

Novel Scenarios for Majorana Neutrino Mass Generation and Leptogenesis from Kalb-Ramond Torsion.

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The Kalb-Ramond (KR) antisymmetric tensor field arises naturally in the gravitational multiplet of string theory. Nevertheless, the respective low-energy field theory action, in which, for reasons of gauge invariance, the only dependence on the KR field is through its field strength, constitutes an interesting model *per se*. In this context, the KR field strength also acts as a totally antisymmetric torsion field, while in four space-time dimensions is *dual* to an (KR) axion-like pseudoscalar field. In this context, we review here first the rôle of quantum fluctuations of the KR axion on the generation of Majorana mass for neutrinos, via a mixing with ordinary axions that may exist in the theory as providers of dark matter candidates. Then we proceed to discuss the rôle of constant in time (thus Lorentz violating) KR torsion backgrounds, that may exist in the early Universe but are completely negligible today, on inducing Leptogenesis by means of *tree-level* CP violating decays of Right Handed Massive Majorana neutrinos in the presence of such H-torsion backgrounds. Some speculations regarding microscopic D-brane world models, where such scenarios may be realised, are also given.

*18th International Conference From the Planck Scale to the Electroweak Scale
25-29 May 2015
Ioannina, Greece*

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1. Introduction and Summary

The discovery [1] of the Higgs boson at the CERN Large Hadron Collider (LHC) in 2012 constitutes an important milestone for the Ultra-Violet (UV) completion of the Standard Model (SM), verifying that a Higgs-like mechanism can explain the generation of most of the particle masses in the SM. Nevertheless, the origin of the small neutrino masses still remains an open issue. One such mechanism providing a rather natural explanation of the observed smallness of the light neutrino masses is the so-called *see-saw* mechanism [2], which necessitates the Majorana nature of the light (active) neutrinos and postulates the presence of heavy right-handed Majorana partners of mass M_R . The right-handed Majorana mass M_R is usually considered to be much larger than the lepton or quark masses. The origin of M_R has been the topic of several extensions of the SM in the literature, within the framework of quantum field theory [2, 3] and string theory [4]. However, up to now, there is no experimental evidence for right-handed neutrinos or, in fact, for any extension of the SM, although some optimism of discovering supersymmetry in the current round of LHC (operating ultimately at 14 TeV energies) exists among particle physicists [5].

Until therefore such extensions of the SM are discovered, it is legitimate to search for alternative mechanisms for neutrino mass generation, that keep the spectrum of SM intact, except perhaps for the existence of right handed neutrinos that are allowed. Such minimal, non supersymmetric extensions of the Standard Model with three in fact right-handed Majorana neutrinos complementing the three active left-handed neutrinos (termed ν MSSM), have been proposed [6], in a way consistent with current cosmology. Such models are characterised by relatively light right-handed neutrinos, two of which are almost degenerate, with masses of order GeV, and a much lighter one, almost decoupled, with masses in the keV range, which may play the role of warm dark matter. The right-handed neutrinos serve the purpose of generating, through seesaw type mechanisms, the active neutrino mass spectrum, consistent with observed flavour oscillations. However, there are no suggestions for microscopic mechanisms for the generation of the right-handed neutrino mass spectrum in such scenarios.

Motivated by these facts we first review in this talk, in section 2, an alternative proposal for dynamical generation of Majorana mass for neutrinos propagating in space-time geometries with quantum-fluctuating torsion [7]. Microscopically this torsion may be provided by the spin-one antisymmetric tensor (Kalb-Ramond (KR)) field [8] of the gravitational multiplet of a string [9, 10], but the low energy theory of the KR torsion is an interesting field theory model that can be studied in its own right. In this scenario, the totally antisymmetric part of the torsion couples, via the gravitational covariant derivative, to all fermions in a way that the resulting interaction resembles that of the Lorentz and CPT-Violating pseudovector background with the axial fermion current in the Standard Model Extension (SME) of Kostelecky and collaborators [11]. The generation of (right-handed, sterile) neutrino masses in that case proceeds, as we shall review below, via chiral anomalous three-loop graphs of neutrinos interacting with the totally antisymmetric torsion quantum field. In four space-time dimensions, the latter is represented as an axion field [9, 10], whose mixing with ordinary axion fields, that in turn interact with the Majorana right-handed neutrinos via chirality changing Yukawa couplings, is held responsible for the right-handed Majorana neutrino mass generation through the aforementioned anomalous graphs. In the second part of the talk, in section 3, we discuss non-trivial backgrounds of the KR torsion field, constant in cosmic time,

and thus Lorentz-violating, whose coupling to the fermion axial current induces extra sources of CP violation, which in the case of right-handed Majorana neutrinos assumed abundant in the early Universe, may lead to Leptogenesis [12, 13]. Finally, in the last part of the talk, section 4, instead of conclusions, I present some speculations as to how the above scenarios of Leptogenesis, which require relatively large backgrounds of the KR torsion in early epochs of the Universe, but negligible today, due to phenomenological reasons, may be realised in some microscopic string/brane models of the Universe, in which our four space-time dimensional brane world propagates in a bulk punctured by populations of point-like D-brane defects, interacting with right-handed neutrinos [14]. Some technical aspects of our approach are outlined in an Appendix.

2. Neutrino Mass Generation due to Quantum (Kalb-Ramond) Torsion

Let us for concreteness consider Dirac fermions in a torsionful space-time [15]. The extension to the Majorana case is straightforward. The relevant action reads ¹:

$$S_\psi = \frac{i}{2} \int d^4x \sqrt{-g} \left(\bar{\psi} \gamma^\mu \bar{\mathcal{D}}_\mu \psi - (\bar{\mathcal{D}}_\mu \bar{\psi}) \gamma^\mu \psi \right) \quad (2.1)$$

where $\bar{\mathcal{D}}_\mu = \bar{\nabla}_\mu + \dots$, is the covariant derivative (including gravitational and gauge-field connection parts (the latter represented by \dots), in case the fermions are charged). The overline above the covariant derivative, i.e. $\bar{\nabla}_\mu$, denotes the presence of torsion,

$$\bar{\nabla}_a \psi = e_a^\mu \left(\partial_\mu + \frac{i}{2} \bar{\omega}_{b\mu c} \Sigma^{bc} \right) \psi. \quad (2.2)$$

In the formula above $\Sigma^{ab} = \frac{i}{4} [\gamma^a, \gamma^b]$ is the generator of the Lorentz group representation on four-spinors, while $\bar{\omega}_{ab\mu}$ is the torsionful spin connection

$$\bar{\omega}_\mu^{ab} = e_\nu^a \bar{\nabla}_\mu e^{b\nu} = e_\nu^a \left(\partial_\mu e^{b\nu} + \bar{\Gamma}_{\mu\lambda}^\nu e^{b\lambda} \right). \quad (2.3)$$

with $\bar{\Gamma}_{\nu\rho}^\mu \neq \bar{\Gamma}_{\rho\nu}^\mu$ the generalised Christoffel symbols in the presence of torsion.

The torsionful spin connection can be decomposed as: $\bar{\omega}_{ab\mu} = \omega_{ab\mu} + K_{ab\mu}$, where $K_{ab\mu}$ is the contorsion tensor. The latter is related to the torsion two-form $\mathbf{T}^a = \mathbf{d} e^a + \bar{\omega}^a \wedge e^b$ via [15, 10]: $K_{abc} = \frac{1}{2} (\mathbf{T}_{cab} - \mathbf{T}_{abc} - \mathbf{T}_{bcd})$. The presence of torsion in the covariant derivative in the action (2.1) leads, apart from the standard terms in manifolds without torsion, to an additional term involving the axial current

$$J_5^\mu \equiv \bar{\psi} \gamma^\mu \gamma^5 \psi. \quad (2.4)$$

The relevant part of the action reads:

$$S_\psi \ni -\frac{3}{4} \int d^4x \sqrt{-g} S_\mu \bar{\psi} \gamma^\mu \gamma^5 \psi = -\frac{3}{4} \int S \wedge *J^5 \quad (2.5)$$

¹The second term in the right-hand-side of (2.1) is usually not written in flat space, as its contribution is equal to the first term plus a surface integral. However, the situation is different when spacetime is not flat. In fact the second term is needed in order preserve unitarity, allowing for the cancellation of an anti-hermitean term involving the trace of the spin-connection coefficients $\bar{\omega}_{ac}^a$.

where $\mathbf{S} = \star\mathbf{T}$ is the dual of \mathbf{T} : $S_d = \frac{1}{3!}\epsilon^{abc} T_{abc}$.

We next remark that the torsion tensor can be decomposed into its irreducible parts [15], of which S_d is the pseudoscalar axial vector: $T_{\mu\nu\rho} = \frac{1}{3}(T_\nu g_{\mu\rho} - T_\rho g_{\mu\nu}) - \frac{1}{3!}\epsilon_{\mu\nu\rho\sigma} S^\sigma + q_{\mu\nu\rho}$, with $\epsilon_{\mu\nu\rho\sigma} q^{\nu\rho\sigma} = q_{\rho\nu}^\nu = 0$. This implies that the contorsion tensor undergoes the following decomposition:

$$K_{abc} = \frac{1}{2}\epsilon_{abcd} S^d + \widehat{K}_{abc} \quad (2.6)$$

where \widehat{K} includes the trace vector T_μ and the tensor $q_{\mu\nu\rho}$ parts of the torsion tensor.

The gravitational part of the action can then be written as: $S_G = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} (R + \widehat{\Delta}) + \frac{3}{4\kappa^2} \int \mathbf{S} \wedge \star\mathbf{S}$, where $\widehat{\Delta} = \widehat{K}^\lambda_{\mu\nu} \widehat{K}^{\nu\mu}_\lambda - \widehat{K}^{\mu\nu}_\nu \widehat{K}_{\mu\lambda}^\lambda$, with the hatted notation defined in (2.6).

