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F-Theory Model Building with Discrete Symmetry

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A review of some concepts in F-Theory model building was presented, with an overview of the the spectral cover approach given. Non-Abelian monodromy was considered as a mechanism for family symmetry like structure in semi-local F-Theory. An $SU(5)$ GUT model with an A_4 monodromy group was presented, with analysis of the neutrino sector.

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1. Introduction

F-Theory [1] is a 12 Dimensional formulation of Type IIB string theory, with the internal dimensions classified as a complex Calabi-Yau four-fold elliptically fibred over a threefold base. There is a well studied correspondence between the singularities of elliptically fibred space and the gauge groups to which they relate. This is an intriguing feature for theorists, since it facilitates popular GUT groups [4, 5, 6, 7, 8, 9, 10] such as $SU(5)$, $SO(10)$ or E_6, with a maximum symmetry enhancement of the exceptional group E_8.

Elliptic fibration can be described mathematically in terms of the Weierstrass equation:

$$y^2 = x^3 + f(z)x + g(z) \tag{1.1}$$

where x, y and z are complex coordinates and f and g are eighth and twelfth degree polynomials respectively. The form of this equation and its discriminant, $\Delta = 4f^3 + 27g^2$, determine the type of singularities present in the geometry. As already mentioned, due to the work of Kodaira, we have a full classification of the gauge groups supported by the singularities of a particular space. By performing a process known as Tate's algorithm, which entails enforcing vanishing of the discriminant to various orders in z, we may re-cast the Weierstrass equation in the so called Tate form:

$$y^2 + \alpha_1 xy + \alpha_3 y = x^3 + \alpha_2 x^2 + \alpha_4 x + \alpha_6 \,. \tag{1.2}$$

The α_n coefficients in this equation and their vanishing order determine the singularity of the surface. For example, an $SU(5)$ singularity would correspond to[19, 20, 21]:

$$\alpha_1 = -b_5,\ \alpha_2 = b_4 z,\ \alpha_3 = -b_3 z^2,\ \alpha_4 = b_2 z^3,\ \alpha_6 = b_0 z^5 \,. \tag{1.3}$$

Table 1 outlines the conditions required for some of the more interesting groups to be realised in the geometry.

In the semi-local approach to F-theory [17] we may put aside the challenging issue of compactifying the space and focus on a local patch: a D7-brane within the manifold that exhibits the GUT symmetry of choice. Exploiting the properties of elliptic fibrations, we shall also assume that there is a point of E_8 enhancement in the geometry and that all interactions descend from this maximal enhancement, which is broken by an appropriate Higgsing. This will be used to study a particular $SU(5)$ scenario in the subsequent narrative.

The assumption of an $SU(5)$ GUT group requires that the matter representations, which come from the adjoint of E_8, are a bi-fundamental representation with charges under the commutant with E_8, which is a perpendicular $SU(5)$:

$$E_8 \to SU(5)_{GUT} \times SU(5)_{\perp} \tag{1.4}$$

$$248 \to (24,1) + (1,24) + (10,5) + (\overline{10},\bar{5}) + (\bar{5},10) + (5,\overline{10}) \tag{1.5}$$

This breaking informs us that under the GUT group we should have five antisymmetric representations, ten fundamental, and twenty four singlets. These are described in terms of the weights of the perpendicular group as :

$$\begin{aligned}
\Sigma_{10} &: t_i = 0, \\
\Sigma_{\bar{5}} &: t_i + t_j = 0,\ i \neq j \\
\Sigma_1 &: \pm(t_i - t_j) = 0,\ i \neq j
\end{aligned}$$

Group	α_1	α_2	α_3	α_4	α_6	Δ
$SU(2n)$	0	1	n	n	2n	2n
$SU(2n+1)$	0	1	n	n+1	2n+1	2n+1
$SO(10)$	1	1	2	3	5	7
E_6	1	2	3	3	5	8
E_7	1	2	3	3	5	9
E_8	1	2	3	4	5	10

Table 1: The vanishing orders of each of the coefficients of Equation (1.2) in terms of the coordinate z for various various interesting gauge groups to be realised in an elliptically fibred space. The gauge groups are supported by specific types of singular fibre, as classified by Kodaira.

