

## $SU(5)$ Yukawa matrix unification in the General Flavour Violating MSSM

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We investigate the possibility of satisfying the  $SU(5)$  boundary condition  $\mathbf{Y}^d = \mathbf{Y}^{eT}$  at the GUT scale within the renormalizable  $R$ -parity conserving Minimal Supersymmetric Standard Model (MSSM). We consider non-zero flavour off-diagonal entries in the soft SUSY-breaking mass matrices and the  $A$ -terms. We show that a non-trivial flavour structure of the soft SUSY-breaking sector can contribute to achieving precise Yukawa coupling unification for all three families. We then confront the successful unification scenario with a wide set of experimental constraints, including flavour and electroweak observables, Higgs physics and the LHC bounds. We show that simultaneous Yukawa coupling unification of three families is strongly disfavoured by the recent measurement of  $BR(\mu^+ \rightarrow e^+ \gamma)$ , as well as by the condition of the electroweak vacuum stability. On the other hand, unification of the 3rd and the 2nd family is still phenomenologically allowed.

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## 1. Introduction

The simplest realization of a Grand Unified Theory (GUT) is based on the  $SU(5)$  gauge symmetry [1], and its straightforward consequence is equality of the Yukawa couplings of down-type quarks and charged leptons at the GUT scale. While such a condition is easy to satisfy for the third family of fermions, it turns out to be quite a non-trivial task for the remaining two families, at least in the framework of the minimal  $SU(5)$ . In particular, given the experimentally measured values of the fermion masses, one observes  $Y_s(M_{GUT})/Y_\mu(M_{GUT}) = 1/3$  and  $Y_d(M_{GUT})/Y_e(M_{GUT}) = 3$ . Several solutions to this problem have been proposed in the literature, which either considered an extended Higgs sector above the GUT scale [2], or employed higher-dimensional operators [3, 4, 5].

Another approach to the Yukawa coupling unification is based on an observation that SUSY threshold corrections at the superpartner decoupling scale can considerably alter or even generate masses of the light fermions [6]. Several studies have been devoted to a possibility of using the threshold corrections to facilitate unification of the first and second family Yukawa couplings in the framework of Minimally Flavour Violating supersymmetric  $SU(5)$ . They considered either trilinear soft terms proportional to the corresponding Yukawa matrices (Ref. [7]), or general diagonal A-terms (Refs. [8, 9]). The latter, however, led to problems with the electroweak vacuum stability.

In these proceedings we report on the results presented in details in Ref. [10]. In our approach, we abandon the assumption about the Minimal Flavour Violation. We allow for non-zero off-diagonal entries both in the trilinear terms and in the sfermion mass matrices, while the diagonal entries of the trilinear terms have the same hierarchy as the Yukawa couplings. This way the chirally enhanced MSSM threshold corrections to fermion masses, collected in Ref. [11], can be employed to adjust the strange and down Yukawa couplings and facilitate the Yukawa matrix unification at the GUT scale.

## 2. Anatomy of the minimal $SU(5)$ Yukawa unification

In the supersymmetric version of  $SU(5)$ , the MSSM superfields  $Q, U, D, L, E$  are embedded into the 5- and 10-dimensional representations of  $SU(5)$ ,  $\Psi_5$  and  $\Psi_{10}$ , respectively. The Yukawa terms for  $SU(5)$  GUT read [1]

$$W \ni \Psi_{10} \mathbf{Y}^{de} \Psi_5 H_5 + \Psi_{10} \mathbf{Y}^u \Psi_{10} H_5, \quad (2.1)$$

where  $H_5$  and  $H_5$  denote two Higgs multiplets that are coupled to matter. Below the GUT scale, the  $SU(5)$  model is replaced with the MSSM, and the effective superpotential is given by

$$W_{MSSM} = Q \mathbf{Y}^u U H_u + Q \mathbf{Y}^d D H_d + L \mathbf{Y}^e E H_d + \mu H_d H_u. \quad (2.2)$$

