE-based Supersymmetric and Nonsupersymmetric String Vacua,” Phys. Lett. B **683** (2010) 314 [arXiv:0911.5125 [hep-th]].
- [18] C. Angelantonj, M. Cardella, S. Elitzur and E. Rabinovici, “Vacuum stability, string density of states and the Riemann zeta function,” JHEP **1102**, 024 (2011) [arXiv:1012.5091 [hep-th]].
- [19] O. Bergman and M. R. Gaberdiel, “A Nonsupersymmetric open string theory and S duality,” Nucl. Phys. B **499**, 183 (1997) [hep-th/9701137]; “Dualities of type 0 strings,” JHEP **9907**, 022 (1999) [hep-th/9906055];  
R. Blumenhagen and A. Kumar, “A Note on orientifolds and dualities of type 0B string theory,” Phys. Lett. B **464**, 46 (1999) [hep-th/9906234].
- [20] J. D. Blum and K. R. Dienes, “Duality without supersymmetry: The Case of the  $SO(16) \times SO(16)$  string,” Phys. Lett. B **414**, 260 (1997) [hep-th/9707148].
- [21] J. D. Blum and K. R. Dienes, “Strong / weak coupling duality relations for nonsupersymmetric string theories,” Nucl. Phys. B **516**, 83 (1998) [hep-th/9707160].
- [22] A. E. Faraggi and M. Tsulaia, “On the Low Energy Spectra of the Nonsupersymmetric Heterotic String Theories,” Eur. Phys. J. C **54**, 495 (2008) [arXiv:0706.1649 [hep-th]].

- [23] K. R. Dienes, “Statistics on the heterotic landscape: Gauge groups and cosmological constants of four-dimensional heterotic strings,” *Phys. Rev. D* **73**, 106010 (2006) [hep-th/0602286].
- [24] K. R. Dienes, M. Lennek and M. Sharma, “Strings at Finite Temperature: Wilson Lines, Free Energies, and the Thermal Landscape,” *Phys. Rev. D* **86**, 066007 (2012) [arXiv:1205.5752 [hep-th]].
- [25] E. Alvarez and M. A. R. Osorio, “Cosmological Constant Versus Free Energy For Heterotic Strings,” *Nucl. Phys. B* **304**, 327 (1988) [Erratum-ibid. *B* **309**, 220 (1988)]; “Duality Is An Exact Symmetry Of String Perturbation Theory,” *Phys. Rev. D* **40**, 1150 (1989);  
M. A. R. Osorio, “Quantum fields versus strings at finite temperature,” *Int. J. Mod. Phys. A* **7**, 4275 (1992).
- [26] J. J. Atick and E. Witten, “The Hagedorn Transition And The Number Of Degrees Of Freedom Of String Theory,” *Nucl. Phys. B* **310**, 291 (1988).
- [27] M. McGuigan, “Finite Temperature String Theory And Twisted Tori,” *Phys. Rev. D* **38**, 552 (1988);  
I. Antoniadis and C. Kounnas, “Superstring phase transition at high temperature,” *Phys. Lett. B* **261**, 369 (1991);  
I. Antoniadis, J. P. Derendinger and C. Kounnas, “Non-perturbative temperature instabilities in  $N = 4$  strings,” *Nucl. Phys. B* **551**, 41 (1999) [arXiv:hep-th/9902032].
- [28] C. Kounnas and B. Rostand, “Coordinate Dependent Compactifications And Discrete Symmetries,” *Nucl. Phys. B* **341**, 641 (1990).
