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MOEA/D Multi-objective Optimization with Adaptive ε-domination based Random Elitist and Non-uniform Domination Strategy

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MOEA/D multi-objective optimization algorithm features the shortcoming of losing partial of the excellent individuals when it updates sub-problems and its inefficiency of convergence speed. In order to overcome such shortcoming, we propose a multi-objective evolutionary optimization algorithm based on adaptive epsilon-domination and random elitist strategy in this paper. This algorithm uses archive population that is updated by adaptive epsilon-domination to achieve the optimization goals. The algorithm is able to keep non-inferior solutions, reduce the losing of excellent individuals in the evolution process and hold archive population to maintain a certain size, and ensure the convergence speed and the uniformity of the distribution of non-inferior solutions; in addition, the algorithm uses the archive population to update each sub-problem of the evolution population at certain probability and the number of domination so as to speed up the convergence speed. The experiment results show that the new algorithm proposed by us is more effective than MOEA/D and NSGA-II in ensuring the uniformity of the distribution of non-inferior solution and the convergence speed for Multi-objective optimization.

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1. Introduction

In recent years, using Multi-Objective Evolutionary Algorithms (MOEAs) to effectively solve the multi-objective optimization problem in real application has become a hot research topic. In 2007, MOEA/D multi-objective optimization algorithm was proposed by Zhang [1].In MOEA/D. A multi-Objective problem is decomposed to a num of single-objective problems by using a scalarizing function with different weight vectors. Each single-objective problem optimizes the scalarizing function with a different weight vector. As the fitness evalution for each individual is based on scalarizing function calculation, it can be efficiently performed as to many-objective problems. As MOEA/D heavily depended on the design of its weight, Hai-Lin Liu proposed a weight-constructed method, which accessed points uniformly on the hyper sphere to improve the uniformity of the non-inferior solutions distribution[2]. Fang-Qing Gu also gave a dynamically variable weight design method based on projection and equidistant interpolation which was efficient to ameliorate the uniformity of the non-inferior solutions. S-Z Zhao noticed neighbors of MOEA/D algorithm was a fixed size, presented a dynamic algorithm which can adjust the neighbor size, and the new algorithm worked better than the original algorithm. In the aspect of Pareto dominance, Md Nasir applied fuzzy Pareto dominance to MOEA/D, experiments showed that the new algorithm converged significantly fast [3]. Tsung-Che Chiang noted that mating selection was carried out in a uniform and static manner and the mating pool of each individual was determined and fixed based on the distance between weight vectors on the objective space. He proposed an adaptive mating selection mechanism for MOEA/D. Zhang introduced a DE operator to the MOEA/D, which enhanced the ability of solving complex Pareto front [4]. Although the research results have enhanced the ability of MOEA/D's, which has solved multi-objective optimization problem MOEA/D, it may still lose some excellent individuals when it updates the sub-problems; besides, it may take substantial computing resources. The cost can lead to a slowdown of the convergence speed and the distributions of non-inferior solution is not uniform.

2. Related Concept

2.1 Multi-Objective Optimization Problem (MOPs)

Definition 1(Multi-Objective Optimization Problem**)**

The multi-objective optimization problem is generally composed of a group of objective function and some equation or inequality constraints, maximum and minimization problem can be converted to each other, so we only give the definition of minimization problem [5]

$$\begin{gathered} \min_{x\in\Omega} F(x)=(f_1(x), f_2(x), \ldots, f_m(x))^T \\ s.t.\ g_i(x)\geqslant 0, i=1,2,\ldots,k \\ h_j(x)=0, j=1,2,\ldots,l \end{gathered} \tag{2.1}$$

Where $x=(x_1, x_2, \ldots, x_n)^T$ is the decision (variable) space, $F:\Omega\rightarrow R^n$ consists of *n* real-valued objective functions and R^n is called the objective space. $g_i(x)$ denotes the inequality constraints and $h_j(x)$ denotes the equality constraints. They determine the feasible range of decision space.