A straightforward consequence of the GUT symmetry is the equality of the matrices  $\mathbf{Y}^d$  and  $\mathbf{Y}^{eT}$  at  $M_{GUT}$ . Unification conditions take the simplest form in a basis where the superpotential flavour mixing is entirely included in  $\mathbf{Y}^u$ , while  $\mathbf{Y}^d$  and  $\mathbf{Y}^e$  are real and diagonal. In such a case, it is enough to require equality of the diagonal entries at the GUT scale,

$$Y_{ii}^d = Y_{ii}^e, \quad i = 1, 2, 3. \quad (2.3)$$

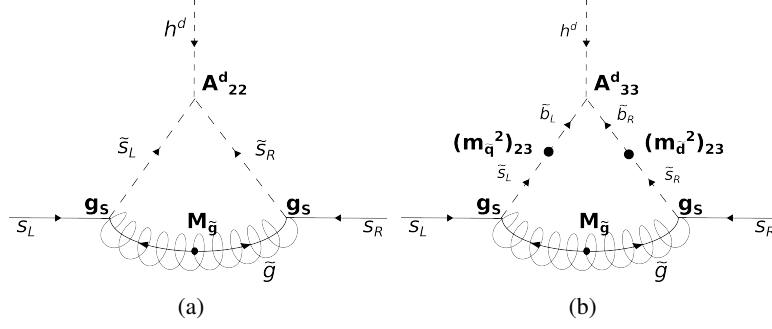


Figure 1: Examples of diagrams that describe threshold corrections to the strange Yukawa coupling at  $\mu_{sp}$ . (a) the flavour-diagonal leading gluino diagram; (b) possibly an even bigger contribution arising in the case of flavour mixing in the soft-mass matrix.

Diagonal entries of the Yukawa couplings are constrained by measurements of the quark and lepton masses at or below the electroweak scale. Consequently, these entries are most easily fixed within the SM. One needs, however, to determine their renormalized values within the MSSM, which is assumed to be an underlying effective theory that connects the electroweak scale with  $M_{GUT}$ . This is done by calculating threshold corrections  $\Sigma_{ii}^f$  at the matching scale  $\mu_{sp}$ . Such corrections depend on values of the soft SUSY-breaking terms,

$$v_f Y_{ii}^{fMSSM} = v_f Y_{ii}^{fSM} - \Sigma_{ii}^f((m_{\tilde{f}}^2)_{ij}, A_{ij}^f, m_{H_i}, M_i). \quad (2.4)$$

Any loop correction to the Yukawa coupling needs to flip chirality. In the framework of the MSSM, there are only two class of objects that allow to do so: Yukawa couplings themselves and the trilinear terms  $A_{ij}^f$ . Therefore,  $\Sigma_{ii}^f$  should be proportional to one of them. The dominant supersymmetric threshold corrections to the Yukawa couplings beyond the small  $\tan\beta$  limit have been calculated in Ref. [11]. They come from sfermion-gluino, sfermion-neutralino and sfermion-chargino diagrams. In order to achieve strict unification of the Yukawa couplings for the first and second families, the threshold corrections have to be of the same order as the leading terms.

In the framework of Minimal Flavour Violation, the gluino-dependent threshold corrections to the Yukawa couplings is given by the diagonal entries of the trilinear terms [11],

$$(\Sigma_{ii}^d)^{\tilde{g}} = \frac{2\alpha_s}{3\pi} m_{\tilde{g}} (v_d A_{ii}^d - v_d Y_{ii}^d \mu \tan\beta) C_0(m_{\tilde{g}}^2, m_{\tilde{q}_L^i}^2, m_{\tilde{q}_R^i}^2), \quad (2.5)$$

The relevant diagram in the strange quark case is depicted in the left panel of Fig. 1. For the first and second generation, the gluino contribution is dominant and can be used to fix the ratios of the corresponding Yukawa couplings at  $\mu_{sp}$ . Since  $C_0$  is always negative,  $A_{22}^d$  should be positive and  $\mu$  negative to maximize the necessary positive correction to the strange quark mass. In the first family case, a contribution from the  $\mu$  term is negligible due to smallness of the corresponding Yukawa coupling. Therefore, a negative  $A_{11}^d$  is sufficient to generate a correction to  $Y_d$  of the right sign. Note, however, that large  $A$ -terms usually lead to problems with the electroweak vacuum stability.

The situation changes in the framework of General Flavour Violation (GFV), when non-zero off-diagonal soft mass entries are allowed (see the right panel of Fig. 1). In such a case, the

dominant contribution to the strange quark self-energy associated with a gluino loop can be written as [11]:

$$(\Sigma_{22}^d)^{\tilde{g}} = \frac{2\alpha_s}{3\pi} m_{\tilde{g}} v_d (A_{33}^d - Y_b \mu \tan \beta) \sum_{m,n=2,3} (-1)^{m+n+1} C_0(m_{\tilde{g}}^2, m_{\tilde{q}_m^L}^2, m_{\tilde{d}_n^R}^2) \times \quad (2.6)$$

$$\frac{(m_{\tilde{q}}^2)_{23}}{\sqrt{[(m_{\tilde{q}}^2)_{22} - (m_{\tilde{q}}^2)_{33}]^2 + 4((m_{\tilde{q}}^2)_{23})^2}} \frac{(m_{\tilde{d}}^2)_{23}}{\sqrt{[(m_{\tilde{d}}^2)_{22} - (m_{\tilde{d}}^2)_{33}]^2 + 4((m_{\tilde{d}}^2)_{23})^2}},$$