In a quantum gravity setting, where one integrates over all fields, the torsion terms appear as non propagating fields and thus they can be integrated out exactly. The authors of [10] have observed though that the classical equations of motion identify the axial-pseudovector torsion field S_μ with the axial current, since the torsion equation yields

$$K_{\mu ab} = -\frac{1}{4} e_\mu^c \epsilon_{abcd} \bar{\Psi} \gamma^5 \tilde{\gamma}^d \Psi. \quad (2.7)$$

From this it follows $\mathbf{d}\star\mathbf{S} = 0$, leading to a conserved ‘‘torsion charge’’ $Q = \int \star\mathbf{S}$. To maintain this conservation in quantum theory, they postulated $\mathbf{d}\star\mathbf{S} = 0$ at the quantum level, which can be achieved by the addition of judicious counter terms. This constraint, in a path-integral formulation of quantum gravity, is then implemented via a delta function constraint, $\delta(d\star\mathbf{S})$, and the latter via the well-known trick of introducing a Lagrange multiplier field $\Phi(x) \equiv (3/\kappa^2)^{1/2} b(x)$. Hence, the relevant torsion part of the quantum-gravity path integral would include a factor

$$\begin{aligned} \mathcal{Z} &\propto \int D\mathbf{S} D b \exp \left[i \int \frac{3}{4\kappa^2} \mathbf{S} \wedge \star\mathbf{S} - \frac{3}{4} \mathbf{S} \wedge \star\mathbf{J}^5 + \left(\frac{3}{2\kappa^2} \right)^{1/2} b d\star\mathbf{S} \right] \\ &= \int D b \exp \left[-i \int \frac{1}{2} \mathbf{d}b \wedge \star\mathbf{d}b + \frac{1}{f_b} \mathbf{d}b \wedge \star\mathbf{J}^5 + \frac{1}{2f_b^2} \mathbf{J}^5 \wedge \star\mathbf{J}^5 \right], \end{aligned} \quad (2.8)$$

where $f_b = (3\kappa^2/8)^{-1/2} = \frac{M_P}{\sqrt{3}\pi}$ and the non-propagating \mathbf{S} field has been integrated out. The reader should notice that, as a result of this integration, the corresponding *effective* field theory contains a *non-renormalizable* repulsive four-fermion axial current-current interaction, characteristic of any torsionful theory [15].

The torsion term, being geometrical, due to gravity, couples universally to all fermion species, not only neutrinos. Thus, in the context of the SM of particle physics, the axial current (2.4) is expressed as a sum over fermion species

$$J_5^\mu \equiv \sum_{i=\text{fermion species}} \bar{\Psi}_i \gamma^\mu \gamma^5 \Psi_i. \quad (2.9)$$

In theories with chiral anomalies, like the quantum electrodynamics part of SM, the axial current is not conserved at the quantum level, due to anomalies, but its divergence is obtained by the one-loop result [16]:

$$\begin{aligned} \nabla_\mu J_5^\mu &= \frac{e^2}{8\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu} - \frac{1}{192\pi^2} R^{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma} \\ &\equiv G(\mathbf{A}, \omega). \end{aligned} \quad (2.10)$$

We may then partially integrate the second term in the exponent on the right-hand-side of (2.8) and take into account (2.10). The reader should observe that in (2.10) the torsion-free spin connection has been used. This can be achieved by the addition of proper counter terms in the action [10], which can convert the anomaly from the initial $G(\mathbf{A}, \bar{\omega})$ to $G(\mathbf{A}, \omega)$. Using (2.10) in (2.8) one can then obtain for the effective torsion action in theories with chiral anomalies, such as the QED part of the SM:

$$\int Db \exp \left[-i \int \frac{1}{2} \mathbf{d}b \wedge \star \mathbf{d}b - \frac{1}{f_b} b G(\mathbf{A}, \omega) + \frac{1}{2f_b^2} \mathbf{J}^5 \wedge \star \mathbf{J}^5 \right]. \quad (2.11)$$

A concrete example of torsion is provided by string-inspired theories, where the totally anti-symmetric component S_μ of the torsion is identified with the field strength of the spin-one anti-symmetric tensor (Kalb-Ramond (KR) [8]) field $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$, where the symbol [...] denotes antisymmetrization of the appropriate indices. The string theory effective action depends only on $H_{\mu\nu\rho}$ as a consequence of the ‘‘gauge symmetry’’ $B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_{[\mu} \Theta_{\nu]}$ that characterises all string theories. Indeed, in the Einstein frame, to first order in the string Regge α' , the bosonic part of the low-energy effective action (in four large target-space-time dimensions) is given by [9]

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(R - 2\partial^\mu \phi \partial_\mu \phi - e^{-4\phi} H_{\lambda\mu\nu} H^{\lambda\mu\nu} - V(\phi) \right), \quad (2.12)$$

where $\frac{1}{\kappa^2} \equiv \frac{M_s^2 \Omega^c}{8\pi} = \frac{1}{8\pi G}$, with G the four-dimensional (gravitational) constant, $M_s^2 = 1/\alpha'$ is the string mass scale, Ω^c the (dimensionless) compactification volume in units of the Regge slope α' of the string and $V(\phi)$ is a dilaton potential.

It can be shown [9] that the terms of the effective action up to and including quadratic order in the Regge slope parameter α' , of relevance to the low-energy (field-theory) limit of string theory, which involve the H-field strength, can be assembled in such a way that only torsionful Christoffel symbols, $\bar{\Gamma}_{\nu\rho}^\mu$ appear:

$$\bar{\Gamma}_{\mu\nu}^\lambda = \Gamma_{\mu\nu}^\lambda + e^{-2\phi} H_{\mu\nu}^\lambda \neq \bar{\Gamma}_{\nu\mu}^\lambda. \quad (2.13)$$

where $\Gamma_{\nu\rho}^\mu = \Gamma_{\rho\nu}^\mu$ is the ordinary, torsion-free, symmetric connection, and κ is the gravitational constant. In four space-time dimensions, the dual of the H-field is indeed the derivative of an axion-like field,

$$\partial^\mu b = -\frac{1}{4} \varepsilon^{abcd} e^{-2\phi} H_{abc}. \quad (2.14)$$

where $b(x)$ is a pseudoscalar field, entirely analogous to the field b above, hence the use of the same letter to describe it. It is termed the Kalb-Ramond (KR) (or ‘‘gravitational’’) axion field, to distinguish from the ordinary axion fields $a(x)$ that can play the rôle of dark matter in particle physics models.

We mention at this point that background geometries with (approximately) constant background H_{ijk} torsion, where Latin indices denote spatial components of the four-dimensional space-time, may characterise the early universe. In such cases, as we shall discuss in section 3, the H-torsion background constitutes extra source of CP violation, necessary for leptogenesis, and through Baryon-minus-Lepton-number (B-L) conserving processes, Baryogenesis, and thus the observed matter-antimatter asymmetry in the Universe [12, 13]. Today of course any torsion background should be strongly suppressed, due to the lack of any experimental evidence for it. Scenarios as to how such cosmologies can evolve so as to guarantee the absence of any appreciable traces of torsion today can be found in [13] and will be briefly reported upon in sections 3 and 4.

In the remainder of this section, we shall consider the effects of the quantum fluctuations of torsion, which survive the absence of any torsion background. An important aspect of the coupling of the torsion (or KR axion) quantum field $b(x)$ to the fermionic matter discussed above is its shift symmetry, characteristic of an axion field. Indeed, by shifting the field $b(x)$ by a constant: $b(x) \rightarrow b(x) + c$, the action (2.11) only changes by total derivative terms, such as $cR^{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma}$ and $cF^{\mu\nu}\tilde{F}_{\mu\nu}$. These terms are irrelevant for the equations of motion and the induced quantum dynamics, provided the fields fall off sufficiently fast to zero at space-time infinity. The scenario for the anomalous Majorana mass generation through torsion proposed in [7], and reviewed here, consists of augmenting the effective action (2.11) by terms that break such a shift symmetry. To illustrate this last point, we first couple the KR axion $b(x)$ to another pseudoscalar axion field $a(x)$. In string-inspired models, such pseudoscalar axion $a(x)$ may be provided by the string moduli [17] and play the rôle of a dark matter candidate. The proposed coupling occurs through a mixing in the kinetic terms of the two fields. To be specific, we consider the action (henceforth we restrict ourselves to right-handed Majorana neutrino fermion fields)

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2}(\partial_\mu b)^2 + \frac{b(x)}{192\pi^2 f_b} R^{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma} + \frac{1}{2f_b^2} J_\mu^5 J^{5\mu} + \gamma(\partial_\mu b)(\partial^\mu a) + \frac{1}{2}(\partial_\mu a)^2 - y_a i a \left(\bar{\Psi}_R^C \Psi_R - \bar{\Psi}_R \Psi_R^C \right) \right], \quad (2.15)$$

where $\Psi_R^C = (\Psi_R)^C$ is the charge-conjugate right-handed fermion Ψ_R , $J_\mu^5 = \bar{\Psi} \gamma_\mu \gamma_5 \Psi$ is the axial current of the four-component Majorana fermion $\Psi = \Psi_R + (\Psi_R)^C$, and γ is a real parameter to be constrained later on. Here, we have ignored gauge fields, which are not of interest to us, and the possibility of a non-perturbative mass M_a for the pseudoscalar field $a(x)$. Moreover, we remind the reader that the *repulsive* self-interaction fermion terms are due to the existence of torsion in the Einstein-Cartan theory. The Yukawa coupling y_a of the axion moduli field a to right-handed sterile neutrino matter Ψ_R may be due to non perturbative effects. These terms *break* the shift symmetry: $a \rightarrow a + c$.