2.2 Pareto Solution and Pareto Optimal Set

Definition 2(Pareto solution**)**

Let $u, v\in R^n$, u is said to dominate v if $u_i\leqslant v_i$ for every $i\in\{1,2,\ldots,m\}$ and $u_i<v_i$ for at least one index $i\in\{1,2,\ldots,m\}$. A point x^* is Pareto optimal to (2.1) if there is no point $x\in\Omega$ so that $f_i(x)$ dominates $f_i(x^*)$. We notice that the Pareto optimal to (2.1) is usually not the only one solution, and this Pareto optimal compose a Pareto optimal set.

Definition 3 (Pareto Optimal Set)

We assume P^* is a sub-set of decision space Ω

$$P^* = \{x^* \in \Omega | \neg \exists x \in \Omega, f_i(x) \leqslant f_i(x^*), i = 1,2,\ldots,m\} \qquad (2.2)$$

If P^* meets constraint (2.2), then P^* is the Pareto optimal set to (2.1).

2.3 Tchebycheff Approach

Definition 4(Tchebycheff approach**)**

The MOEA/D algorithm decomposes a multi-objective optimization problem into a number of scalar optimization sub-problems and optimizes them simultaneously. Several decomposition methods for constructing aggregation functions can be found and the most popular among them is Tchebycheff approach. This work has also carried out research and improvement on the MOEA/D algorithm by using Tchebycheff approach. is a weight vector, and meets for every problems . Then, the optimal solution to the following scalar optimization problem is shown as below:

$$\begin{gathered} minimize\ g^{te}(x, \lambda, z^*) = \max_{1 \leqslant i \leqslant m} \{\lambda_i | f_i(x) - Z_i^*|\} \\ x \in \Omega \end{gathered} \qquad (2.3)$$

Where the optimal solution is the reference point, i.e., $Z_i^* = max\{f_i(x) | x \in \Omega\}$ for each $i \in \{1,\ldots,m\}$. As to each optimal point x^* , there is a weight vector λ so that x^* is the optimal solution of (2.3) and each optimal solution of (2.3) is a Pareto optimal solution of (2.1); therefore, one is able to obtain different Pareto optimal solutions by altering the weight vector.

2.4 ε -dominance

Definition 5($\varepsilon - dominant$ **)**

In 2002, Laumanns *etc*-proposed the concept of ε -dominance, as a new dominance strategy. Let $f \in R^m$, individual Q_1, Q_2 and $\varepsilon > 0$.

$$\begin{gathered} f_j^{Q_1} - \varepsilon \leqslant f_j^{Q_2}, \forall j \in \{1,2,\ldots,m\} \\ f_i^{Q_1} - \varepsilon \leqslant f_i^{Q_2}, \exists i \in \{1,2,\ldots,m\} \end{gathered} \qquad (2.4)$$

then Q_1 is said to dominate Q_2 under the conception of $\varepsilon - dominant$ if and only if they meet the constraint (2.4).

2.5 Self Adaptive ε -dominance

Although $\varepsilon - dominant$ achieves the preset convergence and diversity, it has the following disadvantages: (2.1) the slack variable must be specified by the user; (2.2) in the late process of evolution, due to the sharp increasing number of non-inferior solutions, especially in high-Victoria objective optimization problem, the archive population size must be large enough [6,7]. This paper applies a Self Adaptive ε dominant MOEA/D algorithm. It assumes that NP donates the number of Archive populations. Set a smaller ε in the early evolution. When $NP \geqslant K$ (K is a threshold as assumed by the user) with the algorithm continuing, the approach does the following iteratively until $NP \leqslant t * K (t \in (0,1])$ ： add $\Delta\varepsilon$ to ε and calculate NP[8,9]. It can ensure the size of archive population should be maintained at an appropriate scale and no priori experience for setting slack variables be needed.