In general these weights are not independent, but related by monodromy actions on the roots as we shall later discuss.

The Tate form of the Weierstrass equation will correspond to an $SU(5)$ singularity if the conditions of Equation (1.3) are enforced. This gives a relatively complicated polynomial in the coordinates of the space:

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 y z^2 + b_4 x^2 z + b_5 xy\,. \tag{1.6}$$

However, using homogenous coordinates, $z \to U$, $x \to V^2$, and $y \to V^3$ along with an affine parameter $s = U/V$ can be written as a so-called spectral cover, which is simply a fifth order polynomial in s, the roots of which are identified as the roots of $SU(5)_\perp$, t_i:

$$0 = b_5 + b_4 s + b_3 s^2 + b_2 s^3 + b_1 s^4 + b_0 s^5 \propto \prod_{i=1}^{5}(s + t_i)\,. \tag{1.7}$$

This equation accounts for the antisymmetric representation of the GUT group, while an equation to characterise the fundamental representation should be a tenth order polynomial with roots $t_i + t_j$ as state above. By consistency with Equation (1.7), this can be written in terms of the coefficients of the equation for the antisymmetric representation. The defining equation is the zeroth order part of that polynomial, which can be shown to be:

$$P_5 = b_3^2 b_4 - b_2 b_3 b_5 + b_0 b_5^2 \propto \prod_{i>j}(t_i + t_j) \tag{1.8}$$

These two equations are sufficient to describe the matter content of an $SU(5)$ GUT model in F-theory, however the exact matter content is influenced by monodromy actions on the roots, which we shall discuss in Section 2.

2. Monodromy

The perpendicular group left over after isolating the GUT group within the parent E_8 theory will cause various points of symmetry enhancement on the GUT surface. For example, in the $SU(5)$ case we shall consider, the model essentially has four $U(1)$s intersecting the $SU(5)$ GUT surface at various points,

$$E_8 \to SU(5)_{GUT} \times SU(5)_\perp \to SU(5)_{GUT} \times U(1)^4\,. \tag{2.1}$$

âĂć However, the charges associated to these enhancements must cancel out in the low energy theory, such as for Yukawa couplings.

We shall focus on Yukawa couplings, which are formed in the usual way for an $SU(5)$ GUT theory, with the additional constraint that the 10s have charges t_i and the $5s$ ($\bar{5}$s) have charge $-t_i - t_j$ ($t_i + t_j$). Correspondingly, the Top quark and the Bottom/Tau Yukawa couplings are of the form:

$$\begin{aligned}\text{Top type Yukawas: } & 10_{t_i} \cdot 10_{t_j} \cdot 5_{-t_k - t_l} \\ \text{With: } & k \neq l \\ \text{Bottom/Tau type Yukawas: } & 10_{t_i} \cdot \bar{5}_{t_j + t_k} \cdot \bar{5}_{t_l + t_m} \\ \text{With: } & j \neq k \text{ and } l \neq m\end{aligned}$$

In order for the Top quark to have a tree-level, renormalisable coupling, the diagonal term in the matrix ($10_{t_i} \cdot 10_{t_i} \cdot 5_{-t_j - t_k}$ with $j \neq k$) must be have no residual charge under the perpendicular charge. Clearly there is no way to achieve this as $j \neq k$ for the 5 carrying the Higgs. As a consequence we seem to be unable to write down any terms to give a renormalisable Top, which is a phenomenologically undesirable feature.

However, in general not all the roots are independent and there will be some monodromy action(s) relating two or more roots. An enlightening yet simple example (following the presentation in [15]) is the minimal monodromy action, which is Z_2. Suppose that two of the roots of the spectral cover found in Equation (1.7) cannot be factorised within the same field as the original coefficients of the equation[1], i.e.

$$b_5 + b_4 s + b_3 s^2 + b_2 s^3 + b_1 s^4 + b_0 s^5 = (a_1 + a_2 s + a_3 s^2)(a_4 + a_5 s)(a_6 + a_7 s)(a_8 + a_9 s)\,. \tag{2.2}$$