where we have assumed that  $(m_{\tilde{q}}^2)_{23}$  and  $(m_{\tilde{d}}^2)_{23}$  are the only non-zero off-diagonal elements of the down-squark mass matrix, and that they are real. It follows from Eq. (2.6) that chirality-conserving flavour-changing interactions  $(m_{\tilde{q}}^2)_{23} \tilde{b}_L^* \tilde{s}_L$  and  $(m_{\tilde{d}}^2)_{23} \tilde{b}_R^* \tilde{s}_R$  generate a threshold correction to  $Y_s$  of the order of  $\Delta Y_s \sim \alpha_s A_{33}^d / M_{\text{SUSY}}$ , which in general can be large enough to facilitate Yukawa coupling unification for the second family, even when the coupling  $A_{22}^d$  is small.

### 3. Impact of the GFV threshold corrections on Yukawa unification

We perform a numerical scan to determine those SUSY GFV parameters whose non-zero values are indispensable from the point of view of the considered unification. We assume for simplicity that all the soft SUSY-breaking terms are real, therefore neglecting the possibility of new SUSY sources of CP violation. The GUT-scale  $SU(5)$  boundary conditions for the soft-masses read

$$(m_{\tilde{t}}^2)_{ij} = (m_{\tilde{b}}^2)_{ij} \equiv (m_{\tilde{d}_l}^2)_{ij}, \quad (m_{\tilde{q}}^2)_{ij} = (m_{\tilde{u}}^2)_{ij} = (m_{\tilde{e}}^2)_{ij} \equiv (m_{\tilde{u}_e}^2)_{ij}. \quad (3.1)$$

The off-diagonal elements of the down-squark matrix are normalized to the entry  $(m_{\tilde{d}_l}^2)_{33}$ , and are required to satisfy the upper limit  $(m_{\tilde{d}_l}^2)_{ij} / (m_{\tilde{d}_l}^2)_{33} \leq 1$ . We further assume that  $(m_{\tilde{u}_e}^2)_{ij} = 0$  for  $i \neq j$ . Such an assumption is not expected to cause any significant loss of generality because relatively large off-diagonal elements of  $(m_{\tilde{q}}^2)_{ij}$  are generated radiatively at  $M_{\text{SUSY}}$  due to the RGE running in the super-CKM basis. We also introduce a short-hand notation  $m_{ij}^{dl} \equiv \sqrt{(m_{\tilde{d}_l}^2)_{ij}}$ ,  $m_{ij}^{ue} \equiv \sqrt{(m_{\tilde{u}_e}^2)_{ij}}$ .

The GUT-scale  $SU(5)$  boundary conditions for the trilinear terms are given by

$$A_{ij}^d = A_{ji}^e \equiv A_{ij}^{de}. \quad (3.2)$$

We constrain the relative magnitude of the diagonal entries by the corresponding Yukawa couplings  $\frac{|A_{ii}^f|}{|A_{33}^f|} < \frac{Y_{ii}^f}{Y_{33}^f}$  in order to relax the strong tension between the EW vacuum stability condition and Yukawa unification that arises in the case of large diagonal  $A$ -terms. We also impose that  $A_{ij}^t = 0$  for  $i \neq j$ . On the other hand, the off-diagonal entries in the down-sector trilinear matrix are not constrained in our initial scan. They are only required to satisfy  $|(A_{ij}^{de}) / (A_{33}^{de})| \leq 0.5$ .

Finally, we assume that the gaugino mass parameters are universal at  $M_{\text{GUT}}$ ,

$$M_1 = M_2 = M_3 \equiv M_{1/2}, \quad (3.3)$$

which is the simplest among relations that naturally arise in the framework of SUSY  $SU(5)$  GUTs. The sign of the parameter  $\mu$  is chosen to be negative to facilitate the second family unification, as explained in Sec. 2. All the scanning ranges are collected in Table 1.