2.6 Non-uniform Domination

The fitness evaluation step evaluates each individual from the population. As to each individual, it computes the fitness value iteratively. The number of iterations is determined according to γ, which denotes the number of individuals which dominate the evaluated one. If the number γ is less than 5, the number of evaluation iterations is set as 5; otherwise it is set to be the value of γ.

3. The MOEA/D Algorithm based on the Self Adaptive ε-dominant

In order to overcome the shortcoming of MOEA/D multi-objective optimization algorithm which may lose partial of the excellent individuals when updating sub-problems and the slowdown of the convergence speed. In this paper, a multi-objective evolutionary optimization algorithm based on adaptive epsilon-domination is proposed. We call it $\varepsilon - AMOEA/D$,which accepts the non-inferior solution by self adaptive $\varepsilon - dominant$ and changes the value of ε dynamically as to different problems.

In order to improve the convergence speed, as to each generation, the algorithm uses archive population to update each sub-problem at certain probability to ensure every sub-problem can get the best solution so far and avoid degenerate phenomenon and the sub-problem arises premature convergence; besides, this is also an effective method to ensure the diversity of evolutionary population and make full advantage of the limited computing resources.

3.1 Updating Archive Population based on Self Adaptive ε -dominated

In this paper, Archive population is introduced into MOEA/D to improve the performance of the algorithm. Archive population remains all the non-dominant individuals. When new individuals c joins in the Archive population, we take the algorithm as follows:

Algorithm 1: self Adaptive $\varepsilon - dominant$ Updating Archive population
Input: archive population, individual c
Output: archive population

Step1 based on the $\varepsilon - dominant$ mechanism, if there is individual a in the archive population and a dominate individual c ,then give up c and end the algorithm.

Step2 based on the $\varepsilon - dominant$ mechanism, check the individual a in the archive population one by one. When individual c dominates a , delete the individual a from the archive population; when all individuals dominated by c are removed, add c to the archive population and go to Step5.

Step3 when there is individual a in the archive population and a does not dominate c , at the same time, c does not dominate a , in other words, they are in the same box, consider the original Pareto dominance, preferentially accept the individual which cannot be dominated by the other; otherwise prefer the individual closer to the corner. If c is accepted, go to Step5; otherwise, the algorithm ends.

Step 4 when there doesn't exist individuals a in the archive population and a dominate c , then add c to the archive population and go to Step5.

Step 5 we assume that NP denotes the number of Archive populations. When $NP \geqslant K$ (K is a threshold which is assumed by the user), continue the algorithm and we do as follows iteratively until $NP \leqslant t * K \left(t \in (0,1]\right)$ ： add $\Delta \varepsilon$ to ε and calculate NP . The algorithm ends.

3.2 Update Non-uniform Domination

This paper introduces a non-uniform domination strategy to the MOEA/D algorithm to improve the performance of the algorithm. In every fitness evaluation, it updates the number of evaluation iterations according to the number γ.

Algorithm 2: update non-uniform domination
Input: evolutionary population EP
Output: domination array S[]

Step1 initialize the domination array "S[]" with 0;
Step2 compare each individual i (0 <i <N) and the population of other individuals j (0 <j <N). If the individual i dominants the individual j, the S [i] ++;
Step3 End the algorithm.

3.3 Processes of MOEA/D Algorithm based on the Self Adaptive ε-dominance

This paper attempts to introduce the elite strategy of single-objective optimization into MOEA/D so as to speed up the convergence rate of the algorithm. MOEA/D algorithm decomposes a multi-objective optimization problem into a number of scalar optimization sub-problems and optimizes them simultaneously at the same time. Each decomposed sub-problem has independently and uniquely identified the weight vector $\lambda=\{\lambda_1,\lambda_2,\ldots,\lambda_m\}$, and every individual in the archive population also records its corresponding sub-problem so as to update individual of evolutionary population based on its corresponding sub-problem and avoid premature phenomenon by using the archive population upon each generation evolution. Why we update the evolution population at a certain probability is to avoid the degradation phenomenon.

