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Fermion mass and mixing pattern in a minimal T7 flavor 331 model.

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We present a model based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge symmetry having an extra $T_7 \otimes Z_2 \otimes Z_3 \otimes Z_{14}$ flavor group, which successfully describes the observed SM fermion mass and mixing pattern. In this framework, the light active neutrino masses arise via double seesaw mechanism and the observed charged fermion mass and quark mixing hierarchy is a consequence of the $Z_2 \otimes Z_3 \otimes Z_{14}$ symmetry breaking at very high energy. In our minimal T_7 flavor 331 model, the spectrum of neutrinos includes very light active neutrinos as well as heavy and very heavy sterile neutrinos. The obtained physical observables for both quark and lepton sectors are compatible with their experimental values. The model predicts the absence of CP violation in neutrino oscillations.

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†A footnote may follow.

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1. Introduction

Despite its big experimental success, the Standard Model (SM) has several unexplained features. Some of them are the Dark Matter problem, the fermion mass and mixing hierarchy and the neutrino oscillations. The lack of predictivity of the Standard Model Yukawa sector, motivates to consider extensions of the Standard Model aimed to address its flavor puzzle. Discrete flavor symmetries are important because they generate fermion textures useful to explain the three generation flavor structure. Very recently, discrete groups such as, for example A_4 [2,3], S_4 [4], S_3 [5–8], T_7 [9, 10] and so forth, have been implemented in several extensions of the SM, to explain the observed pattern of fermion masses and mixings.

Furthermore, another unaswered issue in particle physics is the existence of three generations of fermions at low energies. The mixing patterns of leptons and quarks are significantly different; while in the quark sector, the mixing angles are small, in the lepton sector two of the mixing angles are large and one is small. Models having $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ as a gauge symmetry, are vectorlike with three fermion generations and thus do not contain anomalies [11, 12]. Defining the electric charge as the linear combination of the T_3 and T_8 $SU(3)_L$ generators, we have that it is a free parameter, which does not depend on the anomalies (β). The charge of the exotic particles is defined by setting a value for the β parameter. Setting $\beta = -\frac{1}{\sqrt{3}}$, implies that the third component of the weak lepton triplet is a neutral field ν_R^C allowing to build the Dirac Yukawa term with the usual field ν_L of the weak doublet. The 331 models with $\beta = -\frac{1}{\sqrt{3}}$ provide an alternative neutrino mass generation mechanism and include in their neutrino spectrum light active sub-eV scale neutrinos as well as sterile neutrinos which could be dark matter candidates if they are light enough or candidates for detection at the LHC, if they have TeV scale masses. Having TeV scale sterile neutrinos in its neutrino spectrum, makes the 331 models very important since if these sterile neutrinos are detected at the LHC, these models can shed light in the understanding of the electroweak symmetry breaking mechanism.

In this paper we formulate an extension of the minimal $SU(3)_C \times SU(3)_L \times U(1)_X$ model with $\beta = -\frac{1}{\sqrt{3}}$, where an extra $T_7 \otimes Z_2 \otimes Z_3 \otimes Z_{14}$ discrete group extends the symmetry of the model and very heavy extra scalar fields are added with the aim to generate viable and predictive textures for the fermion sector that successfully describe the fermion mass and mixing pattern.

2. The model

We consider a $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes T_7 \otimes Z_2 \otimes Z_3 \otimes Z_{14}$ model where the full symmetry $\mathcal{G}$ is spontaneously broken in three steps as follows:

$$\begin{aligned}\mathcal{G} &= SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes T_7 \otimes Z_2 \otimes Z_3 \otimes Z_{14} \xrightarrow{\Lambda_{int}} \\ &SU(3)_C \otimes SU(3)_L \otimes U(1)_X \xrightarrow{v_\chi} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{v_\eta, v_\rho} \\ &SU(3)_C \otimes U(1)_Q, \end{aligned} \tag{2.1}$$

where the hierarchy $v_\eta, v_\rho \ll v_\chi \ll \Lambda_{int}$ among the symmetry breaking scales is fullfilled.

The electric charge in our 331 model is defined as:

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + XI, \tag{2.2}$$

PoS(PLANCK 2015)023

where T_3 and T_8 are the $SU(3)_L$ diagonal generators and I is the 3×3 identity matrix.

The anomaly cancellation requirement implies that quarks are unified in the following $(SU(3)_C, SU(3)_L, U(1)_X)$ left- and right-handed representations:

$$Q_L^{1,2} = \begin{pmatrix} D^{1,2} \\ -U^{1,2} \\ J^{1,2} \end{pmatrix}_L : (3,3^*,0), \qquad Q_L^3 = \begin{pmatrix} U^3 \\ D^3 \\ T \end{pmatrix}_L : (3,3,1/3), \tag{2.3}$$

$$\begin{array}{ll} D_R^{1,2,3} : (3^*,1,-1/3), & U_R^{1,2,3} : (3^*,1,2/3), \\ J_R^{1,2} : (3^*,1,-1/3), & T_R : (3^*,1,2/3). \end{array} \tag{2.4}$$

Here U_L^i and D_L^i $(i=1,2,3)$ are the left handed up- and down-type quarks in the flavor basis. The right handed SM quarks U_R^i and D_R^i $(i=1,2,3)$ and right handed exotic quarks T_R and $J_R^{1,2}$ are assigned into $SU(3)_L$ singlets representations, so that their $U(1)_X$ quantum numbers correspond to their electric charges.