Parameter	Scanning Ranges: $GFV_{23}$	Scanning Ranges: $GFV_{123}$
$M_{1/2}$	[100, 4000] GeV	[100, 4000] GeV
$m_{H_u}, m_{H_d}$	[100, 8000] GeV	[100, 8000] GeV
$\tan\beta$	[3, 45]	[3, 45]
$A_{33}^u, A_{33}^{de}$	[-9000, 9000], [0, 5000] GeV	[-9000, 9000], [0, 5000] GeV
$A_{11}^{de}/A_{33}^{de}$	[-0.00028, 0.00028]	[-0.00028, 0.00028]
$A_{22}^u/A_{33}^u, A_{22}^{de}/A_{33}^{de}$	[-0.005, 0.005], [-0.065, 0.065]	[-0.005, 0.005], [-0.065, 0.065]
$A_{ij}^{de}/A_{33}^{de}, i \neq j$	0	[-0.5, 0.5]
$m_{ii}^{dl}, m_{ii}^{ue}, i = 1, 2, 3$	[100, 7000] GeV	[100, 7000] GeV
$m_{23}^{dl}/m_{33}^{dl}, m_{13}^{dl}/m_{33}^{dl}, m_{12}^{dl}/m_{33}^{dl}$	[0, 1], 0, 0	[0, 1], [0, 1], [0, 1]

Table 1: Ranges of the input SUSY parameters used in our initial scan. Several omitted soft SUSY-breaking parameters at the GUT scale (namely  $A_{11}^u$  as well as  $A_{ij}^u$  and  $m_{ij}^{ue}$  for  $i \neq j$ ) have been set to zero.  $\text{sgn}\mu = -1$  all over the paper.

Four SM parameters ( $m_t^{\text{pole}}, m_b^{\overline{MS}}(m_b), \alpha_{\text{em}}^{-1}(M_Z)$  and  $\alpha_s^{\overline{MS}}(M_Z)$ ), as well as the elements of the CKM matrix in the Wolfenstein parametrisation, are randomly drawn from a Gaussian distribution centred around their experimentally measured central values and are collected in Table 2.

In our study, we consider two different scenarios for the Yukawa matrix unification:

$GFV_{23}$ : Only the third and the second generation Yukawa couplings are unified at the GUT scale. The only relevant GFV parameter in this case is  $m_{23}^{dl}$ .

$GFV_{123}$ : Yukawa couplings of all the three families are unified at the GUT scale. All the GFV parameters in the down-squark sector can assume non-zero values.

### 3.1 $GFV_{23}$ : unification of the third and second family

In Fig. 2, we present distributions of the collected points in the planes of ( $m_{23}^{dl}/m_{33}^{dl}, Y_b\mu \tan\beta$ ) (a), ( $M_{1/2}, Y_b\mu \tan\beta$ ) (b), and ( $\tan\beta, A_{33}^{de}$ ) (c). All the points that satisfy the Yukawa unification condition for the third generation ( $0.9 < Y_b/Y_\tau < 1.1$ ) are represented as gray stars. Those for which both third and second generations are unified are shown as green dots.

The most characteristic feature of the  $GFV_{23}$  scenario is the lack of points consistent with the Yukawa unification and with a large universal gaugino mass parameter,  $M_{1/2} > 2000$  GeV. This is a straightforward consequence of Eq. (2.6) in which the gluino mass appears both as a multiplicative factor and through the loop function  $C_0$ . The impact of the latter is of particular importance since for  $M_{1/2} > 2000$  GeV, and for the fixed sfermion masses, it leads to strong suppression of the threshold correction ( $\Sigma_{22}^d$ )<sup>8</sup>. At the same time, the correction is somewhat enhanced when the mass of sfermions is decreased with a fixed  $M_{1/2}$ , although this effect is less prominent.

Secondly, strange-muon unification occurs for both large and small values of  $A_{33}^{de}$ , which is of relevance for the minimisation of the scalar potential. This is confirmed by panels (a) and (b) of Fig. 2 where the values of the factor  $Y_b\mu \tan\beta$ , which appears in Eq. (2.6), are depicted for all the

$m_t^{\text{pole}}: 173.34 \pm 0.76$ GeV	$m_b^{\overline{MS}}(m_b): 4.18 \pm 0.03$ GeV	$\alpha_s^{\overline{MS}}(M_Z): 0.1184 \pm 0.0007$	$\alpha_{\text{em}}^{-1}(M_Z): 127.944 \pm 0.015$
$\bar{\rho}: 0.159 \pm 0.045$	$\bar{\eta}: 0.363 \pm 0.049$	$A: 0.802 \pm 0.020$	$\lambda: 0.22535 \pm 0.00065$

Table 2: Standard Model parameters used in our numerical calculations.