Algorithm 3: MOEA/D algorithm based on the adaptive ε-dominance
Input: evolutionary population EP, $X=(X_1,X_2,\ldots,X_i,\ldots,X_N)$, where N donates the size of evolutionary population and individual X_i donates the solution of the $i-th$ sub-problem. The archive population, $Y=(Y_1,\ldots Y_k,\ldots,Y_t)$, where NP donates the size of Archive population and individual Y_k donates the solution of the $k-th$ sub-problem.γ denotes the replacing probability ; neighbor size T ; Max_Evalates donates maximum number of evaluations.
Output: the archive population

Step1 generate the initial population EP randomly.
Step2 use Algorithm 1 to add individuals in the population EP to the archive population.
Step3 Initial weight vector $\lambda=\{\lambda^1,\ldots,\lambda^N\}$, according to the Euclidean distance, calculate the neighbors $B(i)=\{i_1,\ldots,i_T\}$ of each weight vector, where $\lambda^{i_1},\ldots,\lambda^{i_T}$ denotes T neighbors of the weight vector λ^i.
Step4 initialize the reference point $z=\{z^1,\ldots,z^m\}$.
Step5 update: as to all $i=1,\ldots,N$, do the following steps:
Step5.1 update the non-uniform domination;
Step5.2 increase the number of Evolution ： as to each individual $i=1,\ldots,N$, execute S[i] times steps:
Step5.2.1 select two indices k and l randomly from $B(i)$, and then generate a new solution y from x_k and x_l by using genetic operators.
Step5.2.2 update the reference point z via solution y ;
Step5.2.3 update the neighbor sub-problem solution: as to each $j\in B(i)$,use Tchebycheff approach to update its neighbor sub-problem solution;
Step5.3 via Algorithm 1, add the individual y to the archive population.
Step6 use Archive population to update evolutionary population EP. Do the following steps:
Step6.1 set $s=0$;
Step6.2 use Tchebycheff approach to calculate the the fitness of individual Y_l which is the $s-th$ individual in the archive population and the fitness of individual X_l which is the $l-th$ sub-problem in EP population , if the fitness of individual

Y_l is better than the fitness of individual X_l and the random number P which randomly rated in[0,1] is bigger than r, then use Y_l to replace X_l .

Step6.3 $s{+}{+}$, if $s \geqslant NP$, then go to step4; else go to step6.2;

Step6.4 one generation Archive population have finished.

Step7 if the number of evaluations meets $\text{evaluates} < Max\_\text{Evaluates}$, go to Step5; otherwise output the solution set in Archive population.

4. Experiments

4.1 Test Functions and Test Parameter Settings

In order to demonstrate the performance of MOEA/D algorithm based self-adaptive ε-domination, we compare $\varepsilon - AMOEA/D$ with MOEA/D and NSGAII algorithm. We test 12 standard test functions, including five functions of ZDTx and seven functions of DTLZx. All algorithms use the following settings: the size of evolution population is 100, the maximum number of evaluations is 25000; the weights vector takes the uniform design method; the cross of GA operator's is 0.5, the polynomial mutation is the reciprocal of the number of independent variables of variability; the neighbor size T = 20; the maximal size of the archive population is 400, the number of final output of the non-inferior solutions is about 300; ε is initialized to be 0.0005, the increase of ε is $\Delta\varepsilon$ and $\Delta\varepsilon = 0.00005$ and the replacing probability $r = 0.4$.