- [29] M. J. Bowick and L. C. R. Wijewardhana, “Superstrings At High Temperature,” *Phys. Rev. Lett.* **54**, 2485 (1985);  
S. H. H. Tye, “The Limiting Temperature Universe And Superstring,” *Phys. Lett. B* **158**, 388 (1985);  
B. Sundborg, “Thermodynamics Of Superstrings At High-Energy Densities,” *Nucl. Phys. B* **254**, 583 (1985);  
E. Alvarez, “Strings At Finite Temperature,” *Nucl. Phys. B* **269**, 596 (1986);  
E. Alvarez and M. A. R. Osorio, “Superstrings At Finite Temperature,” *Phys. Rev. D* **36**, 1175 (1987);  
“Thermal Heterotic Strings,” *Physica A* **158**, 449 (1989) [Erratum-ibid. *A* **160**, 119 (1989)]; “Thermal Strings In Nontrivial Backgrounds,” *Phys. Lett. B* **220**, 121 (1989);  
M. Axenides, S. D. Ellis and C. Kounnas, “Universal Behavior Of D-Dimensional Superstring Models,” *Phys. Rev. D* **37**, 2964 (1988);  
Y. Leblanc, “Cosmological Aspects Of The Heterotic String Above The Hagedorn Temperature,” *Phys. Rev. D* **38**, 3087 (1988);  
B. A. Campbell, J. R. Ellis, S. Kalara, D. V. Nanopoulos and K. A. Olive, “Phase Transitions In QCD And String Theory,” *Phys. Lett. B* **255**, 420 (1991).
- [30] S. Ferrara, C. Kounnas and M. Porrati, “Superstring Solutions With Spontaneously Broken Four-dimensional Supersymmetry,” *Nucl. Phys. B* **304**, 500 (1988).
- [31] S. Ferrara, C. Kounnas and M. Porrati, “ $N = 1$  Superstrings With Spontaneously Broken Symmetries,” *Phys. Lett. B* **206**, 25 (1988).
- [32] S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, “Superstrings with Spontaneously Broken Supersymmetry and their Effective Theories,” *Nucl. Phys. B* **318**, 75 (1989).
- [33] E. Kiritsis and C. Kounnas, “Perturbative and nonperturbative partial supersymmetry breaking:  $N=4 \rightarrow N=2 \rightarrow N=1$ ,” *Nucl. Phys. B* **503**, 117 (1997) [hep-th/9703059].
- [34] E. Dudas and J. Mourad, “Brane solutions in strings with broken supersymmetry and dilaton tadpoles,” *Phys. Lett. B* **486**, 172 (2000) [hep-th/0004165].



- [35] C. A. Scrucca and M. Serone, “On string models with Scherk-Schwarz supersymmetry breaking,” *JHEP* **0110**, 017 (2001) [hep-th/0107159].
- [36] M. Borunda, M. Serone and M. Trapletti, “On the quantum stability of IIB orbifolds and orientifolds with Scherk-Schwarz SUSY breaking,” *Nucl. Phys. B* **653**, 85 (2003) [hep-th/0210075].
- [37] C. Angelantonj, M. Cardella and N. Irges, “An Alternative for Moduli Stabilisation,” *Phys. Lett. B* **641**, 474 (2006) [hep-th/0608022].
- [38] J. Scherk and J. H. Schwarz, “Spontaneous Breaking of Supersymmetry Through Dimensional Reduction,” *Phys. Lett. B* **82**, 60 (1979).
- [39] D. Lust, “Compactification Of The  $O(16) \times O(16)$  Heterotic String Theory,” *Phys. Lett. B* **178**, 174 (1986).
- [40] W. Lerche, D. Lust and A. N. Schellekens, “Ten-dimensional Heterotic Strings From Niemeier Lattices,” *Phys. Lett. B* **181**, 71 (1986).
- [41] W. Lerche, D. Lust and A. N. Schellekens, “Chiral Four-Dimensional Heterotic Strings from Selfdual Lattices,” *Nucl. Phys. B* **287**, 477 (1987).
- [42] A. H. Chamseddine, J. P. Derendinger and M. Quiros, “Nonsupersymmetric Four-dimensional Strings,” *Nucl. Phys. B* **311**, 140 (1988).
- [43] A. Font and A. Hernandez, “Nonsupersymmetric orbifolds,” *Nucl. Phys. B* **634**, 51 (2002) [hep-th/0202057].
- [44] M. Blaszczyk, S. Groot Nibbelink, O. Loukas and S. Ramos-Sanchez, “Non-supersymmetric heterotic model building,” *JHEP* **1410**, 119 (2014) [arXiv:1407.6362 [hep-th]].
