PROCEEDINGS OF SCIENCE

Progress in off-shell amplitudes

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Off-shell amplitudes allow for off-shell external partons in a gauge-invariant manner. They are relevant for factorization prescriptions for hadron scattering involving parton density function that provide momentum components to the partons that are transversal to the direction of the hadrons. This write-up reports on recent developments and applications.

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1. Introduction

The complexity of calculations involving quantum chromodynamics (QCD) for scattering processes at collider experiments like at the Large Hadron Collider is tamed to a large degree with the help of factorization prescriptions. Contributions to cross sections are factorized according to the scales involved, and/or according to universality and accessibility via perturbation theory. Some factorization prescriptions are proven, which means that perturbation theory and the treatment of possible singularities, along with the occurrence of large logarithms of ratios of scales, can be dealt with in a systematic manner. Many, however, are rather of a heuristic nature.

One of the factors in a factorized calculation is the partonic matrix element, which is the absolute value of the square of the partonic scattering amplitude. Thanks to the Lagrangian of QCD and the Lehmann Symanzik Zimmermann reduction formula, on-shell partonic amplitudes are well defined gauge invariant objects, even though the partonic states are not physical. The non-physical scattering amplitudes are embedded into physical cross sections by the very factorization. Scattering amplitudes involving any number of off-shell external gluons can also be defined in a rigorous manner, as shown in [1]. Such amplitudes are relevant in factorization prescriptions requiring off-shell initial-state partons, like high energy factorization [2, 3]. The off-shellness is caused by momentum components transversal to the direction of the colliding hadrons, and we will refer to it as k_T-factorization. This factorization requires partonic scattering amplitudes with off-shell initial state partons. Systematic formulations of their calculation at tree-level have been established [4, 5, 6, 7, 8].

2. Construction and calculation of off-shell amplitudes

The effective action approach [4, 5, 6] was devised to access the high-energy behavior of scattering amplitudes. It is a tool to systematically extract the leading contributions in the total energy. The lowest order term in the expansion in this variable is the tree-level amplitude required in k_T-factorization. The method of [7, 8] leads to the same tree-level amplitude, but only involves simple Feynman rules that allow for well-known efficient evaluation techniques. The proposal in [7] to arrive at gauge invariant amplitudes with off-shell gluons is to embed the desired scattering process into a process in which each off-shell gluons is replaced by an auxiliary on-shell quark-anti-quark pair. Let the desired momentum of the off-shell gluon be $k = xp + k_T$, where p is the hadron momentum. In order to ensure that the sum of the momenta $p_A, p_{A'}$ of the auxiliary quark-anti-quark pair is equal to k, it was proposed to decompose the transversal momentum k_T of the gluon into polarization vectors

$$k_T^\mu = -\frac{\kappa}{2}\,\varepsilon^\mu - \frac{\kappa^*}{2}\,\varepsilon^{*\mu} \quad , \quad \varepsilon^\mu = \frac{\langle p|\gamma^\mu|q]}{[pq]} \quad , \quad \kappa = \frac{\langle q|\not{k}|p]}{\langle qp\rangle} \tag{2.1}$$

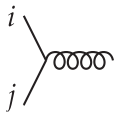
where q must be a light-like momentum with $p\cdot q \neq 0$ and $q\cdot k_T = 0$, and choose

$$p_A^\mu = \Lambda p^\mu - \frac{\kappa^*}{2}\,\varepsilon^{*\mu} \quad , \quad p_{A'}^\mu = -(\Lambda - x)p^\mu - \frac{\kappa}{2}\,\varepsilon^\mu \ , \tag{2.2}$$

so that

$$p_A^2 = p_{A'}^2 = 0 \quad \text{and} \quad p_A^\mu + p_{A'}^\mu = xp^\mu + k_T^\mu \ , \tag{2.3}$$

for any value of the auxiliary parameter Λ. In the limit of large Λ the longitudinal component Λp becomes dominant in the (anti-)quark momentum, and this limit corresponds to the high-energy limit. This limit was analyzed in [7] for tree-level amplitudes, and it was observed that the auxiliary quark-anti-quark pair satisfies eikonal Feynman rules:



$$\mu, b \;=\; -\mathrm{i}\, p^\mu\, T^b_{i,j} \qquad , \qquad \xrightarrow{\quad K \quad} \;=\; \frac{\mathrm{i}}{p\cdot K} \quad . \tag{2.4}$$

Alternatively, given an exact tree-level expression in terms of particle momenta for an on-shell matrix element involving a $q\bar{q}$-pair, the momenta (2.2) may be assigned and Λ may be taken to infinity, after multiplying the expression with $|k_T^2|/\Lambda^2$, to obtain the expression for the matrix element with an off-shell gluon instead of the $q\bar{q}$-pair. For example, taking the expression for $qg \to qgg$, *eg.* from [10], and applying this procedure will lead to the expression for $g^*g \to gg$, *eg.* from [9].