4.2 Evaluation Index

The vector hydrophone should be calibrated before measurement; the frequency response is shown in Fig. 5. In order to discriminate performance of different algorithms in solving the multi-objective optimization problem, some researchers also carried on the relevant researches and proposed many effective methods. Generally, the distribution of non-inferior solution and the convergence are two indicators [10,11,12]. In this paper, we estimate the approximation ratio of true Pareto front and the Pareto front as found by the proposed algorithm. Assume Q be the non-inferior solution set which is got by our algorithm, P* is the real Pareto front approximation set of problem .

$$IGD = \frac{\sqrt{\sum_{i=1}^{N} d_i^2}}{N} \tag{4.1}$$

Where d_i denotes the minimal Euclidean distance of the $i-th$ individual in Q and the individuals in P *. When IGD is smaller, it shows the Pareto front Q which is got by our algorithm is closer to the real Pareto front and the speed of convergence is faster.

The distribution of non-inferior solution is discriminated by the distributivity evaluation method proposed by K. Deb and it is used to evaluate the distribution width and the uniformity degree of the non-inferior solution set Q:

$$\Delta = \left(d_f + d_l + \sum_{i=1}^{|Q|-1} |d_i - \bar{d}|\right) / \left(d_f + d_l + (|Q|-1)\bar{d}\right) \tag{4.2}$$

Where d_i donates the Euclidean distance of two continuous non-inferior solution in Q. $\bar{d}$ is the average of all d_i. Both d_f and d_l donate the Euclidean distance between the boundary point in true solution set P* and the boundary point in non-inferior solution set Q. If Δ is smaller, it shows the diversity of algorithm is better.

4.3 Experimental Results and Discussion

All algorithms in the test functions run 30 times independently. The Runtime environment is a PC, clocked at 2.3GHZ, 2GB memory. Now, we give the statistical results of

IGD and Δ of $\varepsilon-AMOEA/D$, MOEA/D and NSGAII, in Table 1 and Table 2 (best value in bolding).

Algorithm	ZDT1	ZDT2	ZDT3	ZDT4	ZDT6	DTLZ1
NSGAII	2.39E-04	3.18E-04	3.68E-04	4.59E-04	7.10E-04	6.35E-04
	1.71E-05	1.28E-04	6.49E-05	5.11E-04	1.66E-04	4.61E-05
MOEA/D	2.19E-04	4.62E-04	5.95E-04	3.07E-04	2.40E-04	3.81E-04
	1.02E-04	8.29E-04	1.43E-03	7.41E-05	3.90E-05	1.79E-06
ε-AMOEA/D	**5.48E-05**	**7.81E-05**	**1.39E-05**	**5.33E-05**	**6.22E-05**	**3.06E-04**
	1.01E-05	7.94E-06	1.18E-04	6.11E-05	3.38E-05	1.69E-05
Algorithm	DTLZ2	DTLZ3	DTLZ4	DTLZ5	DTLZ6	DTLZ7
NSGAII	8.05E-04	5.08E-02	1.25E-03	2.87E-05	8.71E-03	**3.76E-03**
	4.94E-05	3.69E-02	1.12E-04	5.17E-05	6.85E-04	4.24E-03
MOEA/D	4.13E-04	6.82E-04	2.99E-03	**2.25E-05**	9.80E-04	6.04E-03
	1.64E-06	1.62E-05	3.61E-03	7.19E-08	2.66E-04	5.47E-03
ε-AMOEA/D	**4.07E-04**	**6.63E-04**	**1.24E-03**	9.29E-05	**8.56E-04**	7.83E-03
	1.49E-05	1.75E-05	8.07E-05	1.66E-06	2.16E-04	2.95E-04

Table1: IGD. Mean (first line) and Standard Deviation (Second Line)