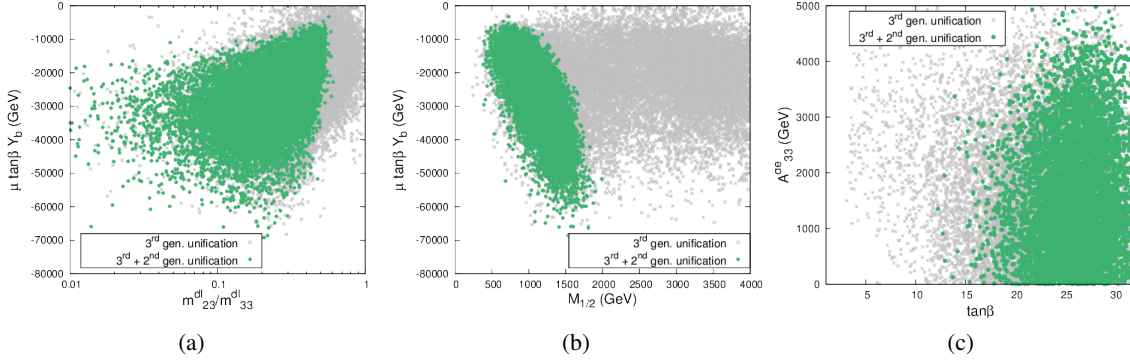


Figure 2: Scatter plot of the  $GFV_{23}$  points in the planes  $(m_{23}^{dl}/m_{33}^{dl}, Y_b \mu \tan \beta)$  (a),  $(M_{1/2}, Y_b \mu \tan \beta)$  (b), and  $(\tan \beta, A_{33}^{de})$  (c). Gray stars: all points satisfying the Yukawa unification condition for the third generation; green dots: points for which both heavier generations are unified.

model points satisfying the Yukawa unification condition. One can observe, by comparing with panel (c) of Fig. 2, that  $|Y_b \mu \tan \beta| \gg A_{33}^{de}$  and the impact of  $A_{33}^{de}$  in Eq. (2.6) is indeed negligible.

The strange-muon Yukawa coupling unification can be achieved through an interplay of several mechanisms. For small gluino masses (that correspond to the upper-left corner of Fig. 2(b)), the threshold correction  $(\Sigma_{22}^d)^{\bar{g}}$  can be enhanced either by large values of the GFV parameter  $m_{23}^{dl}$  (upper-right corner of Fig. 2(a)) or by large values of the factor  $Y_b \mu \tan \beta$  with  $m_{23}^{dl}$  as small as 0.01. The latter effect is additionally enhanced for moderate values of  $\tan \beta$  which are preferred in the  $GFV_{23}$  scenario, as shown in panel (c) of Fig. 2. For large gluino masses, that correspond to  $M_{1/2} \geq 1500$  GeV, the threshold correction of Eq. (2.6) becomes suppressed by the loop function  $C_0$ . As a consequence, both large  $m_{23}^{dl}$  and an even larger  $\mu$ -term are needed to achieve Yukawa unification for the second family. This case correspond to the green points in the lower part of Fig. 2(a).

### 3.2 $GFV_{123}$ : unification of all three families

In Fig. 3, we present distributions of points in the planes  $(m_{13}^{dl}/m_{33}^{dl}, m_{12}^{dl}/m_{33}^{dl})$  (a), and  $(m_{23}^{dl}/m_{33}^{dl}, m_{12}^{dl}/m_{33}^{dl})$  (b), and  $(A_{12}^{de}/A_{33}^{de}, A_{21}^{de}/A_{33}^{de})$  (c). All the points that satisfy the Yukawa unification condition for the third generation are depicted as gray stars, while those that additionally fulfil unification of the first family as green diamonds. Finally, black dots correspond to those points for which all three generations are unified.

The functional form of the threshold correction in Eq. (2.6) might suggest that non-zero soft-mass elements  $m_{23}^{dl}$  and  $m_{13}^{dl}$  are sufficient to allow the Yukawa unification in both the second and first family cases. Such a simplistic picture, however, is not true, as can be seen from the panel (a) of Fig. 3 where large  $m_{12}^{dl}$  is clearly favoured. To understand what happens, let us note that the GFV corrections  $(\Sigma_{22}^d)^{\bar{g}}$  and  $(\Sigma_{11}^d)^{\bar{g}}$  (obtained from Eq. (2.6) by replacing indices “2” with “1”) are determined by overlapping sets of parameters, in particular  $M_{1/2}$  and  $A_{33}^{de}$ . On the other hand, sizes of those corrections as required by the Yukawa coupling unification differ by two orders of magnitude. Let us now assume that  $(\Sigma_{11}^d)^{\bar{g}}$  is fixed by the unification of the first family. Thus  $M_{1/2}$  and  $A_{33}^{de}$ , already constrained by unification of the third family, are even more limited. But