Besides corresponding to the high-energy limit, taking $\Lambda \to \infty$ is also supported by the fact that it makes the unnatural imaginary momentum components in (2.2) irrelevant. One may, however, also construct a momentum decomposition without imaginary momentum components, namely

$$p_A^\mu = \Lambda p^\mu + \alpha q^\mu + \beta k_T^\mu \quad , \quad p_{A'}^\mu = -(\Lambda - x)p^\mu - \alpha q^\mu + (1-\beta)k_T^\mu \ , \tag{2.5}$$

where

$$\alpha = \frac{-\beta^2 k_T^2}{\Lambda(p+q)^2} \quad , \quad \beta = \frac{1}{1+\sqrt{1-x/\Lambda}} \ . \tag{2.6}$$

Also defined like this, the momenta $p_A, p_{A'}$ satisfy (2.3) for any value of Λ. One advantage of the previous decomposition is that with (2.2), the (anti-)quark spinors are directly given by

$$|p_A] = \sqrt{\Lambda}\,|p] \quad , \quad |p_{A'}\rangle = \sqrt{\Lambda - x}\,|p\rangle \ , \tag{2.7}$$

while with (2.5) this relation only holds approximately for large Λ.

2.1 On-shell recursion

One of the remarkable properties of tree-level off-shell amplitudes is that they can be calculated via "on-shell" recursion [11]. It just requires the generalization of the concept of "on-shellness" for off-shell gluons to the condition that besides its momentum k, a light-like momentum, or *direction*, p is associated with it, satisfying the relation $p\cdot k = 0$. This, of course, is just the longitudinal momentum component.

On-shell recursion is based on the fact a tree-level amplitude has a pole whenever the sum of a set of external momenta becomes light-like, with a residue that is the product of two on-shell amplitudes. Schematically:

$$\mathscr{A}(k_1,\ldots,k_n) \overset{K_{1,l}^2 \to 0}{\to} \frac{\mathscr{A}(k_1,\ldots,k_l,-K_{1,l})\mathscr{A}(K_{1,l},k_{l+1},\ldots,k_n)}{K_{1,l}^2} \quad , \quad K_{1,l} = \sum_{i=1}^{l} k_i \ . \tag{2.8}$$

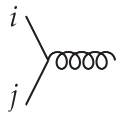
In terms of Feynman graphs, when the internal momentum $K_{1,l}$ becomes light-like, the internal line with that momentum becomes a cut on-shell gluon. For "off-shell" gluons, the condition $K_{1,l}^2 = 0$ is replaced by $p \cdot K_{1,l} = 0$. It corresponds to a pole for an internal eikonal line associated with direction p, and it corresponds to the generalized "on-shell" condition for an external gluon.

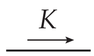
On-shell recursion leads to compact expressions for so-called dual amplitudes, which are the helicity-and momentum-dependent coefficients in a color-decomposition of the amplitude. In the definition an off-shell amplitude, the off-shell gluon is replaced by a $q\bar{q}$-pair. Regarding the color-content this only makes sense if the trace of the amplitude with respect of the color indices of the $q\bar{q}$-pair vanishes. In [12] it is argued that this is indeed the case, so that the color decomposition is independent of whether gluons are on-shell or off-shell.

Regarding helicity, the analytical calculation of off-shell amplitudes reveals that a dual amplitude with an off-shell gluon is a sum of terms, where the ones leading in the limit of $|k_T^2| \to 0$ correspond to the two possible helicity amplitudes if the off-shell gluon were on-shell. Only at the level of the cross section, this coherent sum of amplitudes becomes an incoherent sum of squared helicity amplitudes in the on-shell limit. The remnant angular integral over ϕ in $\vec{k}_T = |k_T|(cos\phi, sin\phi)$ eliminates interference terms.