Algorithm	ZDT1	ZDT2	ZDT3	ZDT4	ZDT6	DTLZ1
NSGAII	4.85E-01	5.35E-01	9.48E-01	8.43E-01	7.06E-01	8.74E-01
	6.06E-02	3.22E-01	8.93E-02	4.2E-01	3.26E-01	2.01E-01
MOEA/D	**3.73E-01**	**4.00E-01**	1.00E+00	**4.66E-01**	**2.08E-01**	8.26E-01
	1.62E-01	4.50E-01	1.13E-02	1.24E-01	2.76E-02	1.31E-02
ε-AMOEA/D	9.26E-01	9.26E-01	**9.43E-01**	9.33E-01	1.01E-00	**6.93E-01**
	4.39E-02	3.86E-2	2.27E-02	6.37E-02	3.87E-02	1.64E-02
Algorithm	DTLZ2	DTLZ3	DTLZ4	DTLZ5	DTLZ6	DTLZ7
NSGAII	7.05E-01	1.05E-00	6.66E-01	**5.87E-01**	8.27E-01	**7.85E-01**
	6.24E-02	1.12E-01	6.62E-02	3.26E-02	5.72E-02	6.65E-02
MOEA/D	8.38E-01	8.55E-01	8.84E-01	1.14E+00	1.06E+00	1.23E+00
	1.16E-02	2.81E-02	6.37E-02	2.79E-03	2.07E-02	5.41E-02
ε-AMOEA/D	**5.88E-01**	**5.76E-01**	**5.85E-01**	1.07E-00	**6.52E-01**	9.05E-01
	3.15 E-02	1.96E-02	6.88E-02	2.97E-02	5.62E-02	6.37E-02

Table2 : Mean (first line) and Standard Deviation (Second Line)

Via statistical results of IGD from Table1, $\varepsilon-AMOEA/D$ has much faster convergence than the MOEA/D method and the NSGAII method. It converges faster on the vast majority of the series ZDT and DTLZ test functions, and its variance is also very small, indicating that the sought non-inferior solutions can be very close to the real Pareto front and showing that the new algorithm has better robustness at the same time. On DTLZ2, DTLZ3 test function, the convergence performance is unsatisfactory. It is only a little bit worse when compared with the original MOEA/D, and its convergence is still better than NSGAII. The convergence performance of $\varepsilon-AMOEA/D$ is mainly due to the Archive population updated by self adaptive domination. The archive population retains the vast majority of the best individual so far, then introduces the elite strategy with a random strategy, both of which take advantage of the best individual to guide the searching of evolution and avoid the premature convergence. These measures can accelerate the convergence of the algorithm to a certain extent.

Via statistical results of the distribution from Table2, $\varepsilon-AMOEA/D$ have much better distribution than MOEA/D; and NSGAII, the non-inferior solutions sought by $\varepsilon-AMOEA/D$ is more uniform. It is outstanding on the ZDT3 test and the six test functions of DTLZx series, and the variance is also smaller, indicating the effectiveness of the algorithm. The performance of distribution on ZDT1, ZDT2, ZDT4 and ZDT6 is not very good mainly because the original MOEA/D uses the uniform design weight which is very suitable for solving multi-objective problem whose real Pareto front are ultra-spherical or approximate hypersphere. Though $\varepsilon-AMOEA/D$ uses uniform design weight method, it utilizes self adaptive dominant to update the Archive population; however, the restrictions of self adaptive dominance still exist. It will lose some of the almost horizontal or near-vertical Pareto front. At the same time, although the elitist strategy updates evolutionary population at certain probability, it can also cause the algorithm into a local convergence in solving some multi-objective problem and impacting the distribution of non-inferior solutions. In the vast majority of test functions, especially as to the high-dimensional multi-objective optimization

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problems, $\varepsilon - AMOEA/D$ is better than MOEA/D and NSGAII, which demonstrates the effectiveness of the self adaptive dominant mechanism for the maintenance of the distribution of non-inferior solutions.

The computational complexity of NSGA-II, MOEA/D and $\varepsilon - AMOEA/D$ is the O(rN^2) (r is the number of objectives); but $\varepsilon - AMOEA/D$ costs a lot time in updating the archive population based on self adaptive ε -dominant. Form the statistical results of IGD and Δ, $\varepsilon - AMOEA/D$ has more advantages out of calculation accuracy.