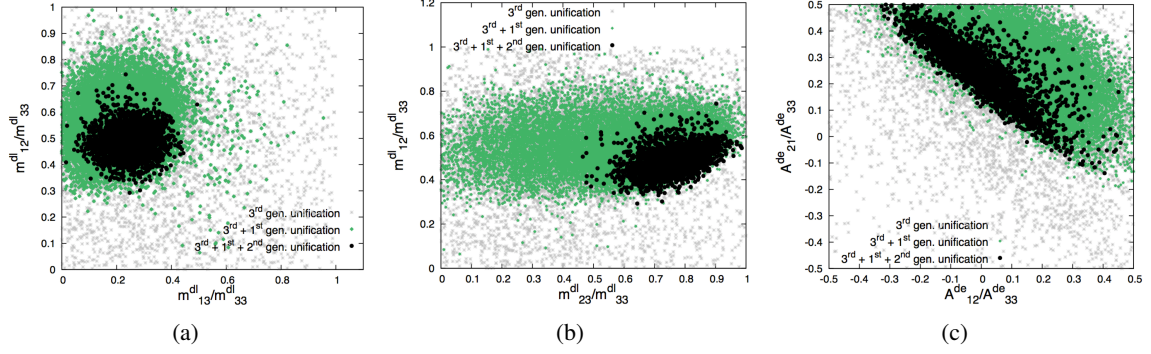


Figure 3: Scatter plot of the  $GFV_{123}$  points in the planes  $(m_{13}^{dl}/m_{33}^{dl}, m_{12}^{dl}/m_{33}^{dl})$  (a), and  $(m_{23}^{dl}/m_{33}^{dl}, m_{12}^{dl}/m_{33}^{dl})$  (b), and  $(A_{12}^{de}/A_{33}^{de}, A_{21}^{de}/A_{33}^{de})$  (c). Gray stars: all the points satisfying the Yukawa unification condition for the third generation; green diamonds: points additionally requiring unification of the first family; black dots: points for which all the three generations are unified.

with such a choice of parameters the correction  $(\Sigma_{22}^d)^{\tilde{g}}$  is still too small to allow unification of the second family, and needs to be further enhanced by another contribution. Such a contribution comes from a diagram like the one shown in Fig. 1(b), but with the trilinear term  $A_{21}^{de}$  in the vertex and  $m_{12}^{dl}$  mixing in the right-handed sector. However, a similar diagram also exists for the first family, and the corresponding contribution should be added to the one driven by  $m_{13}^{dl}$ . That explains why all the five parameters  $m_{12}^{dl}, m_{13}^{dl}, m_{23}^{dl}, A_{12}^{de}$  and  $A_{21}^{de}$  must be adjusted simultaneously. Note also that  $A_{12}^{de}/A_{21}^{de}$  can be kept relatively low, as this contribution is always enhanced by a large  $m_{12}^{dl}$ .

We conclude this section with summarising the allowed ranges of the non-zero GFV parameters that characterise the  $SU(5)$  GUT scenario with the full Yukawa coupling unification:

$$0.5 < m_{23}^{dl}/m_{33}^{dl} < 1, \quad 0 < m_{13}^{dl}/m_{33}^{dl} < 0.5, \quad 0.3 < m_{12}^{dl}/m_{33}^{dl} < 0.7, \quad 0 < A_{12}^{de}/A_{21}^{de} < 0.2. \quad (3.4)$$

#### 4. Phenomenology of the unification scenarios

In order to find points satisfying both the Yukawa coupling unification conditions at the GUT scale and the experimental constraints, we scan the parameter space of the unification scenarios given in Table 1. The most relevant experimental constraints applied in the analysis are listed in

Measurement	Mean or range	Error [ exp., th.]	Reference
$\Omega_{\tilde{\chi}} h^2$	0.1199	[0.0027, 10%]	[12]
$BR(\bar{B} \rightarrow X_s \gamma) \times 10^4$	3.43	[0.22, 0.23]	[13]
$BR(B_s \rightarrow \mu^+ \mu^-) \times 10^9$	2.8	[0.7, 0.23]	[14]
$BR(\mu^+ \rightarrow e^+ \gamma) \times 10^{13}$	< 5.7	[0,0]	[15]
$BR(\tau^\pm \rightarrow \mu^\pm \gamma) \times 10^8$	< 4.4	[0,0]	[16]
$BR(\mu^+ \rightarrow e^+ e^+ e^-) \times 10^{12}$	< 1.0	[0,0]	[17]
$BR(\tau^\pm \rightarrow \mu^\pm \mu^+ \mu^-) \times 10^8$	< 2.1	[0,0]	[18]

Table 3: The most relevant experimental constraints applied in the analysis.

