

Shifted Focus Point and Gluino Mass Bound in the Minimal Mixed Mediation of SUSY Breaking*

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In order to significantly reduce the fine-tuning associated with the electroweak symmetry breaking in the minimal supersymmetric standard model (MSSM), we consider both the minimal gravity- and the minimal gauge mediation effects at the grand unified theory (GUT) scale for a common supersymmetry breaking source at a hidden sector. The minimal forms for the Kähler potential and the gauge kinetic function are employed at tree level, and the MSSM gaugino masses are radiatively generated through the gauge mediation. In such a “minimal mixed mediation model,” a “focus point” of the soft Higgs mass parameter, $m_{h_u}^2$ emerges at 3-4 TeV energy scale, which is exactly the stop mass scale needed for explaining the 126 GeV Higgs boson mass without the “A-term” at the three loop level. As a result, $m_{h_u}^2$ can be quite insensitive to various trial stop masses at low energy, reducing the fine-tuning measures to be much smaller than 100 even for a 3-4 TeV low energy stop mass and $-0.7 < A_t/m_0 \lesssim +0.5$ at the GUT scale. The naturalness of the small $m_{h_u}^2$ is more closely associated with the gluino mass rather than the stop mass unlike the conventional scenario. The requirements of various fine-tuning measures much smaller than 100 and $|\mu| < 600 \text{ GeV}$ constrain the gluino mass to be $1.6 \text{ TeV} \lesssim m_{\tilde{g}} \lesssim 2.2 \text{ TeV}$, which is well-inside the discovery potential range of LHC Run II.

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Although the standard model (SM) has been extremely successful in the experimental side, it doesn't provide reasonable answers to some theoretical puzzles such as the naturalness of the electroweak (EW) scale and the Higgs boson mass. The main motivation of the low energy supersymmetry (SUSY) was to resolve the naturalness problem associated with the EW phase transition raised in the SM, since SUSY can protect the small Higgs boson mass against large quantum corrections [3, 4]. Because of it, the minimal supersymmetric standard model (MSSM) has been believed the most promising theory beyond the SM, guiding the SM to a grand unified theory (GUT) or string theory. However, any evidence of the low energy SUSY has not been observed yet at the large hadron collider (LHC): the mass bounds on the SUSY particles have gradually increased, and now they seem to start threatening the traditional status of SUSY as a prominent solution to such a naturalness problem of the SM.

Actually, a barometer of the naturalness of the MSSM is the mass of the superpartner of the top quark ("stop"): a stop mass lighter than 1 TeV is quite essential for keeping the naturalness of the EW scale and the Higgs boson mass. Because of the reason, many SUSY models for explaining the Higgs mass assume a relatively light stop, $\tilde{m}_t \lesssim 1$ TeV [5]. However, the experimental mass bound on the stop has already exceeded 700 GeV [6]. Thus, it would be very timely to ask whether the low energy SUSY can still remain natural even with a somewhat heavy stop mass greater than 1 TeV. On the other hand, the gluino is not directly involved in this issue, because it does not couple to the Higgs boson at tree level. Instead, the gluino mass dominantly influences the renormalization group (RG) evolution of the stop mass parameters. In this sense, the gluino affects the Higgs mass parameter $m_{h_u}^2$ just indirectly in the ordinary MSSM.

We will attempt to investigate another possibility: the gluino can play a more important role in the naturalness of the small Higgs boson mass. As a consequence, the stop mass can be much less responsible for it: it can be much heavier than the present experimental bound. Indeed, the gluino can be more easily explored than the stop at the Large Hadron Collider (LHC). Thus, if a relatively light gluino mass turns out to be needed, this scenario could readily be tested at LHC Run II.

Due to the large top quark Yukawa coupling (y_t), the top and stop make the dominant contributions to the radiative physical Higgs mass squared and also the renormalization of a soft mass squared of the Higgs boson ($m_{h_u}^2$) in the MSSM. The renormalization effect on $m_{h_u}^2$ would linearly be sensitive to the stop mass squared \tilde{m}_t^2 [3],

$$\Delta m_{h_u}^2 \approx \frac{3y_t^2}{8\pi^2} \tilde{m}_t^2 \log\left(\frac{\tilde{m}_t^2}{\Lambda^2}\right) + \dots, \quad (1)$$

while it depends just logarithmically on an ultraviolet (UV) cutoff Λ . Since the Higgs mass parameters, $m_{h_u}^2$ and $m_{h_d}^2$ are related to the Z boson mass m_Z together with the "Higgsino" mass μ [3],

$$\frac{1}{2}m_Z^2 = \frac{m_{h_d}^2 - m_{h_u}^2 \tan^2\beta}{\tan^2\beta - 1} - |\mu|^2, \quad (2)$$

$\{m_{h_u}^2, m_{h_d}^2, |\mu|^2\}$ should be finely tuned to yield $m_Z^2 = (91 \text{ GeV})^2$ for a given $\tan\beta \equiv \langle h_u \rangle / \langle h_d \rangle$, ratio of the vacuum expectation values (VEVs) of the two MSSM Higgs fields], if they are excessively large. According to the recent analysis based on the three-loop calculations, the stop mass required for explaining the 126 GeV Higgs boson mass [7] without any other helps is about 3-4 TeV [8]. Thus, a fine-tuning of order 10^{-3} or smaller looks unavoidable in the MSSM for $\Lambda \sim 10^{16}$ GeV.