In order to show the degree of convergence of the algorithm and the distribution of non-dominated solutions more visually, we draw non-inferior solutions based on the obtained approximate Pareto front, shown in Fig. 1 to 12.

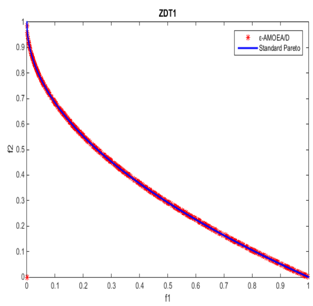


Figure1: ZDT1

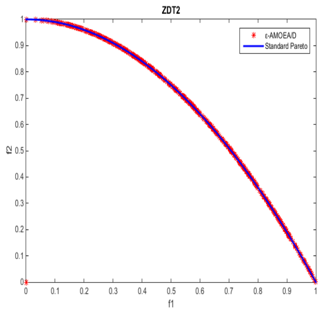


Figure 2:ZDT2

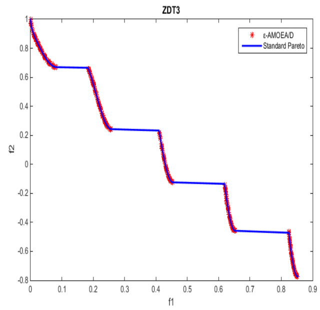


Figure3: ZDT3

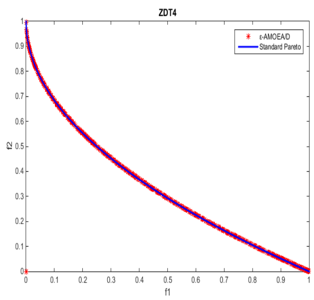


Figure 4: ZDT4

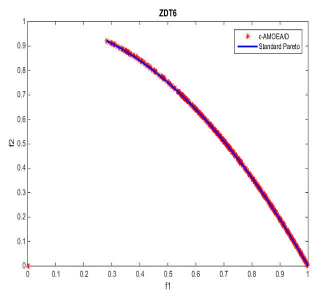


Figure 5: ZDT6

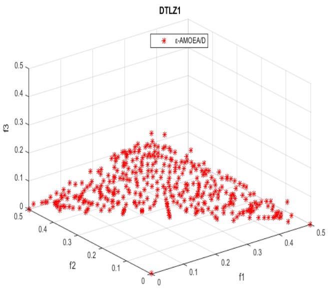


Figure 6: DTLZ1

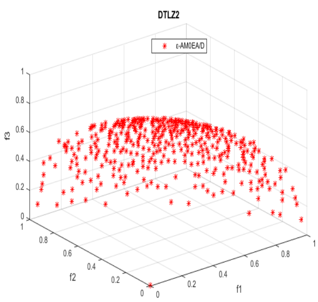


Figure 7: DTLZ2

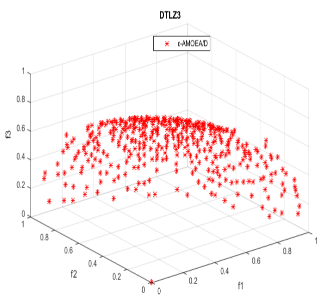


Figure 8: DTLZ3

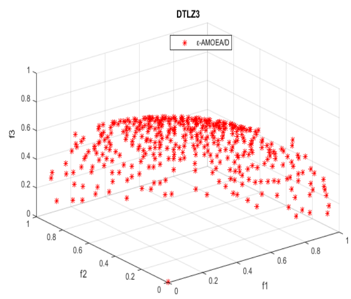


Figure 9: DTLZ4

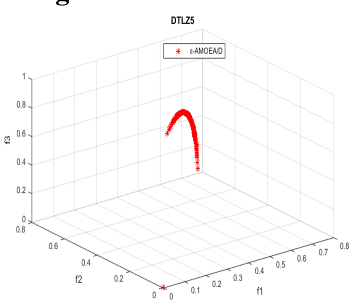


Figure 10: DTLZ5

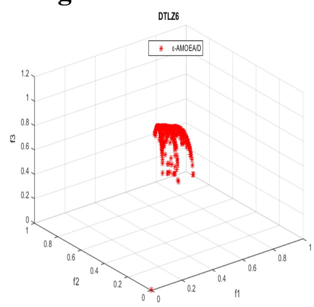


Figure 11: DTLZ6

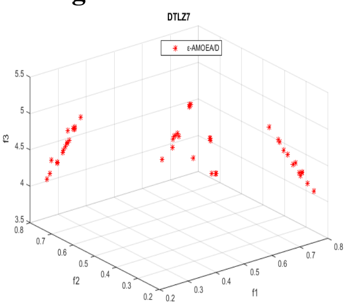
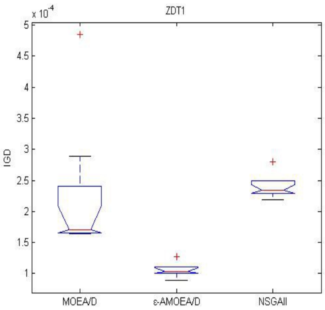
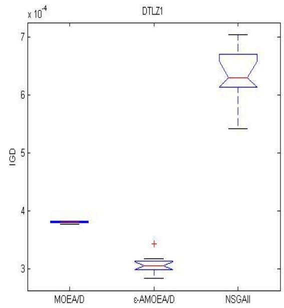


Figure 5: DTLZ7

From the above figures of approximate Pareto front sought by $\varepsilon - AMOEA/D$, the distribution of the Pareto front solving 12 test functions is more uniform and very approximation of the true Pareto front, indicating the effectiveness and superiority of the $\varepsilon - AMOEA/D$ algorithm for multi-objective optimization problem. This paper also used boxplot which is the statistical analysis method of the economics to describe the statistical results of each algorithm. Due to limited space, we only give the boxplot , as shown in Fig. 13 to 15 (where SP represents the value of Δ).





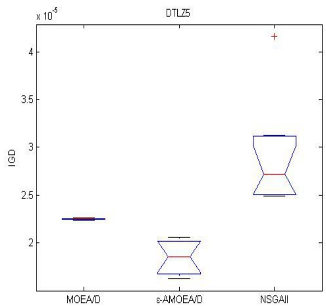


Figure 13 : Boxplot of ZDT1 **Figure 14:** Boxplot of DTLZ1 **Figure 15:** Boxplot of ZDT3