It is convenient to diagonalize the axion kinetic terms by redefining the KR axion field as follows: $b(x) \rightarrow b'(x) \equiv b(x) + \gamma a(x)$. This implies that the effective action (2.15) becomes:

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2}(\partial_\mu b')^2 + \frac{1}{2}(1 - \gamma^2)(\partial_\mu a)^2 + \frac{1}{2f_b^2} J_\mu^5 J^{5\mu} + \frac{b'(x) - \gamma a(x)}{192\pi^2 f_b} R^{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma} - y_a i a \left(\bar{\Psi}_R^C \Psi_R - \bar{\Psi}_R \Psi_R^C \right) \right]. \quad (2.16)$$

Thus we observe that the b' field has decoupled and can be integrated out in the path integral, leaving behind an axion field $a(x)$ coupled both to matter fermions and to the operator $R^{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma}$, thereby playing now the rôle of the torsion field. We observe though that the approach is only valid for $|\gamma| < 1$, otherwise the axion field would appear as a ghost, *i.e.* with the wrong sign of its kinetic terms, which would indicate an instability of the model. This is the only restriction of the parameter γ . In this case we may redefine the axion field so as to appear with a canonical normalised kinetic term, implying the effective action:

$$\mathcal{S}_a = \int d^4x \sqrt{-g} \left[\frac{1}{2}(\partial_\mu a)^2 - \frac{\gamma a(x)}{192\pi^2 f_b \sqrt{1 - \gamma^2}} R^{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma} - \frac{i y_a}{\sqrt{1 - \gamma^2}} a \left(\bar{\Psi}_R^C \Psi_R - \bar{\Psi}_R \Psi_R^C \right) + \frac{1}{2f_b^2} J_\mu^5 J^{5\mu} \right]. \quad (2.17)$$

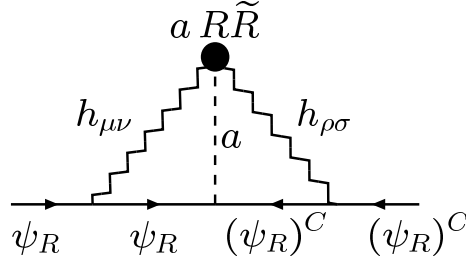


Figure 1: Typical Feynman graph giving rise to anomalous fermion mass generation [7]. The black circle denotes the operator $a(x)R_{\mu\nu\lambda\rho}\tilde{R}^{\mu\nu\lambda\rho}$ induced by torsion. The fields $h_{\mu\nu}$ denote graviton fluctuations.

Evidently, the action \mathcal{S}_a in (2.17) corresponds to a canonically normalised axion field $a(x)$, coupled both to the curvature of space-time, à la torsion, with a modified coupling $\gamma/(192\pi^2 f_b \sqrt{1-\gamma^2})$, and to fermionic matter with chirality-changing Yukawa-like couplings of the form $y_a/\sqrt{1-\gamma^2}$.

The mechanism for the anomalous Majorana mass generation is shown in Fig. 1. We may now estimate the two-loop Majorana neutrino mass in quantum gravity with an effective UV energy cut-off Λ . Adopting the effective field-theory framework of [18], the gravitationally induced Majorana mass for right-handed neutrinos, M_R , is estimated to be:

$$M_R \sim \frac{1}{(16\pi^2)^2} \frac{y_a \gamma \kappa^4 \Lambda^6}{192\pi^2 f_b (1-\gamma^2)} = \frac{\sqrt{3} y_a \gamma \kappa^5 \Lambda^6}{49152\sqrt{8}\pi^4 (1-\gamma^2)}. \quad (2.18)$$

In a UV complete theory such as strings, the cutoff Λ and the Planck mass scale M_P are related.

It is interesting to provide a numerical estimate of the anomalously generated Majorana mass M_R . Assuming that $\gamma \ll 1$, the size of M_R may be estimated from (2.18) to be

$$M_R \sim (3.1 \times 10^{11} \text{ GeV}) \left(\frac{y_a}{10^{-3}} \right) \left(\frac{\gamma}{10^{-1}} \right) \left(\frac{\Lambda}{2.4 \times 10^{18} \text{ GeV}} \right)^6. \quad (2.19)$$

Obviously, the generation of M_R is highly model dependent. Taking, for example, the quantum gravity scale to be $\Lambda = 10^{17}$ GeV, we find that M_R is at the TeV scale, for $y_a = 10^{-3}$ and $\gamma = 0.1$. However, if we take the quantum gravity scale to be close to the GUT scale, i.e. $\Lambda = 10^{16}$ GeV, we obtain a right-handed neutrino mass $M_R \sim 16$ keV, for the choice $y_a = \gamma = 10^{-3}$. This is in the preferred ballpark region for the sterile neutrino ψ_R to qualify as a warm dark matter [6].

In a string-theoretic framework, many axions might exist that could mix with each other [17]. Such a mixing can give rise to reduced UV sensitivity of the two-loop graph shown in Fig. 1. In such cases, the anomalously generated Majorana mass may be estimated to be:

$$M_R \sim \frac{\sqrt{3} y_a \gamma \kappa^5 \Lambda^{6-2n} (\delta M_a^2)^n}{49152\sqrt{8}\pi^4 (1-\gamma^2)} \text{ for } n \leq 3, \text{ and } M_R \sim \frac{\sqrt{3} y_a \gamma \kappa^5 (\delta M_a^2)^3}{49152\sqrt{8}\pi^4 (1-\gamma^2)} \frac{(\delta M_a^2)^{n-3}}{(M_a^2)^{n-3}} \text{ for } n > 3.$$

It is then not difficult to see that three axions $a_{1,2,3}$ with next-to-neighbour mixing as discussed above would be sufficient to obtain a UV finite result for M_R at the two-loop level. Of course, beyond the two loops, M_R will depend on higher powers of the energy cut-off Λ , i.e. $\Lambda^{n>6}$, but if $\kappa\Lambda \ll 1$, these higher-order effects are expected to be subdominant.

In the above n -axion-mixing scenarios, we note that the anomalously generated Majorana mass term will only depend on the mass-mixing parameters δM_a^2 of the axion fields and not on their masses themselves, as long as $n \leq 3$. As a final comment we mention that the values of the Yukawa couplings y_a may be determined by some underlying discrete symmetry [19], which for

instance allows two of the right-handed neutrinos to be almost degenerate in mass, as required for enhanced CP violation of relevance to leptogenesis [13], or in general characterises the ν MSSM [6]. These are interesting issues that deserve further exploration.

We stress at this point that, although above we kept our discussion general, and included right-handed neutrinos in the spectrum of our models, assuming that the Yukawa-ordinary-axion interactions with them are the dominant ones, nevertheless our mechanism applies also to models with *no right-handed* neutrinos. Indeed, all one has to do in such cases is to assume that the ordinary-axion-Yukawa interactions occur in the active neutrino sector directly. At any rate, as in the next section our interest lies on producing Leptogenesis using (heavy) right handed neutrinos, from now on we do assume the existence of such heavy particles. However, as we shall argue below, for producing sufficient Leptogenesis using a KR torsion background in the early Universe, we need only one generation of heavy right handed Majorana (RHM) neutrinos. It goes without saying that the extension of our scenario to three RHM neutrino species, *e.g.* as required in seesaw models [2, 3], is straightforward.

3. Kalb-Ramond-Torsion Backgrounds in the Early Universe and Leptogenesis

In this section we consider first the interaction of massive fermion species ψ_i , $i = 1, \dots, N$, of mass $m^{(i)}$, with non-trivial gravitational and Kalb-Ramond H-torsion backgrounds, which we assume characterise the early Universe [12, 13]. From the considerations in the previous section 2, it becomes clear that the physical content of the effects of an H-torsion background on Leptogenesis is contained in the following fermionic action (we start with Dirac fermions, the generalisation to Majorana ones, of relevance to us here, is straightforward and will be considered later on):

$$S_{Dirac} = \frac{1}{2} \int d^4x \sqrt{-g} i \left(\bar{\psi}_i \gamma^a \nabla_a \psi_i - (\bar{\nabla}_a \bar{\psi}_i) \gamma^a \psi_i + 2im^{(i)} \bar{\psi}_i \psi_i \right) = \int d^4x \sqrt{-g} \bar{\psi}_i \left(i\gamma^a \partial_a + \hat{B}_d \gamma^5 \gamma^d - m^{(i)} \right) \psi_i \equiv S_{Dirac}^{\text{free}} - \int d^4x \sqrt{-g} \hat{B}_\mu J^{5\mu}, \quad (3.1)$$

where summation over the species index i is implied, $J^{5\mu} \equiv \bar{\psi}_i \gamma^\mu \gamma^5 \psi_i$, and the axial vector \hat{B}^d is defined by

$$\hat{B}^d = \frac{1}{4} \varepsilon^{abcd} e_a^\mu \bar{\omega}_{b\mu c} = \frac{1}{4} \varepsilon^{abcd} e_a^\mu e_{bv} \left(\partial_\mu e_c^v + e^{-2\phi} H_{\mu\lambda}^v e_c^\lambda \right). \quad (3.2)$$

In this last step we have used (2.3), (2.13) and the symmetry $\Gamma_{\mu\nu}^\lambda = \Gamma_{\nu\mu}^\lambda$ of the *torsion-free* Christoffel symbol. In the special cases of interest here, either flat (Minkowski) or Robertson-Walker space-times (which do not contain off-diagonal metric elements mixing temporal and spatial components), the axial vector \hat{B}^d is non-trivial and constitutes just the dual of the torsion tensor

$$\hat{B}^d = -\frac{1}{4} \varepsilon^{abcd} e^{-2\phi} H_{abc}. \quad (3.3)$$

In four space-time dimensions $\hat{B}^\mu = \partial^\mu b$, where $b(x)$ is the Kalb-Ramond (KR) axion field (2.14).

For a bosonic string theory (with four uncompactified dimensions) in non-trivial cosmological backgrounds, a world-sheet description has been provided by a sigma model that can be identified with a Wess-Zumino-Witten type conformal field theory [20]. This construction has led to exact solutions (valid to all orders in the Regge slope, α') for cosmological bosonic backgrounds with

non-trivial metric, antisymmetric tensor and dilaton fields. Such solutions, in the Einstein frame, consist of (i) a Robertson-Walker metric with a scale factor $a(t) \sim t$ where t is the cosmic time, (ii) a dilaton field ϕ that scales as $\phi(t) \sim -\ln a(t)$, and (iii) a KR axion field scaling linearly with the cosmic time, $\bar{b} \propto t$ with \bar{b} denoting the background value of b (cf. (2.14)). The resulting background axial vector \overline{B}^d has only a non-trivial temporal component

$$\bar{b} \sim \text{const } t, \quad \partial^\mu \bar{b} \sim \varepsilon^{\mu\nu\rho\sigma} e^{-2\phi} H_{\nu\rho\sigma}, \quad (3.4)$$

and

$$\overline{B}^0 \propto \dot{\bar{b}} = \text{constant} \neq 0, \quad \overline{B}^i = 0, \quad i = 1, 2, 3, \quad (3.5)$$

in the Robertson-Walker frame.