In order to more clearly see the UV dependence of $m_{h_u}^2$ and properly discuss this “little hierarchy problem”, however, one should suppose a specific UV model and analyze its resulting full renormalization group (RG) equations. If the UV model is simple enough, addressing this problem successfully with SUSY, the low energy SUSY could still be regarded as an attractive solution to the naturalness problem.

One nice idea is the “focus point (FP) scenario” [9]. This scenario is based on the minimal gravity mediation (mGrM) of SUSY breaking. So the soft mass squareds such as $m_{h_{u,d}}^2$ and those of the left handed (LH) and right handed (RH) stops, $(m_{q_3}^2, m_{u_3^c}^2)$ as well as the gaugino masses M_a ($a = 3, 2, 1$) are given to be *universal* at the GUT scale, $m_{h_u}^2 = m_{h_d}^2 = m_{q_3}^2 = m_{u_3^c}^2 = \dots \equiv m_0^2$ and $M_3 = M_2 = M_1 \equiv m_{1/2}$. As pointed out in [9], if the holomorphic soft SUSY breaking terms (“A-terms”) in the scalar potential are zero at the GUT scale and the unified gaugino mass $m_{1/2}$ is just a few hundred GeV, $m_{h_u}^2$ converges to a small negative value around the Z boson mass scale in this setup, *regardless of its initial values given by m_0^2 at the GUT scale* [9]: a FP of $m_{h_u}^2$ appears around the m_Z scale. In the RG solution of $m_{h_u}^2$ at the m_Z scale, namely,

$$m_{h_u}^2(Q = m_Z) = C_s m_0^2 - C_g m_{1/2}^2, \quad (3)$$

where the dimensionless numbers C_s, C_g (> 0) can numerically be estimated using RG equations, C_s happens to be quite small with the above universal soft masses, and the EW symmetry is broken dominantly by the C_g term. On the other hand, stop masses are quite sensitive to m_0^2 . As a result, in the FP scenario m_Z^2 could remain small enough even with a relatively heavy stop mass in contrast to the naive expectation from Eq. (1).

However, the experimental bound on the gluino mass M_3 has already exceeded 1.3 TeV [10]. As expected from Eqs. (2) and (3), a too large $m_{1/2}$ needed for $M_3 > 1.3$ TeV at low energy would require a fine-tuned large $|\mu|$ for m_Z of 91 GeV particularly for relatively light stop mass ($\lesssim 1$ TeV) cases. When the stop mass is around 3-4 TeV, the stop should decouple from the RG equations below 3-4 TeV, which makes C_s *sizable* in Eq. (3) [11]. Then, a much larger $m_{1/2}$ is necessary for EW symmetry breaking. Since the RG running interval between 3-4 TeV and m_Z scale, to which modified RG equations should be applied, is too large, the FP behavior is seriously spoiled with such heavy SUSY particles.

The best way to rescue the FP idea is to somehow push up the FP to the stop decoupling scale [11]: C_s needs to be made small enough before stops are decoupled. Then $m_{h_u}^2$ at the m_Z scale can be estimated using the Coleman-Weinberg potential [3, 12]:

$$\begin{aligned} m_{h_u}^2(m_Z) &\approx m_{h_u}^2(\Lambda_T) + \frac{3|y_t|^2}{16\pi^2} \left[m_{q_3}^2 \left\{ \log \frac{m_{q_3}^2}{\Lambda_T^2} - 1 \right\} + m_{u_3^c}^2 \left\{ \log \frac{m_{u_3^c}^2}{\Lambda_T^2} - 1 \right\} - 2m_t^2 \left\{ \log \frac{m_t^2}{\Lambda_T^2} - 1 \right\} \right] \\ &\approx m_{h_u}^2(\Lambda_T) - \frac{3|y_t|^2}{16\pi^2} \left\{ m_{q_3}^2 + m_{u_3^c}^2 \right\} \left[1 - \frac{m_{q_3}^2 - m_{u_3^c}^2}{2(m_{q_3}^2 + m_{u_3^c}^2)} \log \frac{m_{q_3}^2}{m_{u_3^c}^2} \right] \Big|_{\Lambda_T}, \end{aligned} \quad (4)$$

where the cutoff Λ_T is set to the stop decoupling scale [$\approx (m_{q_3} m_{u_3^c})^{1/2}$], and the top quark mass (m_t) contributions are relatively suppressed. Since the m_0^2 dependence of stop masses would be loop-suppressed, $m_{h_u}^2$ needs to be well-focused around Λ_T . Due to the additional negative contribution to $m_{h_u}^2(m_Z)$ below Λ_T , a small positive $m_{h_u}^2(\Lambda_T)$ would be more desirable.

In order to shift the FP upto the desired stop mass scale 3-4 TeV, we suggest to combine the two representative SUSY breaking mediation scenarios, the mGrM and the minimal gauge mediation (mGgM) in a single supergravity (SUGRA) framework with a *common* SUSY breaking source. We will call it “minimal mixed mediation.”