Furthermore, cancellation of anomalies implies that leptons are grouped in the following $(SU(3)_C, SU(3)_L, U(1)_X)$ left- and right-handed representations:

$$L_L^{1,2,3} = \begin{pmatrix} \nu^{1,2,3} \\ e^{1,2,3} \\ (\nu^{1,2,3})^c \end{pmatrix}_L : (1,3,-1/3), \tag{2.5}$$

$$\begin{array}{lll} e_R : (1,1,-1), & \mu_R : (1,1,-1), & \tau_R : (1,1,-1), \\ N_R^1 : (1,1,0), & N_R^2 : (1,1,0), & N_R^3 : (1,1,0). \end{array} \tag{2.6}$$

where ν_L^i and e_L^i (e_L, μ_L, τ_L) are the neutral and charged lepton families, respectively. Let's note that we assign the right-handed leptons as $SU(3)_L$ singlets.The exotic leptons of the model are: three neutral Majorana leptons $(\nu^{1,2,3})_L^c$ and three right-handed Majorana leptons $N_R^{1,2,3}$.

The scalar sector the 331 models includes: three 3's irreps of $SU(3)_L$, where one triplet χ gets a TeV scale vaccuum expectation value (VEV) v_χ, that breaks the $SU(3)_L \times U(1)_X$ symmetry down to $SU(2)_L \times U(1)_Y$, thus generating the masses of non SM fermions and non SM gauge bosons; and two light triplets η and ρ acquiring electroweak scale VEVs v_η and v_ρ, respectively and thus providing masses for the fermions and gauge bosons of the SM.

Regarding the scalar sector of the minimal 331 model, we assign the scalar fields in the following $[SU(3)_L, U(1)_X]$ representations:

$$\chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(\upsilon_\chi + \xi_\chi \pm i\zeta_\chi) \end{pmatrix} : (3,-1/3), \qquad \rho = \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(\upsilon_\rho + \xi_\rho \pm i\zeta_\rho) \\ \rho_3^+ \end{pmatrix} : (3,2/3),$$
$$\eta = \begin{pmatrix} \frac{1}{\sqrt{2}}(\upsilon_\eta + \xi_\eta \pm i\zeta_\eta) \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} : (3,-1/3). \tag{2.7}$$

We extend the scalar sector of the minimal 331 model by adding the following eleven very heavy $SU(3)_L$ scalar singlets:

$$\sigma \sim (1,0), \qquad \tau \sim (1,0), \quad \xi_j : (1,0), \quad \zeta_j : (1,0), \quad S_j : (1,0), \quad j=1,2,3. \tag{2.8}$$

We assign the scalars into T_7 triplet, T_7 antitriplet and T_7 singlet represention s. The $T_7\otimes Z_2\otimes Z_3\otimes Z_{14}$ assignments of the scalar fields are:

$$\begin{aligned}&\eta\sim\left(\mathbf{1}_0,1,e^{\frac{2\pi i}{3}},1\right),\quad \rho\sim\left(\mathbf{1}_0,1,e^{-\frac{2\pi i}{3}},1\right),\quad \chi\sim(\mathbf{1}_0,1,1,1),\quad \tau\sim(\mathbf{1}_1,-1,1,1),\\ &\xi\sim\left(\mathbf{3},1,e^{\frac{2\pi i}{3}},1\right),\quad \zeta\sim\left(\overline{\mathbf{3}},1,1,1\right),\quad S\sim\left(\mathbf{3},1,e^{-\frac{2\pi i}{3}},1\right),\quad \sigma\sim\left(\mathbf{1}_0,1,1,e^{-\frac{i\pi}{7}}\right).\end{aligned}\tag{2.9}$$

In the concerning to the lepton sector, we have the following $T_7\otimes Z_2\otimes Z_3\otimes Z_{14}$ assignments:

$$\begin{aligned}&L_L\sim\left(\mathbf{3},1,e^{\frac{2\pi i}{3}},1\right),\quad e_R\sim\left(\mathbf{1}_0,1,e^{\frac{2\pi i}{3}},-1\right),\quad \mu_R\sim\left(\mathbf{1}_1,1,e^{\frac{2\pi i}{3}},e^{\frac{4i\pi}{7}}\right),\\ &\tau_R\sim\left(\mathbf{1}_2,1,e^{\frac{2\pi i}{3}},e^{\frac{2i\pi}{7}}\right),\quad N_R\sim\left(\mathbf{3},1,e^{\frac{2\pi i}{3}},1\right),\end{aligned}\tag{2.10}$$