The quadratic part of this equation has two roots,

$$r_1 = \frac{-a_2 + \sqrt{a_2^2 - 4a_1 a_3}}{2a_3},\, r_2 = \frac{-a_2 - \sqrt{a_2^2 - 4a_1 a_3}}{2a_3} \tag{2.3}$$

which are identical up to the sign in front of the discriminant. Let $w = a_2^2 - 4a_1 a_3$, then it we may also write without loss of generality $w = e^{i\theta}|w|$, which is invariant under $\theta \to \theta + 2\pi$. However, since we deal with $\sqrt{w} = e^{i\theta/2}\sqrt{|w|}$, the roots $r_{1,2}$ are not invariant, but instead interchange $r_1 \leftrightarrow r_2$, implying that the D7-branes associated to those roots are interchangeable under this action. In terms of the Top quark coupling, the consequence is that the charge of the 5 carrying the Up-type Higgs may have charge $(-t_j - t_k) \to -2t_j$, while the Top quark coupling becomes:

$$10_{t_i} \cdot 10_{t_i} \cdot 5_{-t_i - t_j} \to 10_{t_i} \cdot 10_{t_i} \cdot 5_{-2t_i}\,, \tag{2.4}$$

which is trivially invariant under the perpendicular $U(1)$s allowing a renormalisable Top Yukawa.

This example is the simplest and minimal monodromy choice available, however there are a large number of monodromy options. Since we do not have any knowledge of the global geometry, we must select any monodromy groups for our model by hand. In the case of an $SU(5)$ theory, the monodromy group can range from the simple Z_2 case already discussed to the Weyl group of the

[1]Note that due to tracelessness of $SU(5)$, $b_1 = 0$

b_i	a_j coefficients for 4+1
b_0	$a_5 a_7$
b_1	$a_5 a_6 + a_4 a_7$
b_2	$a_4 a_6 + a_3 a_7$
b_3	$a_3 a_6 + a_2 a_7$
b_4	$a_2 a_6 + a_1 a_7$
b_5	$a_1 a_6$

Table 2: Summary of the relationships between the coefficients of $\mathscr{C}_5$ and $\mathscr{C}_4 \times \mathscr{C}_1$

five weights, S_5. The latter is one of the many non-Abelian monodromy groups, which until recent years [32] have been largely ignored as it is unclear how these may be realised in the geometry.

In this work we discuss the case of A_4 as a monodromy group, which is the group of all even permutations of four elements, which are the symmetries of a tetrahedron. While in the Abelian cases of Z_N monodromies we consider the roots to be directly identified, for this non-Abelian case we treat the weights as representations of the monodromy group as the group structure is more rich. This is conjectured to be generated by some non-Abelian, internal flux that is not yet fully understood.

3. An F-$SU(5)$ Model with A_4 Monodromy

Let us now examine a model based on $SU(5)$ with an A_4 monodromy action upon four of the weights of the perpendicular group. In order to have this choice realised, the spectral cover must factorise into a quartic part and a linear part.

$$\mathscr{C}_4 \times \mathscr{C}_1 : (a_1 + a_2 s + a_3 s^2 + a_4 s^3 + a_5 s^4) \times (a_6 + a_7 s) = 0$$

These coefficients must be consistent with the original b_k coefficients of Equation (1.7), as shown in Table 2, while also being in the same field - avoiding branch-cuts.

These coefficients can in general be any monodromy action on four weights, the most general of which would be S_4. The exact group can be constrained via Galois theory, which deals with transitive groups and polynomials. However, in this instance we shall simply take the monodromy to be A_4 and proceed to examine the consequences.

