PROCEEDINGS OF SCIENCE

Electroweak corrections to top pair production near threshold

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I review the impact of electroweak corrections on the total production cross section of a non-relativistic top-antitop pair at a future electron-positron collider. I outline the implications for the measurement of the top quark mass, width, and Yukawa coupling.

Loops and Legs in Quantum Field Theory
24-29 April 2016
Leipzig, Germany

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PoS(LL2016)049

1. Introduction

The properties of the top quark can be measured with unprecedented precision at a future electron-positron collider, like the proposed International Linear Collider [1]. A key quantity in this respect is the total production cross section near the top-antitop threshold.

In order to match the projected experimental precision, it is mandatory to take into account the combined effects of bound-state interactions and higher-order perturbative corrections. These effects are incorporated systematically in the effective field theory framework of potential non-relativistic quantum chromodynamics (PNRQCD) [2]. Within PNRQCD, the cross section is expanded simultaneously in the small non-relativistic velocity v and the strong coupling constant α_s, adopting the power counting $\alpha_s \sim v \ll 1$. The exchange of Coulomb gluons yields contributions scaling with powers of $\alpha_s/v \sim 1$, which are resummed to all orders.

The PNRQCD Lagrangian at a given order can be derived systematically by matching to non-relativistic quantum chromodynamics (NRQCD) [3], which in turn is obtained by matching to the fundamental theory of QCD.[1] The top-antitop production cross section within PNRQCD has been computed to third order [4]; the power counting up to this order is given by

$$\sigma \sim v \sum_k \left(\frac{\alpha_s}{v}\right)^k \times \begin{cases} 1 & \text{LO} \\ \alpha_s, v & \text{NLO} \\ \alpha_s^2, \alpha_s v, v^2 & \text{N}^2\text{LO} \\ \alpha_s^3, \alpha_s^2 v, \alpha_s v^2, v^3 & \text{N}^3\text{LO} \end{cases} . \tag{1.1}$$

Corrections beyond QCD can be taken into account by generalising the aforementioned two-step matching procedure to the case where the fundamental theory is given by the full Standard Model. It is customary to adopt the power counting $\alpha \sim \alpha_s^2$ for the QED coupling constant α. For the Higgs sector, we will count powers of the Yukawa coupling to be of the same order as the strong coupling constant, i.e. $y_t \sim \alpha_s$. The value of the Higgs boson mass m_H lies between the hard scale given by the top quark mass m_t and its nonrelativistic momentum $m_t v$, which defines the soft scale. Since the numerical value is much closer to the former, we use the power counting $m_H \sim m_t$.

2. Classification of electroweak corrections

In the following we will consider different classes of effects beyond QCD. Apart from the electroweak corrections to the production all of them are discussed in more detail in [5].

2.1 Higgs effects

In a first step, we incorporate Higgs effects by adding

$$\mathscr{L}_{\text{Higgs}} = \frac{1}{2}(\partial_\mu H)^2 - \frac{1}{2}m_H^2 H^2 - \sqrt{\frac{\lambda}{2}} m_H H^3 - \frac{\lambda}{4}H^4 - \frac{y_t}{\sqrt{2}}\bar{t}tH \tag{2.1}$$

with $\lambda = \frac{\pi\alpha m_H^2}{2m_W^2 s_w^2}$ to the QCD Lagrangian. Corrections then arise from the exchange of a virtual Higgs boson between the top and the antitop (see figure 1).

[1] At electron-positron colliders, quark-antiquark pairs are mainly produced via a virtual photon or Z boson. Thus, the corresponding vertices should also be added to the QCD Lagrangian.