The chiral SUGRA Lagrangian is basically described in terms of the Kähler potential K , superpotential W , and gauge kinetic function. First, let us consider the minimal Kähler potential, and a superpotential where the observable and hidden sectors are separated as in the ordinary mGrM [3]:

$$K = \sum_i |z_i|^2 + \sum_r |\phi_r|^2, \quad W = W_H(z_i) + W_O(\phi_r) \quad (5)$$

where z_i [ϕ_r] denotes fields at the hidden [observable] sector, carrying hidden [SM or GUT] gauge quantum numbers. The kinetic terms of z_i and ϕ_r , thus, take the canonical form. We assume non-zero VEVs for z_i s [4]:

$$\langle z_i \rangle = b_i M_P, \quad \langle \partial_{z_i} W_H \rangle = a_i^* m M_P, \quad \langle W_H \rangle = m M_P^2, \quad (6)$$

where a_i and b_i are dimensionless numbers, while M_P ($\approx 2.4 \times 10^{18}$ GeV) denotes the reduced Planck mass. Then, $\langle W_H \rangle$ or m gives the gravitino mass, $m_{3/2} = e^{K/2M_P} \langle W \rangle / M_P^2 = e^{|b_i|^2/2} m$. The soft terms can read from the scalar potential of SUGRA:

$$V_F = e^{\frac{K}{M_P^2}} \left[\sum_i |F_{z_i}|^2 + \sum_r |F_{\phi_r}|^2 - \frac{3}{M_P^2} |W|^2 \right] \quad (7)$$

where the “ F -terms,” F_i ($= D_i W = \partial_i W + \partial_i K W / M_P^2$) are given by

$$F_{z_i} = M_P \left[(a_i^* + b_i^*) m + b_i^* \frac{W_O}{M_P^2} \right], \quad F_{\phi_r} = \frac{\partial W_O}{\partial \phi_r} + \phi_r^* \left(m + \frac{W_O}{M_P^2} \right). \quad (8)$$

The vanishing cosmological constant (C.C.) requires a fine-tuning between $\langle F_{z_i} \rangle$ and $\langle W_H \rangle$, i.e. from Eq. (7) $\sum_i \langle |F_{z_i}|^2 \rangle = 3 |\langle W_H \rangle|^2 / M_P^2$, or $\sum_i |a_i + b_i|^2 = 3$. Neglecting the non-renormalizable terms suppressed with $1/M_P^2$, Eq. (7) is rewritten as [4]

$$V_F \approx \left| \partial_{\phi_r} \tilde{W}_O \right|^2 + m_0^2 |\phi_r|^2 + m_0 \left[\phi_r \partial_{\phi_r} \tilde{W}_O + (A_\Sigma - 3) \tilde{W}_O + \text{h.c.} \right]. \quad (9)$$

where A_Σ is defined as $A_\Sigma \equiv \sum_i b_i^* (a_i + b_i)$. m_0 is identified with the gravitino mass $m_{3/2}$ ($= e^{|b_i|^2/2} m$) and \tilde{W}_O ($\equiv e^{|b_i|^2/2} W_O$) denotes the rescaled W_O . From now on, we will drop out the “tilde” for a simple notation. The first term of Eq. (9) corresponds to the F -term potential in global SUSY, the second term is the universal soft mass term, and the remaining terms are A -terms. The *universal* A -parameter here ($\equiv A_0 = A_t$) does not include Yukawa coupling constants, but it is proportional to m_0 . If there is no quadratic term or higher powers of ϕ_r in W_O , one can get negative (positive) A -terms with $A_\Sigma < 2$ ($A_\Sigma > 2$). With the vanishing C.C. condition, the universal soft mass parameter, m_0 ($= e^{\langle K \rangle / 2M_P^2} \langle W_H \rangle / M_P^2$) can be recast to $e^{\langle K \rangle / 2M_P^2} (\sum_i \langle |F_{z_i} \rangle|^2)^{1/2} / \sqrt{3} M_P$, which is the conventional form in the mGrM scenario.

Next, let us introduce one pair of messenger superfields $\{\mathbf{5}, \bar{\mathbf{5}}\}$, which are the SU(5) fundamental representations, protecting the gauge coupling unification. Through their coupling with a SUSY breaking source S , which is an MSSM singlet superfield,

$$W_m = y_S S \mathbf{5} \bar{\mathbf{5}}, \quad (10)$$

the soft masses of the MSSM gauginos and scalar superpartners are generated at one- and two-loop levels, respectively [3]:

$$M_a = \frac{g_a^2}{16\pi^2} \frac{\langle F_S \rangle}{\langle S \rangle}, \quad m_i^2 = 2 \sum_{a=1}^3 \left[\frac{g_a^2}{16\pi^2} \frac{\langle F_S \rangle}{\langle S \rangle} \right]^2 C_a(i) \quad (11)$$