The defining equation for the antisymmetric representation of the GUT group (the 10s) is for the s^0 term of Equation (1.7), while the defining equation for the fundamental representation (the $5/\bar{5}$s) is Equation (1.8). Inputting the relevant a_i coefficients, we find that the polynomials for the 10s and $5/\bar{5}$s respectively are:

$$P_{10} = a_1 a_6 \tag{3.1}$$

$$P_5 = (a_2^2 a_7 + a_2 a_3 a_6 \mp a_0 a_1 a_6^2)(a_3 a_6^2 + (a_2 a_6 + a_1 a_7) a_7)\,. \tag{3.2}$$

We have also exploited the property of $SU(5)$ tracelessness, solving $b_1 = a_4 a_7 + a_5 a_6 = 0$ with $a_4 = \pm a_0 a_6$ and $a_5 = \mp a_7$. These equations cannot be further factorised within the same field without creating branch-cuts. As such, each factor is interpreted as defining the properties of a D7-brane intersecting the GUT surface, which we call a Matter Curve. The homologies of the original

Curve	Equation	Homology	N	M
10_a	a_1	$\eta - 5c_1 - \chi$	$-N$	M_{10_a}
10_b	a_6	χ	$+N$	M_{10_b}
5_c	$a_2^2 a_7 + a_2 a_3 a_6 \mp a_0 a_1 a_6^2$	$2\eta - 7c_1 - \chi$	$-N$	M_{5_c}
5_d	$a_3 a_6^2 + (a_2 a_6 + a_1 a_7) a_7$	$\eta - 3c_1 + \chi$	$+N$	M_{5_d}

Table 3: Table of matter curves, their homologies, charges and multiplicities.

b_k are well defined in terms of the Chern classes of the tangent bundle (c_1) and the normal bundle (t):

$$\begin{aligned} [b_k] =& \eta - kc_1 \\ \eta =& 6c_1 - t\,. \end{aligned}$$

We can use this to determine the homologies of the a_i coefficients up to some unknown homology of one of the coefficients, usually taken to be $[a_6] = \chi$. The homologies of the Matter Curves are presented in Table 3.

Examining Table 3, we can see that we have a limited spectrum for model building: two 10s and two 5s. This seems insufficient to construct any realistic model, however we may exploit a property of A_4 to improve the situation. Since we are dealing with a monodromy action on four weights we have a quadruplet under the group action, which is a reducible representation of A_4. Using a unitary transformation the weights, $t_{i=1,\ldots,4}$ can be rotated into a basis which transforms as a singlet (t_s) and a triplet ($\{t_a, t_b, t_c\}$).

$$\begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{pmatrix} \rightarrow \begin{pmatrix} t_1 + t_2 + t_3 + t_4 \\ t_1 + t_2 - t_3 - t_4 \\ t_1 - t_2 + t_3 - t_4 \\ t_1 - t_2 - t_3 + t_4 \end{pmatrix} = \begin{pmatrix} t_s \\ t_a \\ t_b \\ t_c \end{pmatrix} \tag{3.3}$$

This can be applied to the 10s, bifurcating one of the preexisting matter curves and giving us a total of three 10s. A similar process can be applied to the $5/\bar{5}$s, increasing the number of matter curves to four. The price we pay for this improvement is that we do not have knowledge of the homologies of the new matter curves, though they must respect the same flux restrictions over all.

The initially massless states on the Matter curves are complete vector multiplets, which must become chiral in the low energy spectrum. This can be achieved by switching on fluxes in the internal geometry, splitting the doublets and triplets. We impose hypercharge flux as follows:

$$\begin{aligned} \mathscr{F}_Y \cdot \chi &= N \\ \mathscr{F}_Y \cdot c_1 &= \mathscr{F}_Y \cdot \eta = 0 \end{aligned}$$

This flux naturally allows us to split the GUT representations to those of the Standard Model, leaving only the multiplicities of the matter on each matter curve to the model builders choice. The $5/\bar{5}$s split as:

$$\begin{aligned} n(3,1)_{-1/3} - n(\bar{3},1)_{+1/3} &= M_5\,, \\ n(1,2)_{+1/2} - n(1,2)_{-1/2} &= M_5 + N\,, \end{aligned} \tag{3.4}$$