In the absence of fermions, such constant H-torsion background in a Robertson-Walker space time *do not* constitute solutions equations of motion derived from the perturbative (in α') action (2.12) (cf. Appendix). However, in the presence of fermions coupled to the torsion H -field as in (3.1), the four-dimensional low-energy effective action gives the following equations of motion for the graviton and antisymmetric tensor:

$$\begin{aligned} \text{graviton :} \quad R_{\mu\nu} - \frac{1}{4} H_\mu^{\alpha\beta} H_{\nu\alpha\beta} &= 8\pi G \left(T_{\mu\nu}^\Psi - \frac{1}{2} g_{\mu\nu} T^\Psi + \text{dilaton - derivative terms} + \dots \right), \\ \text{antisymmetric tensor :} \quad \partial^\mu \left(\sqrt{-g} e^{-2\phi} [H_{\mu\nu\rho} - \varepsilon_{\mu\nu\rho\sigma} J^{5\sigma} + \dots] \right) &= 0, \end{aligned} \quad (3.6)$$

where \dots denotes higher order terms in α' in the gravitational part of the action, $T_{\mu\nu}^\Psi$ is the stress-energy tensor of fermionic matter and $T^\Psi = g^{\mu\nu} T_{\mu\nu}^\Psi$. There is of course an equation of motion for the dilaton which provides additional constraints for the background. In order to simplify the analysis we will assume a constant dilaton below. A full analysis is given in the Appendix.

It is conceivable, as argued in [13], that in the presence of high temperature and densities of fermions (relevant for the early universe), a condensate of the temporal component of the fermion axial current may be formed, in which case one may have (Lorentz-violating) *perturbative* fixed points corresponding to a constant H -torsion. Indeed, from the equation for the antisymmetric tensor field (assuming a constant dilaton), (3.6), we observe that it can be solved upon using the pseudoscalar dual field \bar{b} defined in (3.4):

$$\partial^\mu \left(\sqrt{-g} [\varepsilon_{\mu\nu\rho\sigma} (\partial^\sigma \bar{b} - \tilde{c} J^{5\sigma}) + \mathcal{O}((\partial \bar{b})^3)] \right) = 0, \quad (3.7)$$

where \tilde{c} is a constant of proportionality. From the above equations (in truncated form) it is clear that the fermion condensate can be a source of torsion. Hence the non-perturbative solution (3.5), derived in [20] is still qualitatively valid since, from (3.7), we have

$$\dot{\bar{b}} = \tilde{c} \langle J_0^5 \rangle = \tilde{c} \langle \psi_i^\dagger \gamma^5 \psi_i \rangle = \text{constant} \neq 0, \quad (3.8)$$

where i runs over appropriate fermion species.

In [21] a calculation in strong coupling gauge theory supported the formation of axial vector fermion condensates. At weak gauge coupling the condensates cease to form. The gauge coupling and string coupling are related in string model building of the fundamental interactions [9]. If the dilaton, rather than being a constant becomes more negative with time (as in the explicit solutions

from bosonic string theory considered in [20]), the string coupling and gauge coupling decrease with time; so there will be a time (and a critical value of g , g_c) when the gauge coupling will be too weak to support a condensate. This is of course qualitative: currently we can only speculate that the value of g_c is achieved in the era of leptogenesis. For a fundamental mechanism we would need to be quantitative but this has not been achieved as yet. Nevertheless, we note that in late era of the Universe, when the axial current condensate becomes vanishingly small, from (3.7) we find (upon ignoring (as subleading) the higher order $\mathcal{O}((\partial\bar{b})^3)$ terms), that the rate of change of the \bar{b} field diminishes with the cosmic time as the cube of the scale factor

$$\dot{\bar{b}} \sim 1/a^3(t). \quad (3.9)$$

At this stage we notice that there is another challenging problem at the microscopic level related to the cosmological constant and the need for fine-tuning, a generally unsolved problem at present. The time dependent pseudoscalar, with constant rate (3.8) induces a vacuum energy term of the type of a positive cosmological constant once fluctuations around the background are allowed (cf. Appendix, Eq. (4.14)). There are ways by means of which such positive contributions can be cancelled by negative vacuum energy contributions; for instance, in brane world models propagating in higher-dimensional bulk space times, one may obtain negative contributions on the brane vacuum energy from the bulk dynamics [22, 14], as we shall discuss in section 4. Such negative contributions do not constitute an instability in the presence of the H-torsion constant background as they are cancelled by it. The real issue, however, is how, once the fermion condensates have disappeared, the brane vacuum energy acquires the present tiny value consistent with observational cosmology. One possible way to achieve this is associated with the dynamics of time dependent dilatons, on which we make some speculative remarks in the last part of this section. Hence, at present we do not have a microscopic derivation but rather a microscopic motivation for postulating our H-torsion background. Of course such issues do not apply to the neutrino mass generation mechanism of section 2, which assumes zero H-torsion backgrounds.

A final remark before moving onto a discussion of leptogenesis concerns the path integration of quantum fluctuations of the antisymmetric torsion background. Such fluctuations need to be included for an accurate estimate of the energy budget of the universe. Indeed, as we have seen in section 2, such a process leads to repulsive axial current four fermion interactions in the action that may contribute to the energy budget, especially in eras where condensation of the fermion axial current occurs. To take formal account of such fluctuations, a split of the \widehat{B} field can be made explicit into background \overline{B}^μ and quantum fluctuation $\widehat{\mathcal{B}}^\mu$ parts, $\widehat{B}^\mu = \overline{B}^\mu + \widehat{\mathcal{B}}^\mu$, where the background satisfies (3.5). The result for the relevant factor of the path integral after integration over the quantum fluctuations \widehat{B} reads

$$\mathcal{Z} \propto \int D\psi D\bar{\psi} e^{i\widetilde{S}(\overline{B}) + i \int d^4x \sqrt{-g} \frac{3}{16} \kappa^2 J_\mu^5 J^{5\mu}} \quad (3.10)$$

where $\widetilde{S}(\overline{B}) = S(\overline{B}) + S_{Dirac}^{free} - \int d^4x \sqrt{-g} \overline{B}_\mu J^{5\mu}$ is the action in the presence of the background torsion, given by the sum of (2.12), and (3.1). For a more detailed discussion we refer the reader to ref. [13]. The presence of a fermion axial condensate of the type (3.8) implies in general a

generalised background for the fermions in a Hartree-Fock approximation

$$B_\mu \equiv \left(\widetilde{B}_\mu + \frac{3\kappa^2}{8} \mathcal{F}_\mu \right), \quad (3.11)$$

where, on account of our previous discussion (*cf.* (3.5), (3.8)) only temporal components of B_μ will be non zero, with $\mathcal{F}_0 = \langle \psi_\ell^\dagger \gamma^5 \psi_\ell \rangle \equiv \rho_R - \rho_L \neq 0$, $\mathcal{F}_i = 0$. The reader should notice that the sum over fermion species $\ell = 1, \dots, N$, *does not* include Majorana neutrinos that yield zero contributions to the condensate [13]. Hence, it is the other fermion fields of the SM, such as quarks, that could contribute to such condensates.

We commence our discussion on H-torsion-background induced leptogenesis by considering a minimal extension of the Standard Model, with one right-handed massive (of mass M) Majorana neutrino field in the presence of constant axial backgrounds, B_μ (3.11). The right-handed neutrino sector of such a model is described by

$$\mathcal{L} = i\bar{N}\not{\partial}N - \frac{m}{2}(\bar{N}^c N + \bar{N}N^c) - \bar{N}\not{B}\gamma^5 N - Y_k \bar{L}_k \tilde{\phi} N + h.c. \quad (3.12)$$

where N is the Majorana field with mass m (that can be itself generated dynamically by the fluctuations of the torsion field in the way described in section 2), L_k is a lepton field of the SM, with k a generation index, and the adjoint of the Higgs field is defined by the relation $\tilde{\phi}_i = \varepsilon_{ij}\phi_j$, with $i, j = 1, 2$ are SU(2) group indices. We note that, since our primary motivation here is to identify the axial background field with the totally antisymmetric part of a torsion background, one should also consider the coupling of the axial field B_μ to all other fermions of the SM sector, ψ_j (j =leptons, quarks) via a *universal* minimal prescription. Hence, the coupling with all fermionic species is the same: $\bar{\psi}_j \gamma^5 / B \psi_j$. Specifically, as we have seen above, the identification of the torsion background with a homogeneous and isotropic cosmological Kalb-Ramond field in a string-theory-inspired model will lead to axial backgrounds with non-trivial temporal components only

$$B_0 = \text{const} \neq 0, B_i = 0, i = 1, 2, 3. \quad (3.13)$$

Since in SM the leptons have definite chirality, the Yukawa interactions can be rewritten as

$$\mathcal{L}_{YUK} = -Y_k \bar{L}_k \tilde{\phi} \left(\frac{1 + \gamma^5}{2} \right) N - Y_k^* \bar{N} \tilde{\phi}^\dagger \left(\frac{1 - \gamma^5}{2} \right) L_k. \quad (3.14)$$

Using the properties of the charge conjugation matrix and the Majorana condition, it is again seen to be equivalent to

$$\mathcal{L}_{YUK} = -Y_k \bar{L}_k \tilde{\phi} \left(\frac{1 + \gamma^5}{2} \right) N - Y_k^* \bar{L}_k^c \tilde{\phi}^\dagger \left(\frac{1 - \gamma^5}{2} \right) N. \quad (3.15)$$