where $C_a(i)$ is the quadratic Casimir invariant for a superfield i , $(T^a T^a)_i^j = C_a(i) \delta_i^j$, and g_a ($a = 3, 2, 1$) denotes the MSSM gauge coupling constants. $\langle S \rangle$ and $\langle F_S \rangle$ are VEVs of the scalar and F -term components of the superfield S . Note that M_a and m_i^2 are almost independent of y_S only if $\langle F_S \rangle \lesssim y_S^2 \langle S \rangle$ [3]. However, such mGgM effects would appear below the messenger scale, $y_S \langle S \rangle$. Here we assume that $\langle S \rangle$ has the same magnitude as the VEV of the SU(5) breaking Higgs v_G : $\langle \mathbf{24}_H \rangle = v_G \times \text{diag.}(2, 2, 2; -3, -3) / \sqrt{60}$. It is possible if a GUT breaking mechanism causes $\langle S \rangle$. We provided a model based on SU(5) GUT in Ref. [2]. Actually, the masses of “X” and “Y” gauge bosons induced by $\langle \mathbf{24}_H \rangle$, $M_X^2 = M_Y^2 = \frac{5}{24} g_G^2 v_G^2$ [13], where g_G is the unified gauge coupling constant, can be identified with the MSSM gauge coupling unification scale, because the unified gauge interactions would become active above the $M_{X,Y}$ scale.

In addition to Eq. (5), the Kähler potential (and hidden local symmetries we don’t specify here) can permit

$$K \supset f(z)S + \text{h.c.}, \quad (12)$$

where $f(z)$ denotes a *holomorphic* monomial of hidden sector fields $z_i S$ with VEVs of order M_P in Eq. (6), and so it is of order $\mathcal{O}(M_P)$. Their kinetic terms still remain canonical. The $U(1)_R$ symmetry forbids $M_P f(z)S$ in the superpotential. Then, the resulting $\langle F_S \rangle$ can be

$$\langle F_S^* \rangle \approx m [\langle f(z) \rangle + \langle S^* \rangle] \quad (13)$$

by including the SUGRA corrections with $\langle W_H \rangle = m M_P^2$. Thus, the VEV of F_S is of order $\mathcal{O}(m M_P)$ like F_{z_i} in Eq. (8). They should be fine-tuned for the vanishing C.C.: a precise determination of $\langle F_S \rangle$ is indeed associated with the C.C. problem. Here we set $\langle F_S \rangle = m_0 M_P$. F_{ϕ_r} is still given by Eq. (8), which induces the universal soft mass terms at tree level.

Thus, the typical size of mGgM effects is estimated as

$$f_G \cdot m_0 \equiv \frac{\langle F_S \rangle}{16\pi^2 \langle S \rangle} = \frac{m_0 M_P}{16\pi^2 M_X} \sqrt{\frac{5}{24}} g_G \approx 0.36 \times m_0. \quad (14)$$

Here we set the unified gauge coupling at the GUT scale [$\approx (1.3 \pm 0.4) \times 10^{16}$ GeV] to $g_G^2/4\pi \approx 1/26$ due to relatively heavy colored superpartners ($\gtrsim 3$ TeV). We will present later the more general results, when f_G is taken to be a free parameter.

The fact that the mGgM effects by Eq. (11) are proportional to m_0 or m_0^2 are important. Moreover, A -terms from Eq. (9) are also proportional to m_0 . In this setup, thus, an (extrapolated) FP of $m_{h_u}^2$ must still exist at a higher energy scale [1, 2]. As C_g is converted to a member of C_s in Eq. (3), the naturalness of $m_{h_u}^2$ and m_Z^2 becomes gradually improved, making C_s smaller and smaller, until the FP reaches the stop decoupling scale.

For $|y_S| \lesssim 1$ in Eq. (10), the messenger scale Q_M drops down below $M_{X,Y}$. Since X and Y gauge sectors have already been decoupled below the messenger scale, the soft masses generated by the

mGgM in Eq. (11) become *non-universal* for $Q_M < M_{X,Y}$. Of course, the beta function coefficients of the MSSM fields should be modified above the scale of $y_S \langle S \rangle$ by the messenger fields $\{\mathbf{5}, \bar{\mathbf{5}}\}$. Thus, the RG equations of the MSSM gauge couplings and gaugino masses are

$$8\pi^2 \frac{dg_a^2}{dt} = b_a g_a^4, \quad 8\pi^2 \frac{dM_a}{dt} = b_a g_a^2 M_a, \quad (15)$$

where $t \equiv \log[Q/\text{GeV}]$, and $b_a = (-2, 2, \frac{38}{5})$ for $Q > Q_M$ while $b_a = (-3, 1, \frac{33}{5})$ for $Q < Q_M$. For the RG equations of the Yukawa couplings of the third generation of quarks and leptons (y_t, y_b, y_τ) and other soft parameters, refer to Appendix of Ref. [11].

The boundary conditions at the GUT scale are given by the universal form as seen in Eq. (9). Unlike the case of the mGrM, we have *additional* non-universal contributions by Eq. (11). They should be imposed at a given messenger scale, and so affect the RG evolutions of MSSM parameters for $Q \leq Q_M$. To see clearly how the original FP scenario is modified by the additional mGgM effects, we don't consider the superheavy RH neutrinos in the RG analysis as in [9], assuming their couplings are small enough, even if they are helpful for improving the naturalness [11, 14].