while the $T_7\otimes Z_2\otimes Z_3\otimes Z_{14}$ assignments for the quark sector are:

$$\begin{aligned}&Q_L^1\sim\left(\mathbf{1}_0,1,1,e^{\frac{2\pi i}{7}}\right),\quad Q_L^2\sim\left(\mathbf{1}_0,1,1,e^{\frac{\pi i}{7}}\right),\quad Q_L^3\sim(\mathbf{1}_0,1,1,1),\\ &U_R^1\sim\left(\mathbf{1}_0,1,e^{-\frac{2\pi i}{3}},e^{\frac{2\pi i}{7}}\right),\quad U_R^2\sim\left(\mathbf{1}_0,1,e^{-\frac{2\pi i}{3}},e^{\frac{\pi i}{7}}\right),\quad U_R^3\sim\left(\mathbf{1}_0,1,e^{-\frac{2\pi i}{3}},1\right),\\ &D_R^1\sim\left(\mathbf{1}_0,-1,e^{-\frac{2\pi i}{3}},e^{\frac{2\pi i}{7}}\right),\quad D_R^2\sim\left(\mathbf{1}_0,-1,e^{-\frac{2\pi i}{3}},e^{\frac{\pi i}{7}}\right),\quad D_R^3\sim\left(\mathbf{1}_0,-1,e^{-\frac{2\pi i}{3}},1\right),\\ &T_R\sim(\mathbf{1}_0,1,1,1),\quad J_R^1\sim\left(\mathbf{1}_0,1,1,e^{\frac{2\pi i}{7}}\right),\quad J_R^2\sim\left(\mathbf{1}_0,1,1,e^{\frac{\pi i}{7}}\right).\end{aligned}\tag{2.11}$$

Here the dimensions of the T_7 irreducible representations are specified by the numbers in boldface.

With the aforementioned field content of our model, the relevant quark and lepton Yukawa terms invariant under the group $\mathscr{G}$, take the form:

$$\begin{aligned}-\mathscr{L}_Y^{(Q)}=\;&y_{11}^{(U)}\overline{Q}_L^1\rho^*U_R^1\frac{\sigma^4}{\Lambda^4}+y_{12}^{(U)}\overline{Q}_L^1\rho^*U_R^2\frac{\sigma^3}{\Lambda^3}+y_{21}^{(U)}\overline{Q}_L^2\rho^*U_R^1\frac{\sigma^3}{\Lambda^3}+y_{22}^{(U)}\overline{Q}_L^2\rho^*U_R^2\frac{\sigma^2}{\Lambda^2}\\ &+y_{13}^{(U)}\overline{Q}_L^1\rho^*U_R^3\frac{\sigma^2}{\Lambda^2}+y_{31}^{(U)}\overline{Q}_L^3\eta U_R^1\frac{\sigma^2}{\Lambda^2}+y_{23}^{(U)}\overline{Q}_L^2\rho^*U_R^3\frac{\sigma}{\Lambda}+y_{32}^{(U)}\overline{Q}_L^3\eta U_R^2\frac{\sigma}{\Lambda}\\ &+y_{33}^{(U)}\overline{Q}_L^3\eta U_R^3+y^{(T)}\overline{Q}_L^3\chi T_R+y_1^{(J)}\overline{Q}_L^1\chi^*J_R^1+y_2^{(J)}\overline{Q}_L^2\chi^*J_R^2+y_{33}^{(D)}\overline{Q}_L^3\rho D_R^3\frac{\tau^3}{\Lambda^3}\\ &+y_{11}^{(D)}\overline{Q}_L^1\eta^*D_R^1\frac{\sigma^4\tau^3}{\Lambda^7}+y_{12}^{(D)}\overline{Q}_L^1\eta^*D_R^2\frac{\sigma^3\tau^3}{\Lambda^6}+y_{21}^{(D)}\overline{Q}_L^2\eta^*D_R^1\frac{\sigma^3\tau^3}{\Lambda^6}+y_{22}^{(D)}\overline{Q}_L^2\eta^*D_R^2\frac{\sigma^2\tau^3}{\Lambda^5}\\ &+y_{13}^{(D)}\overline{Q}_L^1\eta^*D_R^3\frac{\sigma^2\tau^3}{\Lambda^5}+y_{31}^{(D)}\overline{Q}_L^3\rho D_R^1\frac{\sigma^2\tau^3}{\Lambda^5}+y_{23}^{(D)}\overline{Q}_L^2\eta^*D_R^3\frac{\sigma\tau^3}{\Lambda}+y_{32}^{(D)}\overline{Q}_L^3\rho D_R^2\frac{\sigma\tau^3}{\Lambda}+H.c,\end{aligned}\tag{2.12}$$