It should be noted that the two hermitian conjugate terms in the Yukawa Lagrangian are also CPT conjugate. This is to be expected on the basis of the CPT theorem. In fact CPT violation is introduced only by interactions with the background field. In the absence of the background, the squared matrix elements obtained from tree level diagrams for the two decays would be the same [23]. From the form of the interaction Lagrangian in Eq. (3.15), it is straightforward to obtain the Feynman rules for the diagrams giving the decay of the Majorana particle in the two distinct channels. It

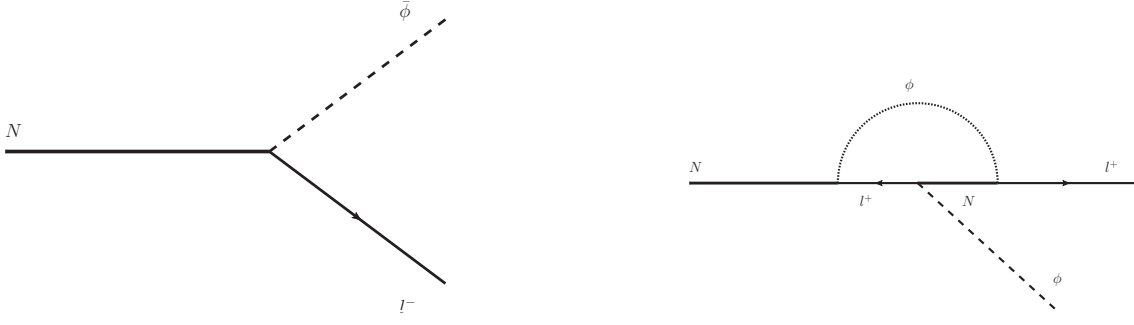


Figure 2: Tree- (left) and one-loop (right) decay amplitudes, corresponding to the Yukawa term that couples a right-handed neutrino to the standard model lepton sector. Analogous diagrams describe the decay in anti-leptons. Continuous undirected lines represent right-handed neutrinos, lines with an arrow are used to represent SM leptons, whilst dashed lines correspond to the SM Higgs. The left diagrams are understood to be evaluated in the presence of a KR background field. The right diagram is the standard result of [24], leading to Leptogenesis.

also allows us to use positive frequency spinors both for the incoming Majorana particle and for the outgoing leptons.

Let us now turn to the study of the *tree-level* decay processes of a Majorana right-handed neutrino into leptons and Higgs fields, depicted in (the left panel of) fig. 2. For the qualitative purposes of the talk we assume that the massive right-handed neutrino is initially at rest. The more general case is studied in ref. [13]. For the channel $N \rightarrow l^- \bar{\phi}$ the decay rate is

$$\Gamma_1 = \sum_k \frac{|Y_k|^2}{32\pi^2} \frac{m^2}{\Omega} \frac{\Omega + B_0}{\Omega - B_0}, \quad \Omega = \sqrt{B_0^2 + m^2}, \quad (3.16)$$

while, for the other channel, $N \rightarrow l^+ \phi$, the respective decay rate reads

$$\Gamma_2 = \sum_k \frac{|Y_k|^2}{32\pi^2} \frac{m^2}{\Omega} \frac{\Omega - B_0}{\Omega + B_0}. \quad (3.17)$$

The reader should notice that the decay rate of one process is obtained from the other upon flipping the sign of B_0 . The total decay rate is

$$\Gamma = \Gamma_1 + \Gamma_2 = \sum_k \frac{|Y_k|^2}{16\pi^2} \frac{\Omega^2 + B_0^2}{\Omega}. \quad (3.18)$$

It is worthwhile observing that this mechanism can produce a lepton asymmetry even with *only one right-handed neutrino*, whereas the standard leptogenesis scenario [24] requires at least three generations. Moreover, the occurrence of leptogenesis here is just due to decay processes at tree level, since the required CP violation is introduced by the background field that enters in the external lines of Feynman diagrams.

The decay process goes out of equilibrium when the total decay rate drops below the expansion rate of the Universe, which is given by the Hubble constant [25]

$$\Gamma \simeq H = 1,66T^2 \mathcal{N}^{1/2} m_P^{-1}. \quad (3.19)$$

Here \mathcal{N} is the effective number of degrees of freedom of all elementary particles and m_P is the Planck mass. From the last equation one can estimate the decoupling temperature T_D , in terms of the unknown parameters Ω , $|Y|$ and B_0 , is

$$T_D \simeq 6.2 \cdot 10^{-2} \frac{|Y|}{\mathcal{N}^{1/4}} \sqrt{\frac{m_P(\Omega^2 + B_0^2)}{\Omega}}. \quad (3.20)$$

In order for the inverse decay to be suppressed by the Boltzmann factor, we have to impose the further requirement that $T_D \leq \Omega$ when $\Gamma \simeq H$ (delayed decay mechanism [25, 24]). From this condition one can determine a lower bound for the mass m . In fact we are lead to the following inequality

$$z(\Omega^2 + B_0^2) \leq \Omega^3, \quad (3.21)$$

where $z = 3.8 \cdot 10^{-3} \frac{m_P |Y|^2}{\mathcal{N}^{1/2}}$. If we require that the bound is satisfied for all values of B_0 we get

$$m^2 \geq 1.09 z^2. \quad (3.22)$$

For us the Yukawa coupling Y is a free parameter. If we assume $|Y| \approx 10^{-5}$, $\mathcal{N} \approx 10^2$, we get an order of magnitude estimate for the lower bound of the right-handed Majorana neutrino mass

$$m \geq \bar{m} \approx 100 \text{ TeV}. \quad (3.23)$$

Clearly in this mechanism for leptogenesis the low-mass right-handed neutrinos of [6] do not fit.

The lepton number density produced can then be estimated in the following way: by assumption all the neutrinos are at rest before the decay; hence the branching ratios of the decays are given by $r = \frac{\Gamma_1}{\Gamma}$ and $1 - r$. The decay of a single neutrino produces the lepton number

$$\Delta L = r - (1 - r) = 2r - 1 = \frac{2\Omega B_0}{\Omega^2 + B_0^2}. \quad (3.24)$$

Multiplying this quantity by the initial abundance of right-handed Majorana neutrinos at the temperature T_D one gets an approximate estimate of the lepton number density. The density of the Majorana neutrinos is given by

$$n_N = \sum_{\lambda} \frac{1}{(2\pi)^3} \int d^3 p f(p, \lambda). \quad (3.25)$$

where as usual β is the inverse temperature, λ denotes the helicity and $f(p, \lambda)$ is the corresponding Fermi-Dirac distribution function. At high temperatures T , as the ones we are dealing with here, this is well approximated by the Maxwell-Boltzmann function $f(p, \lambda) = e^{-\beta \sqrt{m^2 + (p + \lambda B_0)^2}}$. It is now straightforward to see that, upon performing the sum over helicities in (3.25), and the momentum integrals for small $B_0 \ll m, T$ [13], one recovers the usual expression for the density of a non-relativistic species $n_N = e^{-\beta m} \left(\frac{m}{2\pi\beta} \right)^{3/2} + \mathcal{O}(B_0^2)$.

We assume that the right-handed neutrino density distribution follows closely the equilibrium distribution for $T \geq T_D$ and drops rapidly to zero at lower temperatures $T \leq T_D$; furthermore the

density of the sterile neutrino (normalised to the entropy density) is well approximated by a step-function. Therefore, upon multiplying (3.24) by n_N , we obtain that the total lepton asymmetry produced in the full decay of the right-handed neutrino is given by

$$\Delta L^{TOT} = (2r - 1)n_N = \frac{2\Omega B_0}{\Omega^2 + B_0^2}n_N \quad (3.26)$$

If the lepton asymmetry ΔL^{TOT} is communicated to the baryon sector by (Baryon (B))-(Lepton-number (L)) conserving sphaleron processes in the SM sector, then $\frac{\Delta L^{TOT}}{n_\gamma}$ is expected to be of the same order of magnitude of the (observed) baryon asymmetry for temperatures $T > 1$ GeV,

$$Y_{\Delta B} = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.1 \pm 0.3) \times 10^{-10} \quad (3.27)$$

where n_B is the number density of baryons, $n_{\bar{B}}$ is the density of antibaryons in the universe, and n_γ is the density of photons. An order of magnitude estimate of the ratio $\frac{B_0}{m}$ can then be found from (3.26) by making use of the approximation $T_D \simeq m$ and retaining only first order terms in $\frac{B_0}{m}$. Recalling that the photon number density is

$$n_\gamma \simeq \frac{2\zeta(3)}{\pi^2} T^3 \simeq 0.24 T^3 \quad (3.28)$$

and that

$$\frac{\Delta L}{n_\gamma} \simeq 10^{-10}, \quad (3.29)$$

we estimate the ratio of the background field to the mass of the sterile neutrino to be

$$\frac{B_0}{m} \simeq 10^{-8}. \quad (3.30)$$

The small value of this ratio also allows us to justify *a posteriori* the neglect of higher powers of B_0 in the formulae above. From the lower bound for the mass of 100 TeV found in (3.22), for the case where $Y = \mathcal{O}(10^{-5})$, we get an approximation for the smallest possible magnitude of the background field required in order for this mechanism to be effective $B_0 \simeq 1$ MeV. If other mechanisms contributed to the lepton asymmetry in the universe, or the Yukawa couplings assume smaller values, the minimum value of B_0 would be smaller than the one given here. Baryogenesis is then assumed to proceed via B-L conserving processes in the SM sector of the model. In order to get a more accurate estimate of B_0 , the relevant Boltzmann equation will need to be studied. This requires a good approximation for the thermally averaged decay rates of all the relevant processes and will be the subject of future research.

We now remark that, in a microscopic model an H -torsion-induced background B_0 of the above magnitude of 1 MeV would correspond to a *large positive* contribution to the cosmological constant in the effective action (3.10) (*cf.* Appendix, Eq. (4.14)); if uncancelled, this vacuum energy would modify the standard cosmology in the radiation dominated eras of the early Universe, on which the above estimates were based. There are possibilities whereby these fluctuations might be cancelled. As already mentioned, within the context of brane world quantum field theories there may be anti-de Sitter type contributions from the bulk [22]. In such cases, there are negative vacuum energy

contributions to the (four-dimensional) brane vacuum. Such contributions may suppress the B_0 -induced vacuum energy contributions to acceptable levels so that the standard cosmology may apply.