We also suppose that the gaugino masses from the mGrM are relatively suppressed. In fact, the gaugino mass term in SUGRA is associated with the first derivative of the gauge kinetic function [4], and so a constant gauge kinetic function at tree level ($= \delta_{ab}$) can realize it. In fact, it is the simplest case, yielding the canonical gauge kinetic terms in the Lagrangian. Accordingly, the gaugino masses by Eq. (11) dominates over them in this case. Then Eqs. (11), (14), and (15) admit a simple analytic expression for the gaugino masses at the stop mass scale:

$$M_a(t_T) = f_G m_0 \times g_a^2(t_T) \approx 0.36 \times m_0 \times g_a^2(t_T), \quad (16)$$

It does *not* depend on messenger scales.

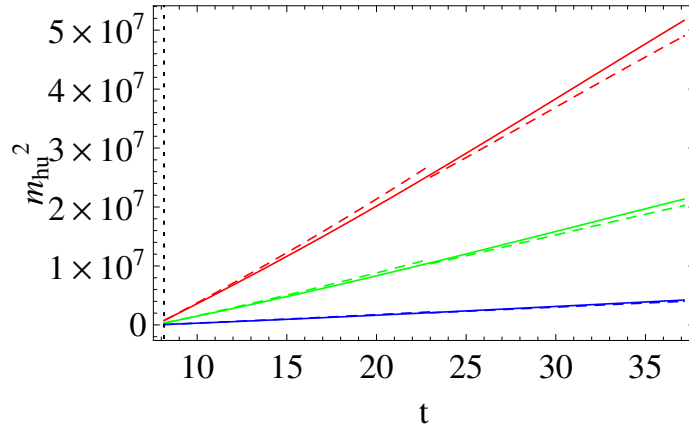


Figure 1: RG evolutions of $m_{h_u}^2$ with $t [\equiv \log(Q/\text{GeV})]$ for $m_0^2 = (7 \text{ TeV})^2$ [Red], $(4.5 \text{ TeV})^2$ [Green], and $(2 \text{ TeV})^2$ [Blue] when $A_t = -0.2 m_0$ and $\tan \beta = 50$. The tilted straight [dotted] lines correspond to the case of $t_M \approx 37$ (or $Q_M \approx 1.3 \times 10^{16} \text{ GeV}$, “Case A”) [$t_M \approx 23$ (or $Q_M = 1.0 \times 10^{10} \text{ GeV}$, “Case B”)]. The vertical dotted line at $t = t_T \approx 8.2$ ($Q_T = 3.5 \text{ TeV}$) indicates the desired stop decoupling scale. The discontinuities of $m_{h_u}^2(t)$ should appear at the messenger scales.

Fig. 1 displays RG evolutions of $m_{h_u}^2$ for various trial m_0^2 s. The straight [dotted] lines correspond to the case of $t_M \approx 37$ (or $Q_M \approx 1.3 \times 10^{16}$ GeV, “Case A”) [$t_M \approx 23$ (or $Q_M = 1.0 \times 10^{10}$ GeV, “Case B”)]. The discontinuities of the lines by additional boundary conditions arise at the messenger scales. As seen in Fig. 1, a FP of $m_{h_u}^2$ appears always at $t = t_T \approx 8.2$ (or $Q_T \approx 3.5$ TeV) regardless of the messenger scales or y_S that we take. Hence, the wide ranges of UV parameters can yield almost the same values of $m_{h_u}^2$ at low energy. Under such a situation, one can guess that $m_0^2 \approx (4.5 \text{ TeV})^2$ happens to be selected, yielding 3-4 TeV stop mass, and so eventually gets responsible for the 126 GeV Higgs mass.

In both cases of Fig. 1, the gluino, wino, and bino masses at low energy are

$$M_{3,2,1} \approx \{1.7 \text{ TeV}, 660 \text{ GeV}, 360 \text{ GeV}\} \quad (17)$$

for $m_0^2 = (4.5 \text{ TeV})^2$. They would be testable at LHC Run II. A_t at low energy is about 1 TeV for Case A and B. Consequently, the contributions of A_t^2/\tilde{m}_t^2 to the radiative Higgs mass are smaller than 2.3 % of those by the stops.

In the above cases, the sleptons and sbottom turn out to be quite heavier than 3 TeV. The first two generations of SUSY particles would be much heavier than them. Hence, the bino is the lightest superparticle (LSP). To avoid overclose of the bino dark matter in the Universe, some entropy production [15] or other lighter dark matter such as the axino and axion is needed.

Figs 2 and 3 show various scatter plots for given ranges of $\{f_G, a_Y (\equiv A_t/m_0)\}$ with $\tan\beta = 50$. Here we regarded f_G as a free parameter. Actually $\tan\beta = 50$ is easily obtained e.g. from the minimal SO(10) GUT [13] or even from the MSSM embedded in a class of the heterotic stringy models [16]. m_0^2 in Figs 2 and 3 are taken, respectively, to be $(4 \text{ TeV})^2$ and $(5 \text{ TeV})^2$. As a result, the stop mass scales are about 3.0 and 3.7 TeV, respectively. Here we set M_G as the scale where the EW gauge couplings, g_2 and g_1 meet. It is approximately 1.7×10^{16} GeV in these cases. They all are drawn using SOFTSUSY-3.6.2 [17]. They have “rainbow” shapes. The two “legs” of the “rainbow” in those figures, which are located in the left and right sides for the figures, are relatively narrow. For a small enough fine-tuning measure $\Delta_{a_Y} (\equiv \left| \frac{\partial \log m_Z^2}{\partial \log a_Y} \right| = \left| \frac{a_Y}{m_Z^2} \frac{\partial m_Z^2}{\partial a_Y} \right|$ [18]), we are more interested in the thick central parts around $a_Y = 0$ in the figures,