$$\begin{aligned}-\mathscr{L}_Y^{(L)}=\;&h_{\rho e}^{(L)}\left(\overline{L}_L\rho\xi\right)_{\mathbf{1}_0}e_R\frac{\sigma^7}{\Lambda^8}+h_{\rho\mu}^{(L)}\left(\overline{L}_L\rho\xi\right)_{\mathbf{1}_2}\mu_R\frac{\sigma^4}{\Lambda^5}+h_{\rho\tau}^{(L)}\left(\overline{L}_L\rho\xi\right)_{\mathbf{1}_1}\tau_R\frac{\sigma^2}{\Lambda^3}\\ &+h_\chi^{(L)}\left(\overline{L}_L\chi N_R\right)_{\mathbf{1}_0}+\frac{1}{2}h_{1N}\left(\overline{N}_RN_R^C\right)_{\mathbf{3}}\xi^*+h_{2N}\left(\overline{N}_RN_R^C\right)_{\overline{\mathbf{3}}}S\\ &+h_\rho\varepsilon_{abc}\left(\overline{L}_L^a\left(L_L^C\right)^b\right)_{\mathbf{3}}\rho^c\frac{\zeta}{\Lambda}+H.c,\end{aligned}\tag{2.13}$$

where $y_{ij}^{(U,D)}$ $(i,j=1,2,3)$, $h_{\rho e}^{(L)}$, $h_{\rho\mu}^{(L)}$, $h_{\rho\tau}^{(L)}$, $h_\chi^{(L)}$, h_{1N}, h_{2N} and h_ρ are $\mathscr{O}(1)$ dimensionless couplings.

In the following we explain the role each discrete group factors of our model. The T_7 and Z_3 discrete groups reduce the number of the $SU(3)_C\otimes SU(3)_L\otimes U(1)_X$ model parameters. We use

T_7 since it is the minimal non-Abelian discrete group having a complex triplet [13], where the three fermion generations can be naturally unified. The Z_3 symmetry determines the allowed entries of the neutrino mass matrix and forbids mixings between SM quarks and exotic quarks. The Z_2 symmetry is responsible for the mass hierarchy between SM up and SM down type quarks. The Z_{14} symmetry generates the hierarchy among charged fermion masses and quark mixing angles that yields the observed charged fermion mass and quark mixing pattern. We use Z_{14} because it is the smallest lowest cyclic symmetry, that allows to build a twelve dimensional charged lepton Yukawa term crucial to explain the smallness of the electron mass, without tuning its corresponding Yukawa coupling.

To get a predictive model that successfully accounts for fermion masses and mixings, we assume that the $SU(3)_L$ singlet scalars have the following VEV pattern:

$$\langle\sigma\rangle = v_\sigma e^{i\phi}, \quad \langle\tau\rangle = v_\tau, \quad \langle\xi\rangle = \frac{v_\xi}{\sqrt{3}}(1,1,1), \quad \langle\zeta\rangle = \frac{v_\zeta}{\sqrt{2}}(1,0,1), \quad \langle S\rangle = \frac{v_S}{\sqrt{3}}(1,1,-1). \tag{2.14}$$

which we have checked to be consistent with the scalar potential minimization equations.

Besides that, the $SU(3)_L$ scalar singlets are assumed to acquire vacuum expectation values at a very high energy $\Lambda_{int} \gg v_\chi \approx \mathcal{O}(1)$ TeV, excepting ζ_j ($j = 1,2,3$), whose vacuum expectation value is much lower than the scale of electroweak symmetry breaking $v = 246$ GeV. Let's note that at the scale Λ_{int}, the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes T_7 \otimes Z_2 \otimes Z_3 \otimes Z_{14}$ symmetry is broken to $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ by the vacuum expectation values of the $SU(3)_L$ singlet scalar fields ξ_j, S_j, σ and τ.

Considering that the charged fermion mass and quark mixing pattern arises from the $Z_2 \otimes Z_3 \otimes Z_{14}$ symmetry breaking, we set the VEVs of the $SU(3)_L$ singlet scalars S, ξ, σ and τ, as follows:

$$v_S \sim v_\xi = v_\sigma = v_\tau = \Lambda_{int} = \lambda\Lambda, \tag{2.15}$$

being $\lambda = 0.225$ one of the Wolfenstein parameters and Λ our model cutoff. Consequently, the VEVs of the scalars in our model have the following hierarchy:

$$v_\zeta << v_\rho \sim v_\eta \sim v << v_\chi << \Lambda_{int}. \tag{2.16}$$

3. Lepton masses and mixings

From Eq. (2.13), it follows that the mass matrix for charged leptons is [9]:

$$M_l = V_{lL}^\dagger P_l \, diag\left(m_e, m_\mu, m_\tau\right), \quad V_{lL} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad P_l = \begin{pmatrix} e^{7i\phi} & 0 & 0 \\ 0 & e^{4i\phi} & 0 \\ 0 & 0 & e^{2i\phi} \end{pmatrix}, \quad \omega = e^{\frac{2\pi i}{3}}, \tag{3.1}$$

where the charged lepton masses read:

$$m_e = h_{\rho e}^{(L)}\lambda^8 \frac{v_\rho}{\sqrt{2}}, \qquad m_\mu = h_{\rho\mu}^{(L)}\lambda^5 \frac{v_\rho}{\sqrt{2}}, \qquad m_\tau = h_{\rho\tau}^{(L)}\lambda^3 \frac{v_\rho}{\sqrt{2}}. \tag{3.2}$$

Taking into account that $v_\rho \approx v = 246$ GeV, it follows that the charged lepton masses are related with the electroweak symmetry breaking scale by their scalings with powers of the Wolfenstein parameter $\lambda = 0.225$, with $\mathcal{O}(1)$ coefficients.