2.2 Off-shell quarks

In [8] it was proposed to define amplitudes with off-shell (anti-)quarks by replacing them by (anti-)quark-photon pairs, assigning the momenta (2.2), and taking the limit of $\Lambda \to \infty$, and it was shown that it leads to the same tree-level amplitudes as the approach of [5]. The auxiliary photon determines the point in a Feynman graph were the eikonal (anti-)quark line turns into a normal (anti-)quark line. It eventually terminates the spinor line and indeed appears as a spinor in the Feynman rules. It has to be noted that the eikonal lines can be replaced by spinor lines in an equivalent formulation. The Feynman rules (2.4) are then replaced with



$\mu, b \;=\; -\mathrm{i}\,\gamma^\mu\, T^b_{i,j}$,


$= \dfrac{\mathrm{i}\not{p}}{p \cdot K}$. (2.9)

Numerically this formulation is slightly more costly, but it is conceptually more natural, since the auxiliary photon is represented by a polarization vector, and the auxiliary (anti-)quark by a spinor. This spinor inherits the opposite helicity of the on-shell end of the spinor line.

In [13] it is shown how to calculate amplitudes with off-shell quarks and/or anti-quarks via on-shell recursion. One observation worth highlighting was that the recursion may involve intermediate amplitudes with off-shell gluons, also when the original amplitude to be calculated does not involve any. Secondly, it appears that there are non-vanishing amplitudes with off-shell (anti-)quarks that would lead to vanishing amplitudes if an (anti-)quark would be on-shell, with any of the possible helicities. These amplitudes are, however, sub-leading in the $|k_T^2|$ of that off-shell (anti-)quark and do vanish smoothly in the on-shell limit. Thirdly, it appears that amplitudes with an off-shell $q\bar{q}$-pair are different for different helicities associated with the off-shell $q\bar{q}$-pair. An off-shell (anti-)quark inherits the opposite helicity of the on-shell (anti-)quark it is associated with for the amplitude to be non-vanishing. If both ends of the spinor line correspond to off-shell (anti-)quarks, both combinations $+-$ and $-+$ can be assigned, and the amplitudes are not equivalent. Both need to be included in the calculation of the cross section.

3. Applications

The on-shell recursion has been implemented in the program AMP4HEF[1]. It numerically evaluates color-ordered helicity amplitudes and matrix elements summed over color and helicity for the processes

$$\begin{array}{lll} \emptyset \to g^* g^* + 5g & \emptyset \to \bar{q} q^* + 3g & \emptyset \to \bar{q}^* q^* + 2g \\ & \emptyset \to \bar{q}^* q + 3g & \emptyset \to g^* \bar{q}^* + q g \\ & \emptyset \to g^* + \bar{q} q + 2g & \emptyset \to q^* g^* + g \bar{q} \end{array} \tag{3.1}$$

plus those with fewer on-shell gluons and those with any off-shell partons replaced by on-shell partons. All amplitudes with up to 5 partons are implemented as analytic expressions, while amplitudes with more partons are evaluated via numerical recursion. The program is written in Fortran 2003, and is straightforward to use both in Fortran and in C++.

For any process within the Standard Model, with any type or number of initial-state partons off-shell, the program AVHLIB[2] can be used. It uses numerical Dyson-Schwinger recursion instead of on-shell recursion. It contains a complete Monte Carlo framework to produce event files for fully differential analyses at the parton level, or to feed to a parton-shower Monte Carlo program. This program has been used for the first calculation within k_T-factorization with up to 4 final-state particles, namely for the process $pp \to c\bar{c}c\bar{c}$ at LHC with massive final-state (anti-)c-quarks [14]. It was in the context of a comparison between double-parton scattering (DPS) and single-parton scattering (SPS), where both were approached with k_T-factorization. The DPS contributions was calculated with the simple factorized "pocket formula"

$$d\sigma^{\mathsf{DPS}}(pp \to c\bar{c}c\bar{c}X) = \frac{1}{2\sigma_{\mathsf{eff}}}\, d\sigma^{\mathsf{SPS}}(pp \to c\bar{c}X_1)\, d\sigma^{\mathsf{SPS}}(pp \to c\bar{c}X_2) \tag{3.2}$$

and with the experimentally confirmed value of 15mb for σ_{eff}. The calculation confirms the observation that DPS gives the major contribution to this process [15, 16, 17], as shown for example in Fig. 1.

More recently, AVHLIB was used for calculations regarding the production of 4 jets at LHC [18, 19]. All processes, except processes with 3 $q\bar{q}$-pairs, were included. The necessary k_T-dependent pdfs, named DLC2016v2, were obtained by applying the Kimber-Martin-Ryskin unintegration technique [20, 21] to the CTEQ10NLO pdfs [22] with NLO running coupling constant (more details can be found the Appendix A of [18]).