$$-0.7 \lesssim a_Y \lesssim 0.5, \quad (18)$$

which satisfies $\Delta_{a_Y} < 100$. Here we confine our discussion to cases of $|\mu| < 600 \text{ GeV}$. In fact, the constraint associated with μ or heavy gluino effects could be relaxed by assuming very heavy masses for the superpartners of the first and second generations of the SM chiral fermions [11]. For simplicity, however, we don’t consider such a possibility here. Below $f_G \approx 0.3$, the EW symmetry breaking does not occur. From Figs 2 and 3, thus, f_G is constrained to

$$0.3 \lesssim f_G \lesssim 0.4, \quad (19)$$

which is consistent with $\Delta_{m_0^2} = \left| \frac{m_0^2}{m_Z^2} \frac{\partial m_Z^2}{\partial m_0^2} \right| < 100$ as seen in Figs 2 and 3. From Figs 2 and 3, we see that the above ranges confine the physical gluino mass to

$$1.6 \text{ TeV} \lesssim m_{\tilde{g}} \lesssim 2.2 \text{ TeV}. \quad (20)$$

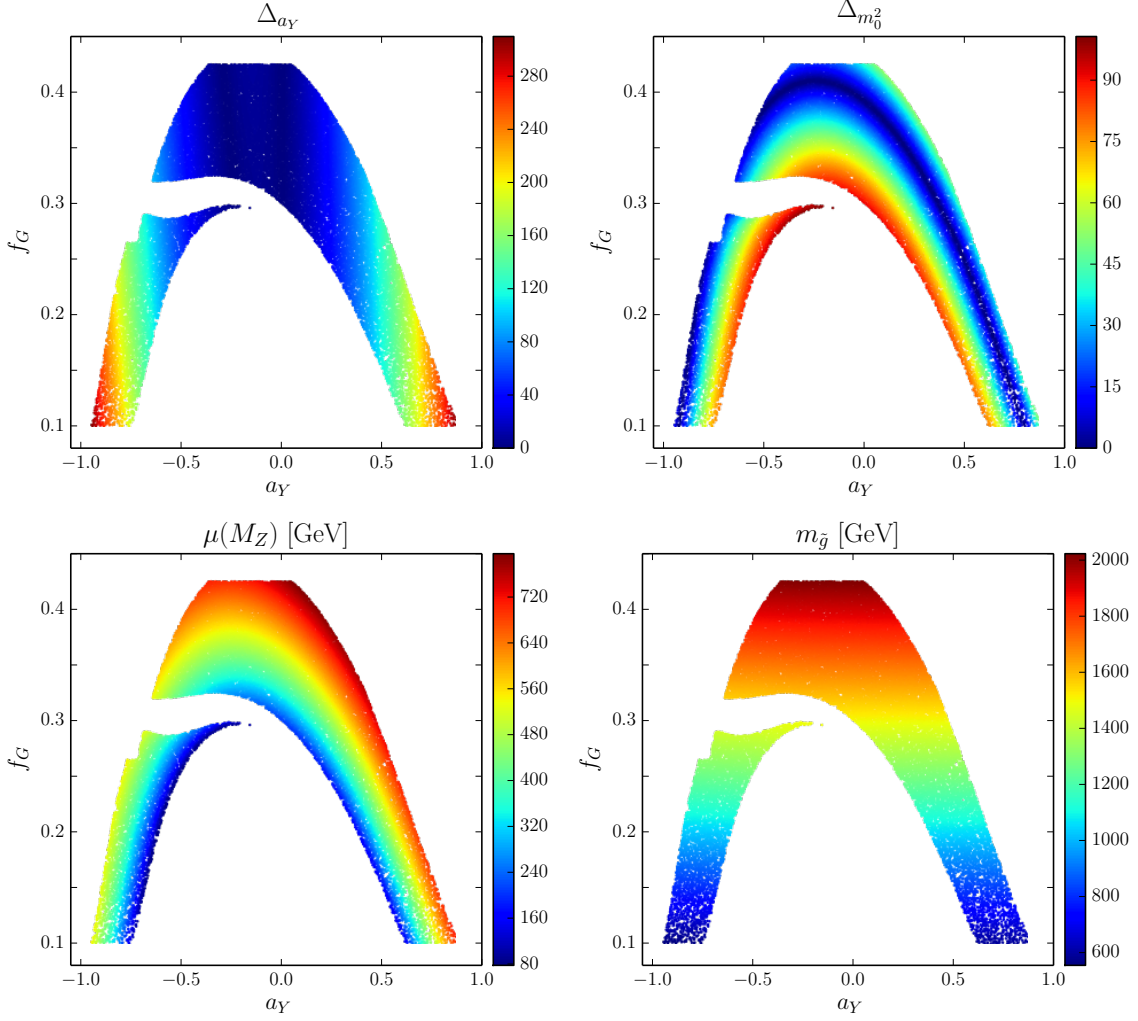


Figure 2: Scatter plots for Δa_Y , Δm_0^2 , and $|\mu|$ at the M_Z scale, and physical gluino mass when $m_0^2 = (4 \text{ TeV})^2$ and $\tan \beta = 50$. The stop mass scale is about 3.0 TeV.