PoS(PLANCK 2015)023

The neutrino mass matrix is given by [9]:

$$M_{\nu} = \begin{pmatrix} 0_{3\times 3} & M_D & 0_{3\times 3} \\ M_D^T & 0_{3\times 3} & M_{\chi} \\ 0_{3\times 3} & M_{\chi}^T & M_R \end{pmatrix}, \qquad M_D = \frac{h_{\rho} v_{\rho} v_{\zeta}}{2\Lambda} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad M_{\chi} = h_{\chi}^{(L)} \frac{v_{\chi}}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$M_R = h_{1N} \frac{v_{\xi}}{\sqrt{3}} \begin{pmatrix} 1 & -x & x \\ -x & 1 & x \\ x & x & 1 \end{pmatrix}, \qquad x = \frac{h_{2N} v_S}{h_{1N} v_{\xi}}. \tag{3.3}$$

Since the $SU(3)_L$ singlet scalars having Yukawa interactions with the right handed Majorana neutrinos acquire VEVs at very high scale, these Majorana neutrinos are very heavy, so that the active neutrinos get small masses via a double seesaw mechanism.

The neutrino mass matrices for the physical states are [14]:

$$M_{\nu}^{(1)} = M_D \left(M_{\chi}^T\right)^{-1} M_R M_{\chi}^{-1} M_D^T, \qquad M_{\nu}^{(2)} = -M_{\chi} M_R^{-1} M_{\chi}^T, \qquad M_{\nu}^{(3)} = M_R, \tag{3.4}$$

being $M_{\nu}^{(1)}$ the mass matrix for light active neutrinos, while $M_{\nu}^{(2)}$ and $M_{\nu}^{(3)}$ are the heavy and very heavy sterile neutrino mass matrices, respectively. Consequently, the double seesaw mechanism gives rise to light active neutrinos as well as to heavy and very heavy sterile neutrinos.

Using Eq. (3.4), we find the following mass matrix for light active neutrinos [9]:

$$M_{\nu}^{(1)} = \begin{pmatrix} A & 0 & A \\ 0 & B & 0 \\ A & 0 & A \end{pmatrix}, \qquad A = \frac{h_{1N} h_{\rho}^2 v_{\rho}^2 v_{\zeta}^2 v_{\xi}}{2\sqrt{3} h_{\chi}^{(L)} v_{\chi}^2 \Lambda^2}, \qquad B = \frac{h_{\rho}^2 v_{\rho}^2 v_{\zeta}^2}{\sqrt{3} h_{\chi}^{(L)} v_{\chi}^2 \Lambda^2} \left(h_{1N} v_{\xi} + h_{2N} v_S\right). \tag{3.5}$$

From Eq. (3.5) it follows that the light active neutrino mass matrix only depends on two effective parameters: A and B, which determine the neutrino mass squared splittings. Let's note that A and B are supressed by their scaling with inverse powers of the high energy cutoff Λ. Furthermore, we have that the smallness of the active neutrino masses arises from their scaling with inverse powers of the high energy cutoff Λ as well as from their quadratic dependence on the very small VEV of the $Z_2 \otimes Z_3 \otimes Z_{14}$ neutral, $SU(3)_L$ singlet and T_7 antitriplet scalar field ζ. From Eq. (3.5) and the relations $v_{\xi} = \lambda\Lambda$, $v_{\rho} \sim 100$ GeV, $v_{\chi} \sim 1$ TeV, we get that the mass scale for the light active neutrinos satisfies $m_{\nu} \sim 10^{-3} \frac{v_{\zeta}^2}{\Lambda}$. Consequently, setting $v_{\zeta} = 1$ GeV, we find for the cutoff of our model the estimate

$$\Lambda \sim 10^5 \text{ TeV}, \tag{3.6}$$

which is of the same order of magnitude of the cutoff of our S_3 lepton flavor 331 model [7]. Consequenty, we find that the heavy and very heavy sterile neutrinos have masses at the $\sim$ MeV and $\sim$ TeV scales, respectively. Then the MeV sterile neutrinos correspond to dark matter candidates. Furthermore, we assume that the lightest of the very heavy sterile neutrinos, i.e., $\xi_{1L}^{(3)}$ has a TeV scale mass and thus corresponds to a candidate for detection at the LHC.