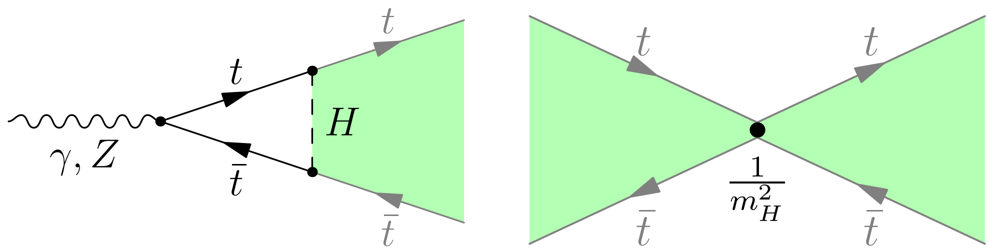

Figure 1: Example diagrams for Higgs corrections to $t\bar{t}$ production. Left: N^{2}LO correction to the production vertex. Right: N^{3}LO potential correction. Grey lines indicate nonrelativistic (anti-)top quarks near their mass shell. The shaded area represents the leading-order colour Coulomb interaction.

A Higgs boson with a hard momentum (of the order of the Higgs boson mass), can only be exchanged if the initially produced top-antitop pair is far off shell. Within the effective theory, this constitutes a correction to the production vertex of the nonrelativistic top-antitop pair. A single Higgs exchange contributes with a factor of y_t^2 and is thus a N^{2}LO correction. We also take into account the exchange of an additional gluon, which contributes at N^{3}LO. These corrections were calculated in [6, 7, 8, 9].

For a Higgs exchange between a top-antitop pair near mass shell, the Higgs momentum $\mathbf{q} \sim m_t v$ is much smaller than its mass and thus can be neglected inside the Higgs propagator. The Higgs exchange then corresponds to a local correction $\delta_H V = -y_t^2/(2m_t^2) \sim v^2/m_t^2$ to the interaction potential. The leading-order colour Coulomb potential is proportional to $\alpha_s/\mathbf{q}^2 \sim 1/(v m_t^2)$. Thus, the Higgs potential is suppressed by a factor v^3 with respect to the leading-order potential and first contributes at N^{3}LO.

2.2 QED Coulomb potential

In order to take into account QED corrections, we add

$$\mathscr{L}_{\text{QED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{l \in \text{leptons}} \overline{\psi}_l i \partial\!\!\!/ \psi_l - \sum_{f \in \text{fermions}} e_f \overline{\psi}_f A\!\!\!/ \psi_f \tag{2.2}$$

to the Lagrangian. The dominant correction is then given by the QED Coulomb potential $\alpha/\mathbf{q}^2$, which is a NLO effect.

2.3 Nonresonant production

Since the top quark is unstable, its decay into a $W^+ b$ pair has to be incorporated consistently into our framework. Within unstable particle effective theory [10, 11], the cross section is given as a sum of resonant top-antitop production and nonresonant production including top decays. At NLO it suffices to consider nonresonant production of the final states $tW^-\bar{b}$ and $\bar{t}W^+b$, which was computed in [12]. The diagrams contributing to the latter final state are shown in figure 2.

2.4 P-wave production

Due to the axialvector coupling of the Z boson the top-antitop pair can be produced in a P-wave state. P-wave production starts to contribute at N^{2}LO and has been found to be numerically small [13, 14, 15, 16].

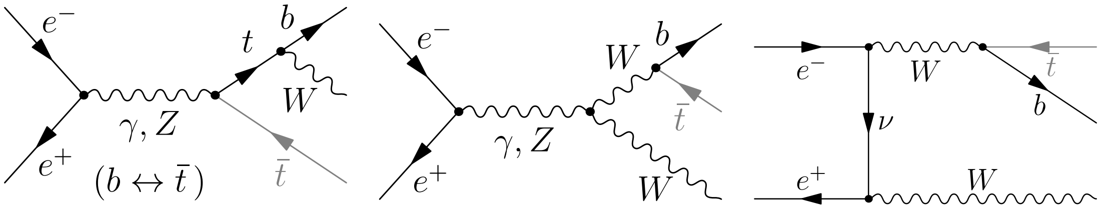

Figure 2: Non-resonant production of $\bar{t}W^+b$. Grey lines indicate on-shell antitop quarks.