Curve	$SU(5)\times A_4\times U(1)_\perp$	N_Y	M	Matter content	R
10_1	$(10,3)_0$	0	M_{T1}	$3\left[M_{T1}Q_L+u^c_L(M_{T1}-N_Y)+e^c_L(M_{T1}+N_Y)\right]$	1
10_2	$(10,1)_0$	$-N$	M_{T2}	$M_{T2}Q_L+u^c_L(M_{T2}-N_Y)+e^c_L(M_{T2}+N_Y)$	1
10_3	$(10,1)_{t_5}$	$+N$	M_{T3}	$M_{T3}Q_L+u^c_L(M_{T3}-N_Y)+e^c_L(M_{T3}+N_Y)$	1
5_1	$(5,3)_0$	0	M_{F1}	$3\left[M_{F1}\bar{d}^c_L+(M_{F1}+N_Y)\bar{L}\right]$	1
5_2	$(5,3)_0$	$-N$	M_{F2}	$3\left[M_{F2}\bar{\bar{D}}+(M_{F2}+N_Y)\bar{H}_d)\right]$	0
5_3	$(5,3)_{t_5}$	$+N$	M_{F3}	$3\left[M_{F3}D+(M_{F3}+N_Y)H_u\right]$	0
5_4	$(5,1)_{t_5}$	0	M_{F4}	$M_{F4}\bar{d}^c_L+(M_{F4}+N_Y)\bar{L}$	1

Table 4: Table showing the possible matter content for an $SU(5)_{\text{GUT}}\times A_4\times U(1)_\perp$, where it is assumed the reducible representation of the monodromy group may split the matter curves. The curves are also assumed to have an R-symmetry

while the 10s split into three representations:

$$\begin{aligned} n(3,2)_{+1/6}-n(\bar{3},2)_{-1/6} &= M_{10}\,,\\ n(\bar{3},1)_{-2/3}-n(3,1)_{+2/3} &= M_{10}-N\,,\\ n(1,1)_{+1}-n(1,1)_{-1} &= M_{10}+N\,. \end{aligned} \tag{3.5}$$

Note that anomaly cancellation requires $\sum M_5+\sum M_{10}=0$. Table 4 shows a prospective matter spectrum for this time of model, with A_4 irreducible representations and doublet-triplet splitting parameters included.

The model presented in [2] is shown in Table 5, which corresponds to a model where $N=0$ and the multiplicities are chosen to be:

$$\begin{aligned} M_{10_1}=&M_{5_4}=0,\\ M_{10_2}=&M_{5_3}=-M_{5_2}=-M_{5_1}=1,\\ M_{10_3}=&2, \end{aligned}$$

which will generate a spectrum with the correct numbers of quarks and charged leptons. The Leptons in particular will be found in a triplet of A_4, which will give the neutrino sector a richer structure. The full discussion of this model can be found in [2], so here we shall merely highlight the neutrino sector for attention.

3.1 Neutrino Sector

The Neutrinos are unique in the SM in that they are the only particles that may be Majorana - that is, the neutrino could be its own anti-particle. This opens up the possibility of explaining their comparatively tiny mass via some form of so-called 'Seesaw' mechanism, whereby the effective mass of the neutrino is given by the Weinberg operator,

$$M_{eff}=M_D M_R^{-1}M_D^{\mathrm{T}}. \tag{3.6}$$

The Dirac matrix (M_D) and the Majoranna mass matrix for the right-handed neutrinos (M_R) are calculated by considering the couplings in Table 6.

Curve	Rep'n	R-sym	Matter content
10_1	$(10,3)_0$	1	-
$10_2 = T_3$	$(10,1)_0$	1	$Q_L + u^c_L + e^c_L$
$10_3 = T$	$(10,1)_{t_5}$	1	$2Q_L + 2u^c_L + 2e^c_L$
$\bar{5}_1 = F$	$(\bar{5},3)_0$	1	$3L + 3d^c_L$
$\bar{5}_2 = H_d$	$(\bar{5},3)_0$	0	$3\bar{D} + 3H_d$
$5_3 = H_u$	$(5,3)_{t_5}$	0	$3D + 3H_u$
5_4	$(5,1)_{t_5}$	1	-
θ_a	$(1,3)_{-t_5}$	0	Higgs Flavons
θ_b	$(1,1)_{-t_5}$	0	Flavon
θ_c	$(1,3)_0$	1	ν_R
θ_d	$(1,3)_0$	0	Flavons

Table 5: Table of Matter content in $N = 0$ model

	Full coupling
Dirac-type mass	$\theta_c \cdot F \cdot H_u \cdot \theta_a$
	$\theta_c \cdot F \cdot H_u \cdot \theta_a \cdot (\theta_d)^n$
	$\theta_c \cdot F \cdot H_u \cdot \theta_b$
	$\theta_c \cdot F \cdot H_u \cdot \theta_b \cdot (\theta_d)^n$
Right-handed neutrinos	$M\theta_c \cdot \theta_c$
	$(\theta_d)^n \cdot \theta_c \cdot \theta_c$

Table 6: Operators allowed in the neutrino sector, assuming the matter content of Table 5. θ_c is selected as the righthanded neutrino, while the charged lepton doublet is found in F.