- [45] C. Angelantonj, I. Florakis and M. Tsulaia, “Universality of Gauge Thresholds in Non-Supersymmetric Heterotic Vacua,” *Phys. Lett. B* **736** (2014) 365 [arXiv:1407.8023 [hep-th]].
- [46] C. Bachas, “A Way to break supersymmetry,” hep-th/9503030;  
J. G. Russo and A. A. Tseytlin, “Magnetic flux tube models in superstring theory,” *Nucl. Phys. B* **461**, 131 (1996) [hep-th/9508068];  
A. A. Tseytlin, “Closed superstrings in magnetic field: Instabilities and supersymmetry breaking,” *Nucl. Phys. Proc. Suppl.* **49**, 338 (1996) [hep-th/9510041];  
H. P. Nilles and M. Spalinski, “Generalized string compactifications with spontaneously broken supersymmetry,” *Phys. Lett. B* **392**, 67 (1997) [hep-th/9606145];  
I. Shah and S. Thomas, “Finite soft terms in string compactifications with broken supersymmetry,” *Phys. Lett. B* **409**, 188 (1997) [hep-th/9705182].
- [47] A. Sagnotti, “Some properties of open string theories,” In \*Palaiseau 1995, Susy 95\* 473-484 [hep-th/9509080].
- [48] A. Sagnotti, “Surprises in open string perturbation theory,” *Nucl. Phys. Proc. Suppl.* **56B**, 332 (1997) [hep-th/9702093].
- [49] C. Angelantonj, “Nontachyonic open descendants of the 0B string theory,” *Phys. Lett. B* **444**, 309 (1998) [hep-th/9810214].
- [50] R. Blumenhagen, A. Font and D. Lust, “Tachyon free orientifolds of type 0B strings in various dimensions,” *Nucl. Phys. B* **558**, 159 (1999) [hep-th/9904069].
- [51] S. Sugimoto, “Anomaly cancellations in type I D-9 - anti-D-9 system and the  $USp(32)$  string theory,” *Prog. Theor. Phys.* **102**, 685 (1999) [hep-th/9905159].

- [52] G. Aldazabal, L. E. Ibanez and F. Quevedo, “Standard - like models with broken supersymmetry from type I string vacua,” *JHEP* **0001**, 031 (2000) [hep-th/9909172].
- [53] C. Angelantonj, “Nonsupersymmetric open string vacua,” *PoS trieste* **99**, 015 (1999) [hep-th/9907054].
- [54] K. Forger, “On nontachyonic  $Z(N) \times Z(M)$  orientifolds of type 0B string theory,” *Phys. Lett. B* **469**, 113 (1999) [hep-th/9909010].
- [55] S. Moriyama, “USp(32) string as spontaneously supersymmetry broken theory,” *Phys. Lett. B* **522**, 177 (2001) [hep-th/0107203].
- [56] C. Angelantonj and I. Antoniadis, “Suppressing the cosmological constant in nonsupersymmetric type I strings,” *Nucl. Phys. B* **676**, 129 (2004) [hep-th/0307254].
- [57] C. Angelantonj, “Open strings and supersymmetry breaking,” *AIP Conf. Proc.* **751**, 3 (2005) [hep-th/0411085].
- [58] E. Dudas and C. Timirgaziu, “Nontachyonic Scherk-Schwarz compactifications, cosmology and moduli stabilization,” *JHEP* **0403**, 060 (2004) [hep-th/0401201].
- [59] B. Gato-Rivera and A. N. Schellekens, “Non-supersymmetric Tachyon-free Type-II and Type-I Closed Strings from RCFT,” *Phys. Lett. B* **656**, 127 (2007) [arXiv:0709.1426 [hep-th]].
- [60] I. Antoniadis, C. Bachas, D. C. Lewellen and T. N. Tomaras, “On Supersymmetry Breaking in Superstrings,” *Phys. Lett. B* **207**, 441 (1988).