Note that this gluino mass bound is a theoretical constraint obtained by considering the naturalness of the EW scale in the Minimal Mixed Mediation scenario. It is well inside the discovery potential range of LHC Run II. Actually the relevant energy scale for the naturalness of the low energy SUSY in the Minimal Mixed Mediation scenario was outside the range of LHC Run I, but it can be covered by LHC Run II. Accordingly, the future exploration for the SUSY particle, particularly, the gluino at the LHC would be more important.

In conclusion, we have studied the SUSY breaking effects by the mGrM parametrized with m_0 , combined with the mGgM parametrized with $f_G \cdot m_0$ for a common SUSY breaking source at a hidden sector, $\langle W_H \rangle (\sim m_0 M_P^2)$ in a SUGRA framework. When the minimal Kähler potential and the minimal gauge kinetic function ($= \delta_{ab}$) are employed at tree level, a FP of $m_{h_u}^2$ appears a bit higher energy scale than m_Z (“shifted FP”), depending on f_G . Basically f_G is a parameter determined by a model. For $0.3 \lesssim f_G \lesssim 0.4$, the FP of $m_{h_u}^2$ emerges at 3-4 TeV scale, which is the stop mass scale desired for explaining the 125 GeV Higgs mass, and so $m_{h_u}^2$ becomes quite

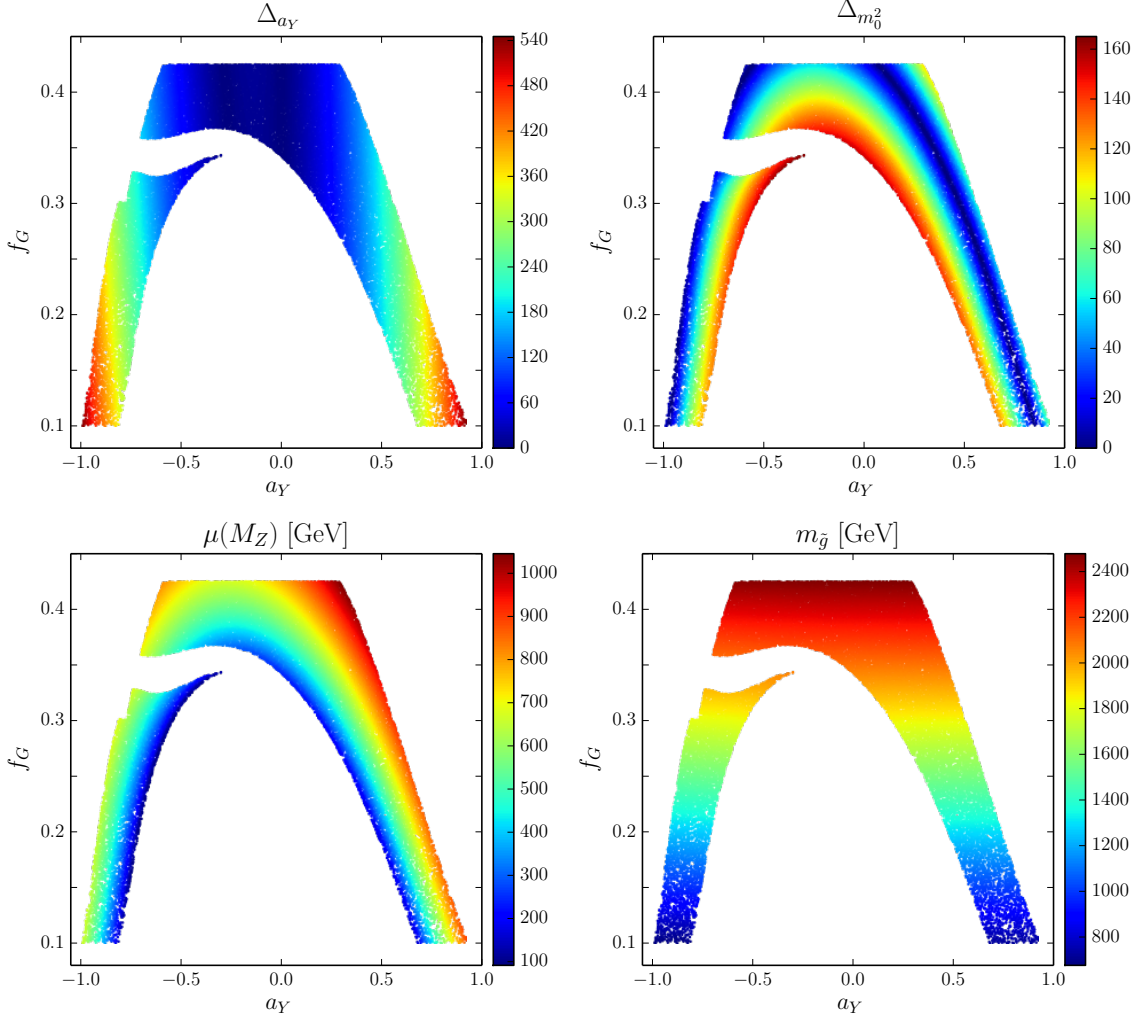


Figure 3: Scatter plots for Δ_{a_Y} , $\Delta_{m_0^2}$, and $|\mu|$ at the M_Z scale, and physical gluino mass when $m_0^2 = (5 \text{ TeV})^2$ and $\tan\beta = 50$. The stop mass scale is about 3.7 TeV .