Parameter	Central value	Min $\to$ Max
$\theta_{12}/^\circ$	33.57	32.82$\to$34.34
$\theta_{23}/^\circ$	41.9	41.5$\to$42.4
$\theta_{13}/^\circ$	8.73	8.37$\to$9.08
$\Delta m^2_{21}/10^{-5}\text{eV}$	7.45	7.29 $\to$ 7.64
$\Delta m^2_{31}/10^{-3}\text{eV}$	2.417	2.403 $\to$ 2.431
$R = \frac{\Delta m^2_{31}}{\Delta m^2_{21}}$	32.0	31.1 $\to$ 33.0

Table 7: Summary of neutrino parameters, using best fit values as found at nu-fit.org

In order to calculate the contributions of these operators we must assign vacuum alignments to the relevant singlets, as well as the Higgs. We assume that the flavons θ_a and θ_b, arising from the singlet sector, get alignments:

$$\langle \theta_a \rangle = (a,0,0)^{\mathrm{T}} \text{ and } \langle \theta_b \rangle = b \tag{3.7}$$

where we have taken the basis where the group element S is diagonal. The Up-type Higgs is chosen in a likewise fashion to be:

$$\langle H_u \rangle = (v,0,0)^{\mathrm{T}} . \tag{3.8}$$

The first four operators correspond to Dirac mass terms, coupling left and right-handed neutrinos.

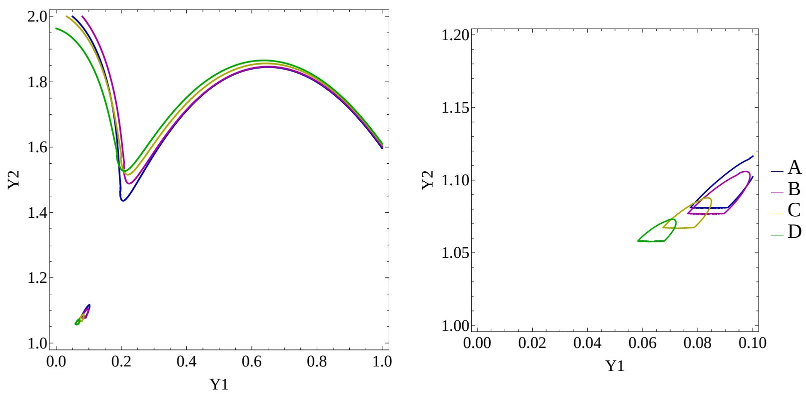

Figure 1: Plots of lines with the best fit value of $R = 32$ in the parameter space of (Y_1, Y_2). Left: The full range of the space examined. Right: A close plot of a small portion of the parameter space taken from the full plot. The curves have (Y_3, Z_1, Z_2) values set as follows: $A = (1.08, 0.05, 0.02)$, $B = (1.08, 0.0, 0.08)$, $C = (1.07, 0.002, 0.77)$, and $D = (1.06, 0.01, 0.065)$.