Moreover, in the context of axial condensates there is a phase transition whereby the gauge coupling is too weak for their formation [21]. In the framework of our string cosmology, the string coupling $g_s = e^\phi$ is proportional to the gauge couplings, and is diminished with the cosmic time, due to the dilaton being a negative logarithmic function of the scale factor [20, 26]. Thus it is likely that, as the Universe cools down, the Lorentz-violating condensates, which require strong couplings and high densities in order to form, are destroyed via a suitable phase transition. At this transition the B field vanishes. In our scenario this temperature needs to be lower than the decoupling one for leptogenesis. The destruction of the fermion condensate at a temperature $T \simeq T_D$, would imply that the Kalb-Ramond-torsion-axion field b no longer varies linearly with the cosmic time but diminishes with the scale factor as in (3.9). If one assumes a cooling law for the Universe of the form $a \sim T^{-1}$, then the B^0 torsion field would scale with the temperature as T^3 for $T \leq T_D$. Taking that into account, the temperature of the Universe today (from the CMB measurements) is $T_{\text{CMB}} = 2.725 \text{ K} = 0.2348 \text{ meV}$, and assuming that the perturbative fixed point is dominant at temperatures of the order of $T \simeq T_D = 100 \text{ TeV}$, we obtain a cooling law for the torsion B_0 -field of the form

$$B_0 = c_0 T^3, \quad c_0 = 1 \text{ MeV}(100 \text{ TeV})^{-3} = 10^{-42} \text{ meV}^{-2}. \quad (3.31)$$

Thus, in such a scenario, the value of B_0 today would be of order

$$B_{0\text{today}} = \mathcal{O}(10^{-44}) \text{ meV}, \quad (3.32)$$

which is much too small for any experimental detection, lying well within the upper bounds on $B_0 \leq \mathcal{O}(10^{-2}) \text{ eV}$, placed by precision atomic experiments within the context of experimental tests of the Standard Model Extension [11, 27, 28]. These and other considerations require further evaluation. For the purposes of this work we will just consider the model with a background field which in the present era, away from the leptogenesis era, is effectively absent. Hence detailed models will need to confirm that the corresponding temperature, at which such a destruction can happen, is much higher than the $\mathcal{O}(100) \text{ GeV}$ temperature at which the lepton asymmetry is transferred to baryon asymmetries due the B-L conserving processes in the SM sector of the model. This is an important issue for future study.

Another important issue is the effect of the constant antisymmetric torsion on the Cosmic Microwave Background (CMB) radiation spectrum. If we assume that the CMB spectrum is largely due to fluctuations at the surface of last scattering, which occurs at redshifts $z = \mathcal{O}(10^3)$, then we observe that at the corresponding temperatures $T = T_{\text{CMB}}(1+z) \sim 10^3 T_{\text{CMB}}$, the value of

$$B_{0\text{last scat}} \sim 10^9 B_{0\text{today}} = \mathcal{O}(10^{-35}) \text{ meV}, \quad (3.33)$$

as follows from (3.31). This is very small to produce any observable effects in the CMB spectrum as can be seen from the following argument: one may consider higher derivative terms in the effective action of photons propagating in a torsion background (as is the case in string-effective theories). One then encounters, among others, higher-covariant-derivative-with-torsion terms of

the form appearing in the Lorentz-violating electrodynamics [27], whose effects on cosmic microwave background radiation have been classified. Among those terms are terms of the form $T_{\rho}^{\alpha\lambda} F_{\alpha\nu} \partial_{\lambda} \tilde{F}^{\rho\nu}$ and $T_{\delta}^{\sigma\gamma} F_{\sigma\nu} \partial_{\gamma} F^{\delta\nu}$, where $T^{\mu\nu\rho}$ is the torsion field and $\tilde{F}_{\mu\nu}$ is the dual of the photon field strength $F_{\mu\nu}$. Such terms may be constrained by the mixing of electric (E-) and magnetic (B-) type polarizations of the CMB due to induced birefringence. With the strength of the Kalb-Ramond torsion in (3.33), the possible effects are well within the corresponding bounds.

In general, the association of the Kalb-Ramond torsion with a pseudoscalar axion-like field, implies constraints of the interactions of this field with electromagnetic fields through the anomaly equation (2.10). In our model, the coupling of this interaction turns out to be the gravitational coupling, and thus such effects are very small, compatible with the current phenomenology.

The cosmological vacuum energy, which is proportional to $\exp(2\phi)$, also decreases with t to very small values in string models, where relevant operators in a (perturbative) world-sheet renormalisation-group sense, drive the theory towards fixed points at which the dilaton field $\phi \sim -\ln a(t)$, where $a(t)$ is the scale factor of the Universe, which asymptotically in time $a \rightarrow \infty$ [26]. However, there are also some non-perturbative scenarios (with large string coupling) associated with brane worlds, which induce such phase transitions, but in which the dilaton tends to $+\infty$. This is considered in the next section.

4. Speculative Applications to Brane Models with Strongly-Coupled Strings

We close the talk by presenting some speculations as to how the above scenarios of a rather large H-torsion background in early eras of the Universe, but negligible today, can be realised in microscopic brane models of our Universe. Specifically, in this section, we argue that there can be solitonic degrees of freedom which can contribute to a Lorentz breaking background and be relevant in terms of the vacuum energy of the Universe at high energies. We consider a toy brane-world model. The essential features of the model are depicted in fig. 3. We consider a brane world (with three of the longitudinal brane directions uncompactified) moving in the bulk adiabatically, which encounters at a certain stage an ensemble of D0-brane defects (D-particles), which depending on the type of string theory can be either truly point-like or “effectively” point-like (e.g. in type II string theory one can have D3-branes wrapped up around three cycles, with a small radius of compactification, of the size of the string length). Models of this type have been studied in [14], where we refer the reader for details.

In what follows we shall only sketch the basic features from the point of view of interest in the current work, namely the phase transition at which the effects of the Kalb-Ramond torsion disappear. The brane universe representing our world moves in an inhomogeneous bulk space punctured on one side of the brane with an ensemble of D-particles. We assume that there is no cosmological expansion in the bulk, only along the longitudinal large directions of the brane universe one has such an expansion. Kalb-Ramond torsion constitutes an excitation restricted on the brane world, for the purposes of our toy example here. When the brane world is relatively far away from the ensemble of the defects then the brane vacuum energy is kept low due to cancelling effects between anti de Sitter contributions of vacuum energy in the bulk [22] and the constant torsion kinetic energy terms (*cf.* Appendix (4.14)).

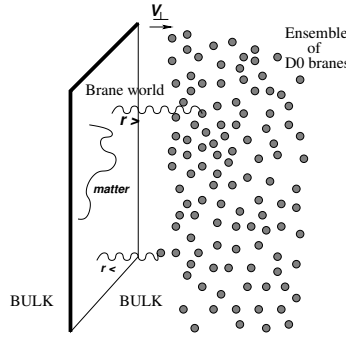


Figure 3: A toy cosmological Brane-world model for the phase transition associated with the destruction of the Kalb Ramond torsion background in the early Universe. A brane world moves adiabatically in the higher-dimensional bulk encountering at a certain epoch, coinciding with the decoupling era of Leptogenesis, an ensemble of D0-brane defects of variable (in time) density, with respect to which the brane is assumed to move with velocity V_{\perp} . There are relative-velocity dependent interactions between the bulk defects and the brane world, described by open strings stretched between the brane and the defect. There are positive contributions to the brane's vacuum energy from defects at distances $r_{>}$ larger than the string length, and negative contributions from defects at distances $r_{<}$ shorter than the string length. The latter can lead to bound states of D0-branes and the brane world which are then responsible for the destruction of the fermion condensates and the Kalb Ramond torsion.

As the brane world approaches the ensemble of defects, and is a few string lengths away ², macroscopic open strings stretched between the brane and the D-particles are in operation, which imply gravitational potential energy contributions on the brane Universe, that are proportional to the sum of the squares of the components of relative velocity v_{\perp} between the brane world and the D-particles transverse to the brane large longitudinal directions.

As discussed in [14], for bulk D-particles at a bulk distance $r > \sqrt{\alpha'}$, larger than the string scale $\sqrt{\alpha'}$, one obtains positive contributions to the brane vacuum energy, while for bulk D-particles near the brane world, at distances $r < \sqrt{\alpha'}$, one has negative contributions to the brane's vacuum energy. In the example of $D0 - D8$ brane interactions discussed in [14], for instance, one has for the long- and short-distance contributions to the potential energy of the brane world

$$V_{D0-D8}^{r > \sqrt{\alpha'}} = + \frac{r}{8\pi\alpha'} \vec{v}_{\perp}^2, \quad V_{D0-D8}^{r < \sqrt{\alpha'}} = - \frac{\pi\alpha'}{12r^3} \vec{v}_{\perp}^2. \quad (4.1)$$

This example is only indicative to demonstrate the different sign of the various contributions to the vacuum energy, for the case at hand of D3-brane universes one should appropriate compactify the pertinent brane worlds, but this will not affect qualitatively our arguments. This difference in sign, then, implies that, one can arrange for the densities of far away and nearby bulk D-particles, which are not in general homogeneous, to be such that the total contribution to the brane world vacuum energy is always subcritical, so that issues such as overclosure of the universe by a significant population of D-particle defects can be avoided. This is rather generic, independent of the type of branes involved.

²There is a screening effect of the ensemble of D-particles due to the fact that open strings can be attached to the D-particles, which can thus cut macroscopic strings. This prevents any meaningful interactions between the D-particles in the ensemble and the brane world to extend beyond the order of a few string lengths.