Moreover, we find that the mass matrix $M_{\nu}^{(1)}$ for light active neutrinos is diagonalized by a rotation matrix V_{ν}, as follows:

$$V_{\nu}^T M_{\nu}^{(1)} V_{\nu} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \quad \text{with} \quad V_{\nu} = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}, \quad \theta = \pm\frac{\pi}{4}, \tag{3.7}$$

where $\theta = +\pi/4$ and $\theta = -\pi/4$ correspond to normal (NH) and inverted (IH) mass hierarchies, respectively. The masses for the light active neutrinos, in the cases of normal (NH) and inverted (IH) mass hierarchies, read:

$$\text{NH} : \theta = +\frac{\pi}{4} : \qquad m_{\nu_1} = 0, \qquad m_{\nu_2} = B, \qquad m_{\nu_3} = 2A, \tag{3.8}$$

$$\text{IH} : \theta = -\frac{\pi}{4} : \qquad m_{\nu_1} = 2A, \qquad m_{\nu_2} = B, \qquad m_{\nu_3} = 0. \tag{3.9}$$

Furthermore, we find that the lepton mixing angles are given by:

$$\sin^2\theta_{12} = \frac{1}{2 \mp \cos 5\phi}, \qquad \sin^2\theta_{13} = \frac{1}{3}(1 \pm \cos 5\phi), \qquad \sin^2\theta_{23} = \frac{1}{2} \pm \frac{\sqrt{3}\sin 5\phi}{4 \mp 2\cos 5\phi}. \tag{3.10}$$

Then, it follows that the limit $\phi = 0$ and $\phi = \pi$ for the inverted and normal neutrino mass hierarchies, respectively, correspond to the tribimaximal mixing, which predicts a vanishing reactor mixing angle. Let's note that the mixing angles for the lepton sector only depend on a single parameter (ϕ), while the neutrino mass squared splittings are controlled by two parameters, i.e., A and B. Furthermore, from the relation $\theta = \pm\frac{\pi}{4}$, we predict $J = 0$ and $\delta = 0$, which implies that our model predicts a vanishing leptonic Dirac CP violating phase.

The parameters A and B for the normal (NH) and inverted (IH) neutrino mass hierarchies read:

$$\text{NH}: \ m_{\nu_1} = 0, \quad m_{\nu_2} = B = \sqrt{\Delta m_{21}^2} \approx 9\text{meV}, \quad m_{\nu_3} = 2A = \sqrt{\Delta m_{31}^2} \approx 50\text{meV}; \tag{3.11}$$

$$\text{IH} \ : \ m_{\nu_2} = B = \sqrt{\Delta m_{21}^2 + \Delta m_{13}^2} \approx 50\text{meV}, \quad m_{\nu_1} = 2A = \sqrt{\Delta m_{13}^2} \approx 49\text{meV}, \quad m_{\nu_3} = 0,$$

which follows from Eqs. (3.9), (3.8) and the definition $\Delta m_{ij}^2 = m_i^2 - m_j^2$. We take the best fit values of Δm_{ij}^2 from Tables 1 and 2 for the normal and inverted neutrino mass hierarchies, respectively.

To reproduce the experimental values of the leptonic mixing parameters $\sin^2\theta_{ij}$ given in Tables 1, 2, we vary the ϕ parameter, finding the following result:

$$\text{NH} \ : \ \phi = 0.576\pi, \quad \sin^2\theta_{12} \approx 0.34, \quad \sin^2\theta_{23} \approx 0.61, \quad \sin^2\theta_{13} \approx 0.0232; \tag{3.12}$$

$$\text{IH} \ : \ \phi = 0.376\pi, \quad \sin^2\theta_{12} \approx 0.34, \quad \sin^2\theta_{23} \approx 0.61, \quad \sin^2\theta_{13} \approx 0.0238. \tag{3.13}$$

Consequently, we find that $\sin^2\theta_{13}$ is in excellent agreement with the experimental data, for both normal and inverted neutrino mass hierarchies, whereas $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$ stay in the experimentally allowed 2σ range. Thus, our predictions for the neutrino mass squared splittings and leptonic mixing parameters, are in very good agreement with the experimental data on neutrino oscillations, for both normal and inverted mass hierarchies.

4. Quark masses and mixing.

From Eq. (2.12), it follows that the SM quark mass matrices have the form [9]:

$$M_U = \begin{pmatrix} a_{11}^{(U)}\lambda^4 & a_{12}^{(U)}\lambda^3 & a_{13}^{(U)}\lambda^2 \\ a_{21}^{(U)}\lambda^3 & a_{22}^{(U)}\lambda^2 & a_{23}^{(U)}\lambda \\ a_{31}^{(U)}\lambda^2 & a_{32}^{(U)}\lambda & a_{33}^{(U)} \end{pmatrix} \frac{v}{\sqrt{2}}, \qquad M_D = \begin{pmatrix} a_{11}^{(D)}\lambda^7 & a_{12}^{(D)}\lambda^6 & a_{13}^{(D)}\lambda^5 \\ a_{21}^{(D)}\lambda^6 & a_{22}^{(D)}\lambda^5 & a_{23}^{(D)}\lambda^4 \\ a_{31}^{(D)}\lambda^5 & a_{32}^{(D)}\lambda^4 & a_{33}^{(U)}\lambda^3 \end{pmatrix} \frac{v}{\sqrt{2}}, \tag{4.1}$$