One of the advantages of k_T-factorization over collinear factorization is the fact that the 2 momenta, obtained by summing all final-state momenta into 2 groups, do not have to be back-to-back, thanks to a momentum imbalance induced by the transversal momenta of the initial-state partons. Consequently, it is possible to calculate distributions of observables constructed with such groupings already at leading order. In case of 4 jets, the following variable [23]

$$\Delta S = \arccos\left(\frac{\vec{p}_T(j_1^{\text{hard}}, j_2^{\text{hard}}) \cdot \vec{p}_T(j_1^{\text{soft}}, j_2^{\text{soft}})}{|\vec{p}_T(j_1^{\text{hard}}, j_2^{\text{hard}})| \cdot |\vec{p}_T(j_1^{\text{soft}}, j_2^{\text{soft}})|}\right) \tag{3.3}$$

[1] http://bitbucket.org/hameren/amp4hef

[2] http://bitbucket.org/hameren/avhlib

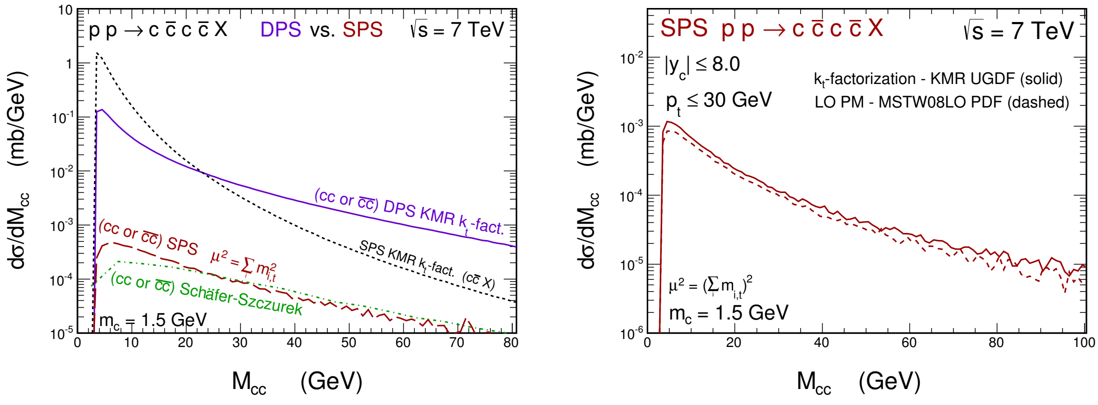


Figure 1: Invariant mass distribution for approximated SPS (Schäfer-Sczcurek), exact SPS within collinear factorization (long dashes on the l.h.s. and dashes on the r.h.s.), exact SPS within k_T-factorization (solid on the r.h.s.), and DPS (solid on the l.h.s.).

is such a candidate. In words, ΔS is the angle between the sum of the two hardest jets and the sum of the two softest jets. It turns out that the set-up of DLC2016v2 in combination with the off-shell matrix elements describes the shape of the available data for this variable remarkably well (Fig. 2). Also, it seems that DPS is not necessary to describe the data.

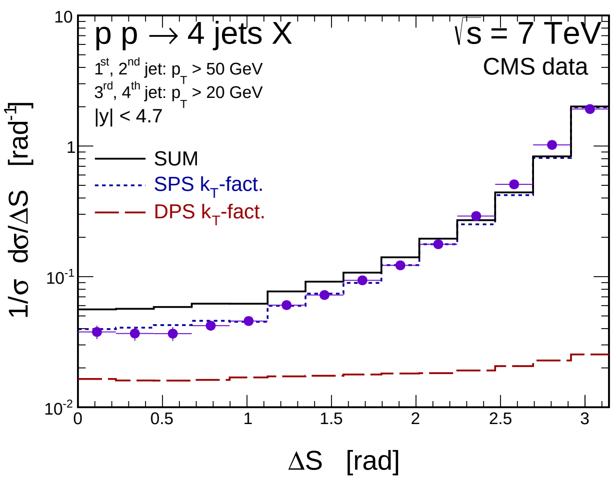


Figure 2: Normalized distribution in the ΔS variable of Eq.(3.3). The data points are from [23].

4. Summary

We reported on the development of off-shell amplitudes, programs for their explicit evaluation, and recent applications. The latter include calculations of process involving a total of up to 6 partons.

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