Drawn from the above boxplot, the non-inferior solutions sought by $\varepsilon - AMOEA/D$ is relatively more uniform than MOEA/D and NSGAII and the robustness of $\varepsilon - AMOEA/D$ is also stronger, indicating its excellent performance when it maintains the distribution of non-inferior solutions.

5. Conclusion

Note the shortcoming that MOEA/D would lose partial of the excellent individuals when it updates sub-problems and the slowdown of convergence speed, we proposed a multi-objective optimization algorithm based on adaptive epsilon-domination in this paper. The algorithm uses the archive population which is updated by adaptive epsilon-domination to retain non-inferior solutions, reduce the losing of excellent individuals in the evolution process and keep the archive population to maintain a certain size, ensure that convergence speed and the uniformity of the distribution of non-inferior solutions. At the same time, in order to speed up the convergence, after every generation, we use the archive population to update each sub-problems of evolution population at certain probability. Through experimental simulation, we tested the new algorithm, NSGAII and MOEA/D on 12 standard test functions of ZDT and DTLZ series. The experimental results show that on the majority of test functions with the same conditions, the convergence of new algorithm is faster than the MOEA/D algorithm and NSGAII algorithm and the new algorithm is more uniform than the two other algorithms. The experiment results support the effectiveness of the new algorithm for multi-objective optimization. In the future, we are going to apply the new algorithm to real applications.

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