There is a new minimal length in the problem, set by $r_{\min} \sim \sqrt{v_{\perp}} \sqrt{\alpha'}$, which for adiabatic brane motion $|v_{\perp}| \ll 1$ can be much smaller than the string length. The important point to notice is that the attractive short-distance contributions for $r < \sqrt{\alpha'}$ can lead to bound states of D0 branes with the brane Universe. The full non-perturbative string theory machinery has to be invoked in order to study in detail the formation and features of such bound states, which we shall not discuss here. However, for our point of view, it suffices to mention that for strings in the neighborhood of such bound states the string coupling $e^{\langle\phi\rangle}$ is now modified to [29]

$$\exp(2\langle\phi\rangle) = g_s^2 \left(1 + \frac{g_s}{(r/\sqrt{\alpha'})^2} \right)^{3/2} \quad (4.2)$$

where g_s is the perturbative string coupling, far away ($r/\sqrt{\alpha'} \rightarrow \infty$) from the D-particle. From (4.2) it becomes obvious that for a string excitation near a D-particle bound to the brane, $r/\sqrt{\alpha'} \rightarrow 0$ the effective string coupling becomes practically infinite. From a σ -model point of view, the corresponding effective actions (2.12), (3.1), describing the propagation of a matter open string (say a standard model fermion) on the brane world near a bound D-particle defect, would be characterised by a $\phi \rightarrow +\infty$ divergent dilaton field, and thus zero Kalb-Ramond torsion (3.2). This implies that upon the encounter of the brane world with the ensemble of bulk D-particles (fig. 3) a phase transition occurs, implying a trivial background for the Kalb-Ramond axion field, $b = 0$, on the four space-time dimensional world. Moreover, the presence of D0-bound states would destroy the fermion condensate, since now the matter fields (represented by open strings) would be attached to the defects, and as a result any long-range coherence would be destroyed.

After the destruction of the fermion axial condensate, the brane continues to move in a bulk region with an ensemble of D-particle defects (until the present era) and hence, the anti de Sitter contributions of the bulk vacuum energy that have been used to cancel the H-torsion contributions during the Leptogenesis era, can now themselves be cancelled by the long-range *positive* contributions to the brane vacuum energy due to the bulk D-particles that are relatively far away from the brane world ($r > \sqrt{\alpha'}$). In this way the vacuum energy of the brane world remains small during the late, post-Leptogenesis, eras of the Universe and the standard cosmology (e.g. Λ CDM model) applies. It goes without saying that, at this stage, we are far from being able to construct phenomenologically realistic cosmological models of the Universe in this framework, but it is encouraging that some toy models, such as the above-described one, exist.

Appendix: Perturbative world-sheet Fixed Points and constant Kalb-Ramond Torsion in first quantised string models

Here we shall discuss the rôle of string matter in ensuring that torsion (provided by the anti-symmetric tensor field strength) remains constant in time, within the context of low-energy string effective field-theory actions. We shall consider perturbation theory in powers of α' (the Regge slope). To lowest-order in α' , the Einstein-frame effective action including matter fields (in four-large-space-time dimensions) reads [9]

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(R - 2\partial^\mu \phi \partial_\mu \phi - e^{-4\phi} H_{\lambda\mu\nu} H^{\lambda\mu\nu} - \frac{2}{3} \delta c \exp(2\phi) \right) + \int d^4x \sqrt{-g} e^{4\phi} \mathcal{L}_{\text{matter}}. \quad (4.3)$$

where δc denotes a central charge deficit [20]. It is important to realise that $\mathcal{L}_{\text{matter}}$ contains the coupling of the antisymmetric tensor field strength $H_{\mu\nu\rho}$ with the axial fermionic current, J_μ^5 ; such terms do exist in our model as a consequence of the interpretation of the $e^{-2\phi}H_{\mu\nu\rho}$ in the Einstein term as contorsion (*cf.* (3.2)). $\mathcal{L}_{\text{matter}}$, to lowest order in α' , depends linearly on $-\frac{1}{4}\varepsilon^{\mu\nu\rho\sigma}e^{-2\phi}H_{\mu\nu\rho}\bar{\psi}\gamma^\sigma\gamma^5\psi$, where ψ are generic fermion fields. Such terms have not been considered explicitly in the literature before.

The corresponding equations of motion for the graviton, antisymmetric tensor fields and dilatons read, respectively:

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= \kappa^2 \left(T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^H + T_{\mu\nu}^\phi + T_{\mu\nu}^c \right), \\ \nabla_\mu \left(e^{-4\phi} H^{\mu\nu\lambda} + \kappa^2 e^{2\phi} \varepsilon^{\mu\nu\lambda\sigma} J_\sigma^5 \right) &= 0, \\ \square\phi - \frac{1}{3}e^{2\phi}\delta c + e^{-4\phi}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + 2\kappa^2 e^{4\phi}\mathcal{L}_{\text{matter}}^{\neq H} - \frac{1}{4}\kappa^2 e^{2\phi}\varepsilon^{\mu\nu\lambda\sigma}H_{\mu\nu\lambda}J_\sigma^5 &= 0, \end{aligned} \quad (4.4)$$

where $J_\mu = \bar{\psi}\gamma_\mu\gamma^5\psi$ and $\mathcal{L}_{\text{matter}}^{\neq H}$ denotes the part of the matter lagrangian independent of couplings to $H_{\mu\nu\rho}$. $T_{\mu\nu}^i$ are the stress tensors pertaining to the various fields, $i = \text{matter}, H, \phi$ and c , with $T_{\mu\nu}^c = (\frac{1}{3}e^{2\phi}\delta c)g_{\mu\nu}$ the vacuum energy contribution. To these equations one should add the equations of motion for the matter fields, which are of no direct relevance for our cosmological considerations.

In four dimensions, in the absence of any matter fields, the equation for the antisymmetric tensor field is solved trivially by the replacement

$$H^{\mu\nu\lambda} = e^{4\phi}\varepsilon^{\mu\nu\lambda\rho}\partial_\rho b, \quad (4.5)$$

(where b is the Kalb-Ramond pseudoscalar). It is then the (classical) Bianchi identity $\partial_{[\mu}H_{\mu\rho\lambda]} = 0$, where [...] indicate antisymmetrization of the indices that serves as a dynamical equation for the b field [20]. Indeed, contracting this identity with $\varepsilon^{\mu\nu\rho\lambda}$ in the Einstein frame, and, on noting that $\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{\mu\nu\rho\lambda} = 6\delta_\sigma^\lambda$, we obtain the following equation for b

$$\square b + 4\partial_\mu\phi\partial^\mu b = 0 \quad \Rightarrow \quad \ddot{b} + \left(3\frac{\dot{a}}{a} + 4\dot{\phi}\right)\dot{b} = 0. \quad (4.6)$$

From this we observe that, to ensure a time-independent \dot{b} solution,

$$\dot{b} = K = \text{constant}, \quad (4.7)$$

we must have a dilaton depending on the scale factor of the FRW Universe, $a(t)$, as

$$\phi = -\frac{3}{4}\ln a. \quad (4.8)$$

Substituting then in the dilaton equation (4.4) we obtain

$$-\frac{3}{2}\left[\frac{1}{2}\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right] = \frac{1}{3}\frac{\delta c}{a^{3/2}} - 2\frac{K^2}{a^3} + \dots \quad (4.9)$$

where the ... denote contributions from matter fields that do not contain H -torsion terms.

This equation can be combined with the Einstein equations for the metric field

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\kappa^2\sum_i\rho_i, \quad \frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = -\frac{1}{2}\kappa^2\sum_i p_i, \quad (4.10)$$

where ρ_i, p_i indicate the energy density and pressure of the various stress energy tensors, assumed to be ideal fluids.

It is straightforward to see that there are no solutions compatible with a simple scaling of the scale factor, as in the radiation era -a conventional cosmology assumed in our leptogenesis scenario.

However, in the presence of gauge fields, such as the electromagnetic potential, colour gauge fields *etc.*, it is known that the Bianchi identity in string theory is modified, such that in the Einstein frame

$$\varepsilon^{\mu\nu\rho\sigma} \partial_\mu H_{\nu\rho\sigma} = \left(\varepsilon^{\mu\nu\rho\sigma} H_{\nu\rho\sigma} \right)_{;\mu} = \text{constant} \times F_{\mu\nu} F_{\rho\sigma} \varepsilon^{\mu\nu\rho\sigma} \equiv \text{constant} \times F_{\mu\nu} \tilde{F}^{\mu\nu}. \quad (4.11)$$

where semicolon denotes covariant derivative with respect to the torsion-free connection; (N.B. $\varepsilon^{\mu\nu\rho\sigma}$, the tensorial density, is covariantly constant). The proportionality constant appearing on the right-hand-side has dimensions of inverse mass squared in our conventions (since it is proportional to α'); the precise numerical coefficients depend on the details of the underlying string/brane model. As mentioned in the text, Eq. (4.11) is also related to the axial anomaly (2.10) [16].