Parameter	$\Delta m_{21}^2(10^{-5}\mathrm{eV}^2)$	$\Delta m_{31}^2(10^{-3}\mathrm{eV}^2)$	$\left(\sin^2\theta_{12}\right)_{\exp}$	$\left(\sin^2\theta_{23}\right)_{\exp}$	$\left(\sin^2\theta_{13}\right)_{\exp}$
Best fit	7.60	2.48	0.323	0.567	0.0234
1σ range	7.42 – 7.79	2.41 – 2.53	0.307 – 0.339	0.439 – 0.599	0.0214 – 0.0254
2σ range	7.26 – 7.99	2.35 – 2.59	0.292 – 0.357	0.413 – 0.623	0.0195 – 0.0274
3σ range	7.11 – 8.11	2.30 – 2.65	0.278 – 0.375	0.392 – 0.643	0.0183 – 0.0297

Table 1: Range for experimental values of neutrino mass squared splittings and leptonic mixing parameters, taken from Ref. [17], for the case of normal hierarchy.

Parameter	$\Delta m_{21}^2(10^{-5}\mathrm{eV}^2)$	$\Delta m_{13}^2(10^{-3}\mathrm{eV}^2)$	$\left(\sin^2\theta_{12}\right)_{\exp}$	$\left(\sin^2\theta_{23}\right)_{\exp}$	$\left(\sin^2\theta_{13}\right)_{\exp}$
Best fit	7.60	2.38	0.323	0.573	0.0240
1σ range	7.42 – 7.79	2.32 – 2.43	0.307 – 0.339	0.530 – 0.598	0.0221 – 0.0259
2σ range	7.26 – 7.99	2.26 – 2.48	0.292 – 0.357	0.432 – 0.621	0.0202 – 0.0278
3σ range	7.11 – 8.11	2.20 – 2.54	0.278 – 0.375	0.403 – 0.640	0.0183 – 0.0297

Table 2: Range for experimental values of neutrino mass squared splittings and leptonic mixing parameters, taken from Ref. [17], for the case of inverted hierarchy.

where $a_{ij}^{(U,D)}$ $(i,j=1,2,3)$ are $\mathcal{O}(1)$ parameters. Furthermore, the exotic quark masses read:

$$m_T = y^{(T)}\frac{v_\chi}{\sqrt{2}}, \qquad m_{J^1} = y_1^{(J)}\frac{v_\chi}{\sqrt{2}} = \frac{y_1^{(J)}}{y^{(T)}}m_T, \qquad m_{J^2} = y_2^{(J)}\frac{v_\chi}{\sqrt{2}} = \frac{y_2^{(J)}}{y^{(T)}}m_T. \tag{4.2}$$

Since the charged fermion mass and quark mixing pattern arises from the breaking of the $Z_2\otimes Z_3\otimes Z_{14}$ discrete group, we asssume an approximate universality in the dimensionless SM quark Yukawa couplings, as follows:

$$\begin{aligned}
a_{11}^{(U)} &= a_1^{(U)}e^{i\phi_1}, \quad a_{22}^{(U)} = a_2^{(U)}, \quad a_{33}^{(U)} = a_3^{(U)}, \\
a_{12}^{(U)} &= a_1^{(U)}\left(1-\frac{\lambda^2}{2}\right)^{-\frac{3}{2}}e^{i\phi_2}, \quad a_{13}^{(U)} = a_2^{(U)}\left(1-\frac{\lambda^2}{2}\right)^{-\frac{3}{2}}e^{i\phi_2}, \quad a_{23}^{(U)} = \left|a_{13}^{(U)}\right|\left(1-\frac{\lambda^2}{2}\right)^{-\frac{3}{2}}, \\
a_{11}^{(D)} &= a_{22}^{(D)}\left(1-\frac{\lambda^2}{2}\right)^{-2}, \quad a_{23}^{(D)} = a_{33}^{(D)}\left(1-\frac{\lambda^2}{2}\right)^{-\frac{1}{2}}, \quad a_{ij}^{(U,D)} = a_{ji}^{(U,D)}, \quad i,j=1,2,3,
\end{aligned} \tag{4.3}$$