In the chosen basis, the triplets of A_4 the triplet product: $3_a \times 3_b = 1 + 1' + 1'' + 3_1 + 3_2$, where:

$$\begin{aligned} 1 &= a_1 b_2 + a_2 b_2 + a_3 b_3 \\ 1' &= a_1 b_2 + \omega a_2 b_2 + \omega^2 a_3 b_3 \\ 1'' &= a_1 b_2 + \omega^2 a_2 b_2 + \omega a_3 b_3 \\ 3_1 &= (a_2 b_3,\, a_3 b_1,\, a_1 b_2)^{\mathrm{T}} \\ 3_2 &= (a_3 b_2,\, a_1 b_3,\, a_2 b_1)^{\mathrm{T}} \end{aligned}$$

Assuming the following alignments $3_a = (a_1, a_2, a_3)^{\mathrm{T}}$ and $3_b = (b_1, b_2, b_3)^{\mathrm{T}}$. The first of these operators will fill out the main diagonal, while the rest of the matrix will be filled out by the remaining operators, giving a Dirac mass matrix (at lowest order) of:

$$M_D = \begin{pmatrix} y_0 v a & z_3 v d_2 b & z_2 v d_3 b \\ z_1 v d_2 b & y_1 v a & y_9 b v \\ z_4 v d_3 b & y_8 b v & y_1 v a \end{pmatrix} \tag{3.9}$$

where $y_{i>0}$ are Yukawa couplings, with $y_0 = y_1 + y_2 + y_3$, and z_i are higher order couplings. Similarly, we may calculate the Majorana mass matrix, which has two contributions that will dominate.

$$M_R = M \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y \begin{pmatrix} 0 & d_3 & d_2 \\ d_3 & 0 & d_1 \\ d_2 & d_1 & 0 \end{pmatrix} \tag{3.10}$$

Between these two matrices we have a neutrino sector that is quite complicated and it will be a challenge to make any predictions due to the large number of parameters. However, we shall make some simplifications to attempt to extract some results. Let us start by setting $z_1 = z_3$ and z_2=z_4, which significantly narrows the parameter space. We may then reduce the number of parameters

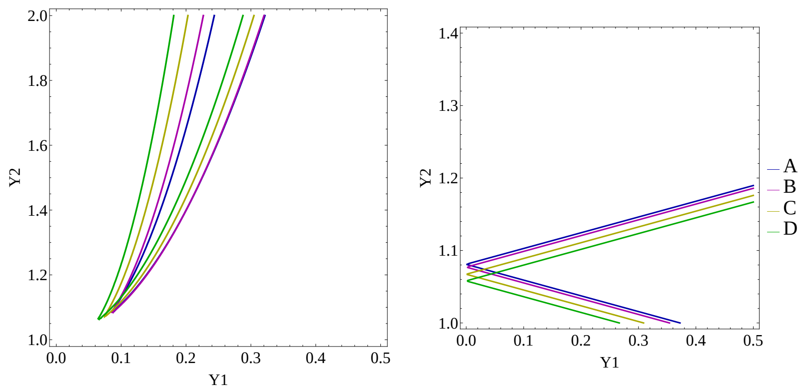

Figure 2: The figures show plots of two large neutrino mixing angles at their current best fit values. Left: Plot of $\sin^2(\theta_{12}) = 0.306$, Right: Plot of $\sin^2(\theta_{23}) = 0.446$. The curves have (Y_3, Z_1, Z_2) values set as follows: $A = (1.08, 0.05, 0.02)$, $B = (1.08, 0.0, 0.08)$, $C = (1.07, 0.002, 0.77)$, and $D = (1.06, 0.01, 0.065)$.

greatly by defining some dimensionless variables to compute with:

$$\begin{gathered} Y_1 = \frac{y_1}{y_0} \leq 1 \;\; Y_{2,3} = \frac{y_{8,9}b}{y_0 a} \\ Z_1 = \frac{z_1 d_2 b}{y_0 a} \quad Z_2 = \frac{z_2 d_3 b}{y_0 a} \end{gathered} \tag{3.11}$$

If we then compute our effective mass matrix, we find that we have five variables and one mass scale, $m_0 = \frac{y_0^2 v^2 a^2}{M}$:

$$\frac{M_{eff}}{m_0} = \begin{pmatrix} 1 + Z_1^2 + Z_2^2 & Y_1 Z_1 + Y_3 Z_2 + Z_1 & Y_2 Z_1 + Y_1 Z_2 + Z_2 \\ Y_1 Z_1 + Y_3 Z_2 + Z_1 & Y_1^2 + Y_3^2 + Z_1^2 & Y_1(Y_2 + Y_3) + Z_1 Z_2 \\ Y_2 Z_1 + Y_1 Z_2 + Z_2 & Y_1(Y_2 + Y_3) + Z_1 Z_2 & Y_1^2 + Y_2^2 + Z_2^2 \end{pmatrix}. \tag{3.12}$$