- [61] I. Antoniadis, “A Possible new dimension at a few TeV,” *Phys. Lett. B* **246**, 377 (1990).
- [62] I. Antoniadis, C. Munoz and M. Quiros, “Dynamical supersymmetry breaking with a large internal dimension,” *Nucl. Phys. B* **397**, 515 (1993) [hep-ph/9211309].
- [63] I. Antoniadis and M. Quiros, “Large radii and string unification,” *Phys. Lett. B* **392**, 61 (1997) [hep-th/9609209].
- [64] K. Benakli, “Phenomenology of low quantum gravity scale models,” *Phys. Rev. D* **60**, 104002 (1999) [hep-ph/9809582].
- [65] C. P. Bachas, “Scales of string theory,” *Class. Quant. Grav.* **17**, 951 (2000) [hep-th/0001093].
- [66] E. Dudas, “Theory and phenomenology of type I strings and M theory,” *Class. Quant. Grav.* **17**, R41 (2000) [hep-ph/0006190].
- [67] W. Fischler and L. Susskind, “Dilaton Tadpoles, String Condensates and Scale Invariance,” *Phys. Lett. B* **171**, 383 (1986).
- [68] W. Fischler and L. Susskind, “Dilaton Tadpoles, String Condensates and Scale Invariance. 2.,” *Phys. Lett. B* **173**, 262 (1986).
- [69] R. Hagedorn, “Statistical Thermodynamics Of Strong Interactions At High-Energies,” *Nuovo Cim. Suppl.* **3**, 147 (1965).
- [70] G. H. Hardy and S. Ramanujan, *Proc. London Math. Soc.* **17**, 75 (1918);  
I. Kani and C. Vafa, “Asymptotic Mass Degeneracies In Conformal Field Theories,” *Commun. Math. Phys.* **130**, 529 (1990).
- [71] K. R. Dienes, “Solving the hierarchy problem without supersymmetry or extra dimensions: An Alternative approach,” *Nucl. Phys. B* **611**, 146 (2001) [hep-ph/0104274].

- [72] H. Kawai, D. C. Lewellen and S. H. H. Tye, “Classification Of Closed Fermionic String Models,” *Phys. Rev. D* **34**, 3794 (1986).
- [73] I. Antoniadis, J. R. Ellis, J. S. Hagelin and D. V. Nanopoulos, “The Flipped  $SU(5) \times U(1)$  String Model Revamped,” *Phys. Lett. B* **231**, 65 (1989).
- [74] I. Antoniadis, G. K. Leontaris and J. Rizos, “A Three generation  $SU(4) \times O(4)$  string model,” *Phys. Lett. B* **245**, 161 (1990).
- [75] A. E. Faraggi, “Hierarchical top - bottom mass relation in a superstring derived standard - like model,” *Phys. Lett. B* **274**, 47 (1992).
- [76] A. E. Faraggi, “A New standard - like model in the four-dimensional free fermionic string formulation,” *Phys. Lett. B* **278**, 131 (1992).
- [77] A. E. Faraggi, “Construction of realistic standard - like models in the free fermionic superstring formulation,” *Nucl. Phys. B* **387**, 239 (1992) [hep-th/9208024].
- [78] A. E. Faraggi, “Custodial nonAbelian gauge symmetries in realistic superstring derived models,” *Phys. Lett. B* **339**, 223 (1994) [hep-ph/9408333].
- [79] K. R. Dienes and A. E. Faraggi, “Gauge coupling unification in realistic free fermionic string models,” *Nucl. Phys. B* **457**, 409 (1995) [hep-th/9505046].
- [80] H. Kawai, D. C. Lewellen and S. H. H. Tye, “Construction of Fermionic String Models in Four-Dimensions,” *Nucl. Phys. B* **288**, 1 (1987).
- [81] I. Antoniadis, C. P. Bachas and C. Kounnas, “Four-Dimensional Superstrings,” *Nucl. Phys. B* **289**, 87 (1987).