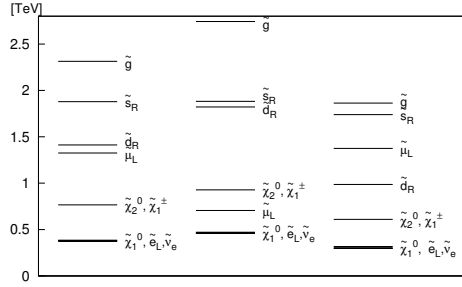


Figure 4: Examples of spectra characteristic of the  $GFV_{23}$  Yukawa unification scenario.

Table 3. Other experimental constraints, as well as a description of our statistical approach and numerical tools used in the analysis, can be found in Ref. [10].

#### 4.1 $GFV_{23}$ : unification of the third and second family

All the points that demonstrate the Yukawa unification and satisfy the experimental constraints listed in Table 3 are characterized by similar spectra. Three examples are shown in Fig. 4. The Next-to-Lightest SUSY particle is the lightest sneutrino, closely followed by one charged slepton. The coloured particles are significantly heavier, as required by the unification condition.

Relic abundance of the dark matter in GUT-constrained SUSY scenarios is usually the most stringent constraint. In Fig. 5(a), we present a distribution of points found by our scanning procedure in the  $(m_{\tilde{\chi}_1^0}, \sigma_p^{SI})$  plane. All the collected points are depicted as gray stars, while those that satisfy at  $3\sigma$  the experimental constraint on the DM relic density appear as brown dots. Blue diamonds correspond to those scenarios for which additionally the Yukawa coupling unification holds. The green dashed line indicates the 90% C.L. exclusion bound on the  $\sigma_p^{SI}$  based on the 85-day measurement by the LUX collaboration, while the purple dashed line is a projection of XENON1T sensitivity. One can observe that in the region where Yukawa unification is achieved, the neutralino LSP is bino-like, which corresponds to a relatively low spin-independent proton-neutralino cross-section. In other words, the condition of Yukawa coupling unification strongly disfavors purely or partly higgsino-like neutralino. This is due to the fact that only the points with

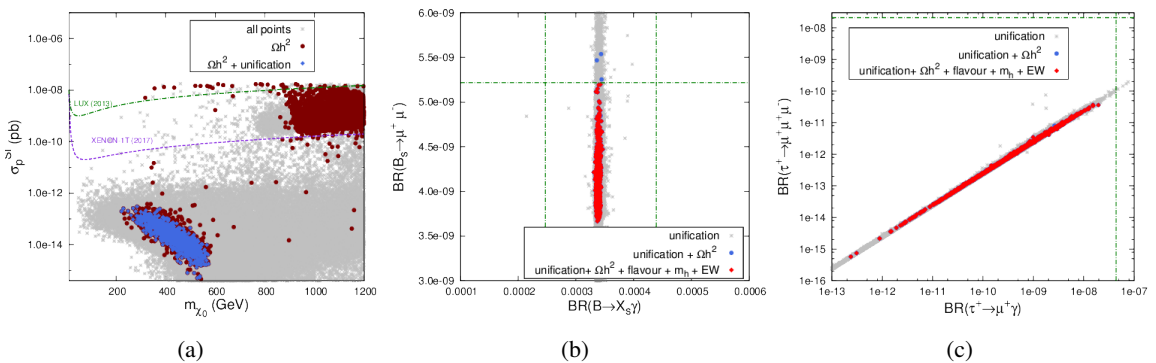


Figure 5: Scatter plot of the  $GFV_{23}$  points in the planes  $(m_{\tilde{\chi}_1^0}, \sigma_p^{SI})$  (a),  $(BR(\bar{B} \rightarrow X_s \gamma), BR(B_s \rightarrow \mu^+ \mu^-))$  (b) and  $(BR(\tau^\pm \rightarrow \mu^+ \gamma), BR(\tau^\pm \rightarrow \mu^\pm \mu^+ \mu^-))$  (c). The colour code is explained in the text.



$|\mu| \lesssim 1$  TeV in Fig. 5(a) correspond to a significant higgsino component of the LSP. In such a case, the  $\mu$ -dependent contribution in Eq. (2.6) is too small to allow for unification of the second family Yukawa couplings even with the maximal off-diagonal term  $m_{23}^{dl}$ .

In Fig. 5(b) we present a distribution of points for  $(\text{BR}(\bar{B} \rightarrow X_s \gamma), \text{BR}(B_s \rightarrow \mu^+ \mu^-))$ , and in Fig. 5(c) for  $(\text{BR}(\tau^\pm \rightarrow \mu^+ \gamma), \text{BR}(\tau^\pm \rightarrow \mu^\pm \mu^+ \mu^-))$ . This time gray stars indicate all the points for which it is possible to achieve the Yukawa coupling unification. Points that satisfy the relic density constraint at  $3\sigma$  are shown as blue dots, while red diamonds additionally met all the other constraints listed in Table 3. Green dashed lines present  $3\sigma$  experimental limits on the corresponding observables. In the considered scenario most of the flavour constraints in the quark sector are quite easily satisfied for the points that have survived imposing the DM experimental limit. This is mainly due to the fact that the coloured sfermions, in particular those of the third generation, are relatively heavy in our setup, while  $\tan\beta$  needs to be low or moderate in order to facilitate the Yukawa coupling unification of the third and second family.