This matrix cannot easily be manipulated by hand to solve for known neutrino parameters, presented in Table 7. Instead the matrix has been computationally diagonalised, with the resulting outputs being fitted to the data. The focus was placed on matching the mass squared ratio,

$$R = \left| \frac{m_3^2 - m_2^2}{m_2^2 - m_1^2} \right|, \tag{3.13}$$

which at the time of this work had a best fit value of 32, as shown in Table 7. Having made our fit as close to this value as possible, we also required that the model give phenomenologically acceptable values of the neutrino mixing angles. While this does not allow prediction of the neutrino mixing angles, it does allow us to read off an absolute mass scale for each of the neutrinos, with the sum of the masses also being known absolutely.

Figure 1 shows a plot of lines in the (Y_1, Y_2) plane for the central value of $R = 32$. It demonstrates that there are large portions of the parameter space that are able to fulfill this starting requirement. Meanwhile Figure 2 plots $\sin^2(\theta_{12})$ and $\sin^2(\theta_{23})$ best fit values in the same plane. Again, the model has regions available that satisfy these constraints from the data.

As a final test of the model, a set of benchmark points has been identified, shown in Table 8, which will give values of all inputs from data within best fit values. The table also shows that

Inputs				
Y_1	0.08	0.09	0.09	0.10
Y_2	1.09	1.10	1.10	1.11
Y_3	1.07	1.08	1.08	1.09
Z_1	0.01	0.01	0.00	0.01
Z_2	0.07	0.08	0.08	0.08
m_0	54.0meV	51.6meV	50.3meV	47.8meV
Outputs				
θ_{12}	33.5	33.2	33.1	32.8
θ_{13}	8.70	8.82	9.05	9.05
θ_{23}	41.9	41.7	41.7	41.5
m_1	53.4meV	51.1meV	49.8meV	47.3meV
m_2	54.1meV	51.8meV	50.5meV	48.1meV
m_3	73.2meV	71.5meV	70.8meV	69.1meV

Table 8: Table of Benchmark values in the Parameter space, where all experimental constraints are satisfied within errors. These point are samples of the space of all possible points, where we assume θ_{23} is in the first octant. All inputs are given to two decimal places, while the outputs are given to 3s.f.

typically the neutrino mass scale is confined to be in a region close to 50meV, with the sum of the neutrino masses being less than 200meV, placing them within cosmological limits on the total neutrino mass contribution.

It is worth noting that this model disfavours a second octant θ_{23}, with no numerical solutions being found that satisfied all the constraints. As such, if θ_{23} were discovered to be in the second octant, this model would be entirely ruled out.

4. Conclusions and Outlook

In this work we examined the implications of using non-Abelian monodromy actions in F-theory GUTs. Specifically we considered an $SU(5)$ model with an A_4 monodromy acting on its roots, which was motivated by an interest in the neutrino sector. In the framework of the spectral cover formalism we were able to discuss the consequences of considering the action on the roots to be a reducible representation of the monodromy group, allowing for non-trivial mass textures to be examined for the neutrino sector. A numerical approach was used to fit the model to results coming from experiment, yielding a model which was able to be compatible with all constraints from experiment.The work presented predicted neutrino masses that were compatible with cosmology, giving a scale for the neutrino mass in the region of 50meV.

Since presenting this work further research has been done to understand these types of model, with a recent paper on the implications of a D_4 model with a geometrically inspired R-parity being examined [3]. In this work we have shown that such models often exhibit distinct types of R-parity violation, with signatures visible in neutron-antineutron oscillations but freedom from proton decay. This geometric R-parity provides a much needed motivation for R-parity, while also giving strong predictions to many models. As such it will be the focus of up-coming works.

To summarise, F-theory provides an excellent playground for motivating discrete symmetries, with promising features for GUT models and flavour physics, as well as other BSM physics. Using the tools developing in this field it may be possible to address many current problems in physics, while making interesting predictions that will be testable at future detectors.

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