- [82] H. Kawai, D. C. Lewellen, J. A. Schwartz and S. H. H. Tye, “The Spin Structure Construction of String Models and Multiloop Modular Invariance,” *Nucl. Phys. B* **299**, 431 (1988).
- [83] A. H. Chamseddine, J. P. Derendinger and M. Quiros, “A Unified Formalism for Strings in Four-dimensions,” *Nucl. Phys. B* **326**, 497 (1989).
- [84] S. E. M. Nooij, “Classification of the chiral  $Z(2) \times Z(2)$  heterotic string models,” hep-th/0603035.
- [85] H. Murayama, Y. Nomura, S. Shirai and K. Tobioka, “Compact Supersymmetry,” *Phys. Rev. D* **86**, 115014 (2012) [arXiv:1206.4993 [hep-ph]].
- [86] S. Dimopoulos, K. Howe and J. March-Russell, “Maximally Natural Supersymmetry,” *Phys. Rev. Lett.* **113**, 111802 (2014) [arXiv:1404.7554 [hep-ph]];  
I. G. Garcia and J. March-Russell, “Rare Flavor Processes in Maximally Natural Supersymmetry,” arXiv:1409.5669 [hep-ph].
- [87] Y. Shadmi and P. Z. Szabo, “Flavored Gauge-Mediation,” *JHEP* **1206**, 124 (2012) [arXiv:1103.0292 [hep-ph]];  
N. Craig, M. McCullough and J. Thaler, “Flavor Mediation Delivers Natural SUSY,” *JHEP* **1206**, 046 (2012) [arXiv:1203.1622 [hep-ph]].
- [88] R. Davies, J. March-Russell and M. McCullough, “A Supersymmetric One Higgs Doublet Model,” *JHEP* **1104**, 108 (2011) [arXiv:1103.1647 [hep-ph]].
- [89] Our conventions are as in J. Polchinski, “String Theory”, Vols. I and II, 1998.  
Many results may also be found in:  
E. Kiritsis, “String Theory in a Nutshell”, 2007 Princeton, NJ: Princeton University Press.  
Additional results for Weierstrass  $P$ -function integrals were derived using identities in:

- S. A. Abel and B. W. Schofield, “One-loop Yukawas on intersecting branes,” JHEP **0506**, 072 (2005) [hep-th/0412206];  
S. A. Abel and M. D. Goodsell, “Intersecting brane worlds at one loop,” JHEP **0602**, 049 (2006) [hep-th/0512072].
- [90] E. Caceres, V. S. Kaplunovsky and I. M. Mandelberg, “Large volume string compactifications, revisited,” Nucl. Phys. B **493**, 73 (1997) doi:10.1016/S0550-3213(97)00129-6 [hep-th/9606036].
- [91] E. Kiritsis, C. Kounnas, P. M. Petropoulos and J. Rizos, “Solving the decompactification problem in string theory,” Phys. Lett. B **385** (1996) 87 [hep-th/9606087].
- [92] E. Kiritsis, C. Kounnas, P. M. Petropoulos and J. Rizos, “String threshold corrections in models with spontaneously broken supersymmetry,” Nucl. Phys. B **540**, 87 (1999) [hep-th/9807067].
- [93] I. Antoniadis and K. Benakli, “Large dimensions and string physics in future colliders,” Int. J. Mod. Phys. A **15**, 4237 (2000) [hep-ph/0007226].
- [94] A. E. Faraggi, C. Kounnas and H. Partouche, “Large volume susy breaking with a chiral solution to the decompactification problem,” arXiv:1410.6147 [hep-th].
- [95] K. R. Dienes, E. Dudas and T. Gherghetta, “Extra space-time dimensions and unification,” Phys. Lett. B **436**, 55 (1998) [hep-ph/9803466]; “Grand unification at intermediate mass scales through extra dimensions,” Nucl. Phys. B **537**, 47 (1999) [hep-ph/9806292]; “TeV scale GUTs,” hep-ph/9807522.
- [96] S. Abel, K. R. Dienes and E. Mavroudi, to appear
- [97] S. Abel, K. R. Dienes and E. Mavroudi, in preparation