insensitive to stop masses or m_0^2 . Thus, this range of f_G and $-0.7 \lesssim a_Y \lesssim 0.3$ can admit the fine-tuning measures and μ to be much smaller than 100 and 600 GeV, respectively. The range $0.3 \lesssim f_G \lesssim 0.4$ is directly translated into e.g. the gluino mass bound, $1.6 \text{ TeV} \lesssim m_{\tilde{g}} \lesssim 2.2 \text{ TeV}$, which could readily be tested at LHC Run II in the near future.

References

- [1] B. Kyae, Phys. Rev. D **92**, no. 1, 015027 (2015) [arXiv:1502.02311 [hep-ph]].
- [2] D. Kim and B. Kyae, Phys. Rev. D **92**, no. 7, 075025 (2015) [arXiv:1507.07611 [hep-ph]].
- [3] For a review, for instance, see M. Drees, R. Godbole and P. Roy, “Theory and phenomenology of sparticles: An account of four-dimensional N=1 supersymmetry in high energy physics,” Hackensack, USA: World Scientific (2004) 555 p. References are therein.
- [4] H. P. Nilles, Phys. Rept. **110**, 1 (1984).

- [5] See, for instance, B. Kyae and J. C. Park, Phys. Rev. D **86**, 031701 (2012) [arXiv:1203.1656 [hep-ph]]; B. Kyae and J. C. Park, Phys. Rev. D **87**, 075021 (2013) [arXiv:1207.3126 [hep-ph]]; B. Kyae and C. S. Shin, Phys. Rev. D **88**, no. 1, 015011 (2013) [arXiv:1212.5067 [hep-ph]]; B. Kyae and C. S. Shin, JHEP **1306**, 102 (2013) [arXiv:1303.6703 [hep-ph]]; B. Kyae, Phys. Rev. D **89**, no. 7, 075016 (2014) [arXiv:1401.1878 [hep-ph]].
- [6] G. Aad *et al.* [ATLAS Collaboration], ATLAS-CONF-2013-024; S. Chatrchyan *et al.* [CMS Collaboration], Eur. Phys. J. C **73**, 2677 (2013) [arXiv:1308.1586 [hep-ex]].
- [7] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716**, 1 (2012) [arXiv:1207.7214 [hep-ex]]; S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716**, 30 (2012) [arXiv:1207.7235 [hep-ex]].
- [8] J. L. Feng, P. Kant, S. Profumo and D. Sanford, Phys. Rev. Lett. **111**, 131802 (2013) [arXiv:1306.2318 [hep-ph]].
- [9] J. L. Feng, K. T. Matchev and T. Moroi, Phys. Rev. Lett. **84**, 2322 (2000) [hep-ph/9908309].
- [10] G. Aad *et al.* [ATLAS Collaboration], JHEP **1409**, 176 (2014) [arXiv:1405.7875 [hep-ex]].
- [11] B. Kyae and C. S. Shin, Phys. Rev. D **90**, no. 3, 035023 (2014) [arXiv:1403.6527 [hep-ph]].
- [12] M. S. Carena, M. Quiros and C. E. M. Wagner, Nucl. Phys. B **461**, 407 (1996) [hep-ph/9508343].
- [13] G. G. Ross, "Grand Unified Theories," Reading, Usa: Benjamin/cummings (1984) 497 P. (Frontiers In Physics, 60)
- [14] K. Kadota and K. A. Olive, Phys. Rev. D **80** (2009) 095015 [arXiv:0909.3075 [hep-ph]]; M. Asano, T. Moroi, R. Sato and T. T. Yanagida, Phys. Lett. B **708** (2012) 107 [arXiv:1111.3506 [hep-ph]].
- [15] M. Asano, T. Moroi, R. Sato and T. T. Yanagida, Phys. Lett. B **708**, 107 (2012) [arXiv:1111.3506 [hep-ph]].
- [16] See, for instance, J. E. Kim and B. Kyae, Nucl. Phys. B **770**, 47 (2007) [hep-th/0608086]; J. E. Kim, J. H. Kim and B. Kyae, JHEP **0706**, 034 (2007) [hep-ph/0702278 [HEP-PH]]; J. H. Huh, J. E. Kim and B. Kyae, Phys. Rev. D **80**, 115012 (2009) [arXiv:0904.1108 [hep-ph]]; K. S. Choi and B. Kyae, Nucl. Phys. B **855**, 1 (2012) [arXiv:1102.0591 [hep-th]].
- [17] B. C. Allanach, Comput. Phys. Commun. **143**, 305 (2002) [hep-ph/0104145].
- [18] J. R. Ellis, K. Enqvist, D. V. Nanopoulos and F. Zwirner, Mod. Phys. Lett. A **1**, 57 (1986); R. Barbieri and G. F. Giudice, Nucl. Phys. B **306**, 63 (1988).