A potential threat to the  $GFV_{23}$  scenario is posed by the LFV observables that severely constrain any non-zero mixing among sleptons. However, in the case of  $m_{23}^{dl}$ , the current constraints on the relevant processes are still easily satisfied, as can be seen in panel (c) of Fig. 5.

#### 4.2 $GFV_{123}$ : unification of all three families

In this case the spectra consistent with the Yukawa matrix unification are very similar to the ones described in the previous subsection. The only difference is the mass of the neutralino LSP, which is now limited to be lighter than 300 GeV, as required by simultaneous unification of all three families. The flavour-related observables in the quark sector are also easily satisfied.

On the other hand, in the  $GFV_{123}$  scenario, the muon-electron conversion observables might be strongly enhanced, as they are influenced by large non-zero values of the parameters  $m_{12}^{dl}$ ,  $A_{12}^{de}$  and  $A_{21}^{de}$ , required by the Yukawa unification of the first and second family. In Fig. 6, we present distributions of model points in the planes  $(\text{BR}(\mu^+ \rightarrow e^+ \gamma), \text{BR}(\mu^+ \rightarrow e^+ e^+ e^-))$  (a), and  $(\text{BR}(\tau^\pm \rightarrow e^+ \gamma), \text{BR}(\tau^\pm \rightarrow e^\pm e^+ e^-))$  (b). The colour code is the same as in Fig. 5(b). It is clear that the 90% C.L. upper bound on  $\text{BR}(\mu^+ \rightarrow e^+ \gamma)$  reported in Ref. [15] is violated in the  $GFV_{123}$  scenario by about five orders of magnitude.

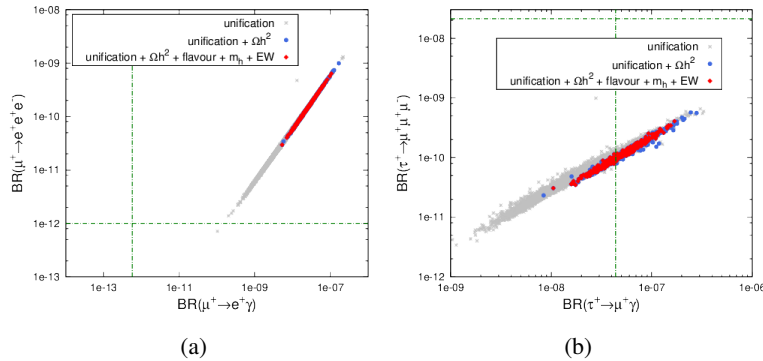


Figure 6: Scatter plot of the  $GFV_{123}$  points in the planes  $(\text{BR}(\mu^+ \rightarrow e^+ \gamma), \text{BR}(\mu^+ \rightarrow e^+ e^+ e^-))$  (a), and  $(\text{BR}(\tau^\pm \rightarrow e^+ \gamma), \text{BR}(\tau^\pm \rightarrow e^\pm e^+ e^-))$  (b). The colour code is explained in the text.

## 5. Conclusions

In this study, we provided evidence that  $SU(5)$  boundary conditions for the Yukawa matrices at the GUT-scale can be satisfied within the renormalizable  $R$ -parity-conserving MSSM if a more general flavour structure of the soft SUSY-breaking sector is allowed.

The scenario with Yukawa unification for the second and third family only, is consistent with a wide set of experimental measurements, including those from the FCNC processes. On the other hand, simultaneous unification of the Yukawa couplings for the second and first family additionally requires the elements  $A_{12/21}^{de}$ ,  $m_{13}^{dl}$  and  $m_{12}^{dl}$  to assume non-zero values. The latter, however, leads to unacceptably large SUSY corrections to some Lepton Flavour Violating processes, in particular to  $\mu \rightarrow e\gamma$  decay.

Note, that it is theoretically possible to evade this constraint while unifying the electron and down quark Yukawa couplings by raising the overall scale of the superpartner masses. However, such a scenario is difficult to test with our present numerical tools which assume  $\mu_{\text{sp}} = M_Z$  and henceforth are not reliable when  $M_{\text{SUSY}}$  is very large.

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