At this stage we would like to make a comment on the consistency of the vacuum current condensates (3.8) with the the anomaly equation (2.10) in a Robertson-Walker background, of interest to us. In such a case, by taking the vacuum expectation values on both sides of the equation, and assuming a Robertson-Walker space-time, we obtain $3H \langle J^5 \rangle \propto \langle \mathcal{G}(\omega, A) \rangle$, $\langle J^5 \rangle = \text{constant}$. This in turn implies that the presence of an axial current condensate would be associated with a condensate involving gauge and gravitational fields, which should scale with cosmic time as the Hubble parameter. In string theory, whose effective action is characterised by highly non-linear terms in gauge field strength, such condensates are possible. On assuming the existence of a (dominant over gravitational) condensate of gauge fields $\langle F_{\mu\nu} \tilde{F}^{\mu\nu} \rangle$ [21, 30] in the Leptogenesis era, it is easy to see from (4.5) that the modified Bianchi identity reads

$$\ddot{b} + 3\frac{\dot{a}}{a}\dot{b} + 4\dot{\phi}\dot{b} \propto e^{-4\phi} \langle F_{\mu\nu} \tilde{F}^{\mu\nu} \rangle \quad (4.12)$$

which implies that a *time independent solution* for b is compatible with a constant dilaton³, as a perturbative fixed point; To ensure a constant dilaton, and thus $\square\phi = 0$ in the dilaton equation of motion (4.4), one needs to impose

$$\frac{1}{4} \kappa^2 e^{2\phi} \varepsilon^{\mu\nu\lambda\rho} H_{\mu\nu\lambda} \langle J_\rho^5 \rangle = e^{-4\phi} H_{\mu\nu\rho}^2 - \frac{1}{3} e^{2\phi} \delta c + \dots \quad (4.13)$$

where ... include contributions from the rest of the matter lagrangian. For consistency with rotational invariance only the temporal component $\langle J_0^5 \rangle \neq 0$ of the vacuum current condensate is non-trivial. During the leptogenesis era we assume a time-independent condensate $\langle J_0^5 \rangle$. After that era, typically the condensate will vary with density [31] as a linear or higher power. Consequently, as the Universe expands, and the matter density decreases as $a(t)^{-3}$, the condensate $\langle J_\rho^5 \rangle$ will also fall-off. Furthermore, as we have also noted earlier, the gauge coupling decreases with time due

³Typically the behaviour of the dilaton can be parametrised [26] as $\phi = \Phi_0 \ln[a(t)]$ with Φ_0 negative. ϕ is a slowly varying function of time which has to decay away before the epoch of Bing-Bang-nucleosynthesis, so as not to disturb its delicate conditions, and so in the era of leptogenesis (that precedes nucleosynthesis in our model) we will for simplicity take ϕ to be a constant $\phi = \Phi_0$.

to the dilaton dependence of the string coupling and so becomes too weak to support a condensate [21]. Substituting (4.13) in the original action (4.3), we obtain a term of the form (the terms containing $\delta c e^{2\phi}$ cancel out):

$$S \ni \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \{-3 e^{-4\phi_0} H_{\mu\nu\rho}^2 + \dots\} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \{-18 e^{4\phi_0} (\dot{b})^2 + \dots\} \quad (4.14)$$

where ϕ_0 is the constant dilaton value. The constant \dot{b} is fixed to produce Leptogenesis, as discussed in section 3, and thus such a term contributes *positively* to the vacuum energy of the respective cosmic fluid. Such terms may be cancelled by bulk dynamics in, *e.g.* brane world models [22, 14], where anti de-Sitter type contributions to the brane world vacuum energy independent of the dilaton may be induced from the bulk. One such scenario is discussed briefly in section 4. The issue is, nonetheless, wide open and much more work is needed before phenomenologically realistic models become available.

References

- [1] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716**, 1 (2012) [arXiv:1207.7214 [hep-ex]]; S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716**, 30 (2012) [arXiv:1207.7235 [hep-ex]].
- [2] P. Minkowski, Phys. Lett. B **67**, 421 (1977); M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, eds. D.Z. Freedman and P. van Nieuwenhuizen (North-Holland, Amsterdam, 1979); T. Yanagida, in Proc. of the *Workshop on the Unified Theory and the Baryon Number in the Universe*, Tsukuba, Japan, 1979, eds. O. Sawada and A. Sugamoto; R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).
- [3] J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980); G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B **181**, 287 (1981); For recent reviews see: R. N. Mohapatra *et al.*, *Theory of Neutrinos: a White Paper*, Rept. Prog. Phys. **70**, 1757 (2007) [hep-ph/0510213] and references therein.
- [4] R. Blumenhagen, M. Cvetič and T. Weigand, Nucl. Phys. B **771**, 113 (2007) [hep-th/0609191]; M. Cvetič, R. Richter and T. Weigand, Phys. Rev. D **76**, 086002 (2007) [hep-th/0703028].
- [5] G. Aad *et al.* [ATLAS Collaboration], Eur. Phys. J. C **75**, no. 7, 318 (2015) [arXiv:1503.03290 [hep-ex]]; G. Barenboim, J. Bernabeu, V. A. Mitsou, E. Romero, E. Torro and O. Vives, arXiv:1503.04184 [hep-ph].
- [6] T. Asaka, M. Shaposhnikov and A. Kusenko, “Opening a New Window for Warm Dark Matter,” Phys. Lett. B **638**, 401 (2006); M. Shaposhnikov, JHEP **0808**, 008 (2008) [arXiv:0804.4542 [hep-ph]].
- [7] N. E. Mavromatos and A. Pilaftsis, Phys. Rev. D **86**, 124038 (2012) [arXiv:1209.6387 [hep-ph]].
- [8] M. Kalb and P. Ramond, Phys. Rev. D **9**, 2273 (1974).
- [9] J. Scherk and J. H. Schwarz, Phys. Lett. B **52**, 347 (1974); R. R. Metsaev and A. A. Tseytlin, Nucl. Phys. B **293**, 385 (1987); D. J. Gross and J. H. Sloan, Nucl. Phys. B **291**, 41 (1987).
- [10] M. J. Duncan, N. Kaloper and K. A. Olive, Nucl. Phys. B **387**, 215 (1992).
- [11] D. Colladay and V. A. Kostelecky, Phys. Rev. D **58**, 116002 (1998) [hep-ph/9809521]; V. A. Kostelecky and N. Russell, Rev. Mod. Phys. **83**, 11 (2011) [arXiv:0801.0287 [hep-ph]].
- [12] N. E. Mavromatos and S. Sarkar, Eur. Phys. J. C **73**, no. 3, 2359 (2013) [arXiv:1211.0968 [hep-ph]]; J. Ellis, N. E. Mavromatos and S. Sarkar, Phys. Lett. B **725**, 407 (2013) [arXiv:1304.5433 [gr-qc]].

- [13] M. de Cesare, N. E. Mavromatos and S. Sarkar, arXiv:1412.7077 [hep-ph].
- [14] J. R. Ellis, N. E. Mavromatos and M. Westmuckett, Phys. Rev. D **70**, 044036 (2004) [gr-qc/0405066]; *ibid.* **71**, 106006 (2005) [gr-qc/0501060]; N. E. Mavromatos, Phys. Rev. D **83**, 025018 (2011) [arXiv:1011.3528 [hep-ph]]; N. E. Mavromatos, V. A. Mitsou, S. Sarkar and A. Vergou, Eur. Phys. J. C **72**, 1956 (2012) [arXiv:1012.4094 [hep-ph]]; J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, JCAP **1411**, no. 11, 014 (2014) [arXiv:1406.0487 [hep-th]].
- [15] T. W. B. Kibble, J. Math. Phys. **2**, 212 (1961); D. W. Sciama, Rev. Mod. Phys. **36**, 463 (1964) [Erratum-*ibid.* **36**, 1103 (1964)]; F. W. Hehl, P. Von Der Heyde, G. D. Kerlick and J. M. Nester, Rev. Mod. Phys. **48**, 393 (1976); I. L. Shapiro, Phys. Rept. **357**, 113 (2002) [hep-th/0103093] and references therein.
- [16] R. Delbourgo and A. Salam, Phys. Lett. B **40**, 381 (1972); see, also: S. Weinberg, *The Quantum Theory of Fields. Volume II: Modern Applications.* (Cambridge University Press 2001) ISBN 0-521-55002-5.
- [17] A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper and J. March-Russell, Phys. Rev. D **81**, 123530 (2010) [arXiv:0905.4720 [hep-th]]; M. Cicoli, M. Goodsell and A. Ringwald, JHEP **1210**, 146 (2012) [arXiv:1206.0819 [hep-th]].
- [18] J. F. Donoghue, Phys. Rev. D **50**, 3874 (1994) [gr-qc/9405057].
- [19] G. K. Leontaris and N. D. Vlachos, Mod. Phys. Lett. A **29**, 0179 (2014) [arXiv:1309.5785 [hep-ph]], but with their results applied to the right-handed neutrino sector, and references therein.
- [20] I. Antoniadis, C. Bachas, J. R. Ellis and D. V. Nanopoulos, Nucl. Phys. B **328**, 117 (1989).
- [21] E. T. Tomboulis, Phys. Rev. D **85** (2012) 024042 [arXiv:1105.3720 [hep-th]].
- [22] N. E. Mavromatos and J. Rizos, Phys. Rev. D **62**, 124004 (2000) [hep-th/0008074]; Int. J. Mod. Phys. A **18**, 57 (2003) [hep-th/0205299].
- [23] E. W. Kolb and S. Wolfram, Nucl. Phys. **B172**, 224 (1980)
- [24] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).
- [25] S. Weinberg, *Cosmology*, Oxford University Press (2008); Phys. Rev. Lett. **42**, 850 (1979).
- [26] A. B. Lahanas, Phys. Rev. D **83**, 103523 (2011) [arXiv:1102.4277 [hep-ph]]; C. M. Bender, N. E. Mavromatos and S. Sarkar, Phys. Rev. D **87** (2013) 5, 055021 [arXiv:1212.6715 [hep-ph]].
- [27] V. A. Kostelecky and M. Mewes, Phys. Rev. Lett. **99**, 011601 (2007) [astro-ph/0702379 [ASTRO-PH]]; M. Das, S. Mohanty and A. R. Prasanna, Int. J. Mod. Phys. D **22**, 1350011 (2013) [arXiv:0908.0629 [astro-ph.CO]].
- [28] V. A. Kostelecky and N. Russell, Rev. Mod. Phys. **83**, 11 (2011) [arXiv:0801.0287 [hep-ph]]; see also: N. E. Mavromatos, Hyperfine Interact. **228**, no. 1-3, 7 (2014) [arXiv:1312.4304 [hep-ph]].
- [29] M. R. Douglas, D. N. Kabat, P. Pouliot and S. H. Shenker, Nucl. Phys. B **485**, 85 (1997) [hep-th/9608024].
- [30] G. E. Volovik Physica B **255**, 86 (1998).
- [31] S. Goda and D. Jido, Phys. Rev. C **88** (2013) 6, 065204 [arXiv:1308.2660 [nucl-th]]. J. P. Blaizot and E. Iancu, *QCD perspectives on hot and dense matter. Proceedings*, NATO Advanced Study Institute, Summer School, Cargese, France, August 6-18, 2001 (NATO science series II: Mathematics, physics and chemistry. 87).