To generate the up, down, strange and charm quark masses, the universality in the quark Yukawa couplings has to be broken. Besides that, for simplicity, we assume that the complex phase responsible for CP violation in the quark sector only arises from up type quark Yukawa terms, as indicated by Eq. (4.3). In addition, for the sake of simplicity, we fix $a_3^{(U)}=1$, which is suggested by naturalness arguments. Let's recall that the quark sector has 10 effective parameters, i.e, λ, $a_3^{(U)}$, $a_1^{(U)}$, $a_2^{(U)}$, $a_{22}^{(D)}$, $a_{12}^{(D)}$, $a_{13}^{(D)}$, $a_{33}^{(D)}$ and the phases ϕ_1 and ϕ_2 to describe the quark mass and mixing pattern, which is determined by 10 observables. Nevertheless, not all these effective parameters are free since the parameters λ and $a_3^{(U)}$ are fixed while the remaining 8 parameters are adjusted to reproduce the physical observables in the quark sector, i.e., 6 quark masses and 4 quark mixing parameters. The results shown in Table 3 correspond to the following values:

$$\begin{aligned}
a_1^{(U)} &\simeq 0.64, \quad a_2^{(U)} \simeq 0.77, \quad \phi_1 \simeq -9.03^\circ, \quad \phi_2 \simeq -4.53^\circ, \\
a_{22}^{(D)} &\simeq 2.03, \quad a_{12}^{(D)} \simeq 1.75, \quad a_{13}^{(D)} \simeq 1.15, \quad a_{33}^{(D)} \simeq 1.40.
\end{aligned} \tag{4.4}$$

PoS(PLANCK 2015)023

Observable	Model value	Experimental value
$m_u(MeV)$	1.59	$1.45^{+0.56}_{-0.45}$
$m_c(MeV)$	673	635 ± 86
$m_t(GeV)$	180	$172.1 \pm 0.6 \pm 0.9$
$m_d(MeV)$	2.9	$2.9^{+0.5}_{-0.4}$
$m_s(MeV)$	59.7	$57.7^{+16.8}_{-15.7}$
$m_b(GeV)$	2.98	$2.82^{+0.09}_{-0.04}$
$\lvert V_{ud}\rvert$	0.975	0.97427 ± 0.00015
$\lvert V_{us}\rvert$	0.224	0.22534 ± 0.00065
$\lvert V_{ub}\rvert$	0.0036	$0.00351^{+0.00015}_{-0.00014}$
$\lvert V_{cd}\rvert$	0.224	0.22520 ± 0.00065
$\lvert V_{cs}\rvert$	0.9736	0.97344 ± 0.00016
$\lvert V_{cb}\rvert$	0.0433	$0.0412^{+0.0011}_{-0.0005}$
$\lvert V_{td}\rvert$	0.00853	$0.00867^{+0.00029}_{-0.00031}$
$\lvert V_{ts}\rvert$	0.0426	$0.0404^{+0.0011}_{-0.0005}$
$\lvert V_{tb}\rvert$	0.999057	$0.999146^{+0.000021}_{-0.000046}$
J	2.98×10^{-5}	$(2.96^{+0.20}_{-0.16}) \times 10^{-5}$
δ	61°	68°

Table 3: Model and experimental values of the quark masses and CKM parameters.

The obtained and experimental values of the quark masses, CKM matrix elements, Jarlskog invariant J and CP violating phase δ are reported in Table 3. We use the experimental values of the quark masses at the M_Z scale, from Ref. ([15]), whereas we use the experimental values of the CKM parameters from Ref. [16]. The obtained values of the quark masses and CKM parameters are in excellent agreement with the experimental data, as indicated by Table 3.

5. Conclusions

We presented an extension of the minimal 331 model with $\beta = -\frac{1}{\sqrt{3}}$, based on the extended $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes T_7 \otimes Z_2 \otimes Z_3 \otimes Z_{14}$ symmetry, compatible with the experimental data on fermion masses and mixing. The T_7 and Z_3 symmetries reduce the number of model parameters. In particular, the Z_3 symmetry determines the allowed entries of the neutrino mass matrix and decouples the SM quarks from the exotic quarks. The Z_2 symmetry generates the hierarchy between SM up and SM down type quark masses. We assumed that the $SU(3)_L$ scalar singlets having Yukawa interactions with the right handed Majorana neutrinos acquire VEVs at very high scale, then providing very large masses to these Majorana neutrinos, and thus giving rise to a double seesaw mechanism of active neutrino masses. Consequently, the neutrino spectrum includes very light active neutrinos as well as heavy and very heavy sterile neutrinos. We find that the heavy and very heavy sterile neutrinos have masses at the $\sim$ MeV and $\sim$ TeV scales, respectively. Thus, the MeV scale sterile neutrinos of our model correspond to dark matter candidates. The smallness of the active neutrino masses is attributed to their scaling with inverse powers of the high energy cutoff $\Lambda \sim 10^5$ TeV as well as well as by their quadratic dependence on the very small VEV of the $Z_2 \otimes Z_3 \otimes Z_{14}$ neutral, $SU(3)_L$ singlet and T_7 antitriplet scalar field ζ. The observed hierarchy of charged fermion masses and quark mixing matrix elements arises from the breaking of the $Z_2 \otimes Z_3 \otimes Z_{14}$ discrete group at a very high energy. Furthermore, our model predicts a vanishing leptonic Dirac CP violating phase.

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