2.5 Electroweak production corrections

At N^{2}LO, the full Standard Model Lagrangian induces further corrections to the resonant top-antitop production, which have been calculated in [6, 7, 17, 8]. Sample diagrams are shown in figure 3. To obtain consistent predictions, these have to be combined with the N^{2}LO nonresonant production, where only partial results are known [18, 19, 20].

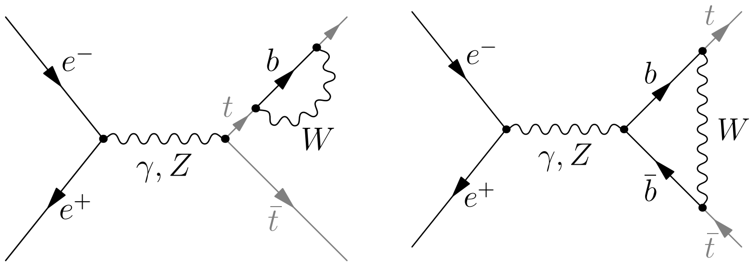

Figure 3: Sample diagrams contributing to the electroweak production corrections. Grey lines indicate on-shell (anti-)top quarks.

3. Phenomenological impact

Figure 4 demonstrates the combined effect of the corrections discussed in section 2 on the total top-antitop production cross section. Excluding the electroweak production corrections outlined in section 2.5, we see a notable change in the cross section of up to 10%. Especially in the peak region the change is significantly larger than the pure QCD scale uncertainty. Once we include the N^{2}LO electroweak corrections (excluding N^{2}LO nonresonant production), the cross section drops again visibly, to the level of the pure QCD result or even below. While the current “full” result does not correspond to a fully consistent theory description, it may indicate the magnitude of the combined N^{2}LO nonresonant and electroweak production corrections.

In figure 5 we compare the theory uncertainty estimated from scale variation between 50 GeV and 350 GeV to the change in the cross section upon variation of various input parameters. The size and shape of the variation suggests that the top-quark mass in the potential-subtracted (PS) scheme [21] can be determined to an accuracy of better than 100 MeV at a future high-energy electron-positron collider. This estimate is backed up by a recent preliminary experimental analysis [22]. In contrast, the cross section is not very sensitive to modifications of the Yukawa coupling and the change in the shape is hard to distinguish from the one induced by a variation of the strong coupling constant.

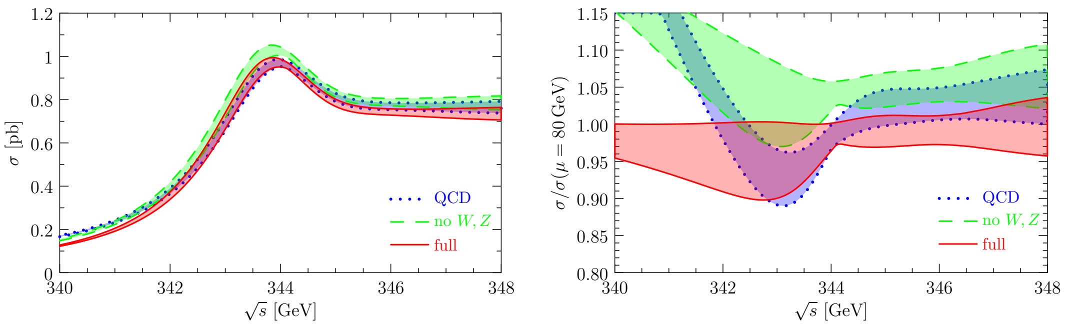

Figure 4: Cross section for $t\bar{t}$ production with $m_t^{\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}\,,\Gamma_t = 1.33\,\mathrm{GeV}\,,m_H = 125\,\mathrm{GeV}\,,\alpha_s(m_Z) = 0.1185\,,\alpha(m_Z) = 1/128.944$. Error bands result from scale variation between 50 GeV and 350 Gev. The dotted blue line shows the N^{3}LO QCD-only cross section, whereas for the dashed green line also Higgs, QED, and non-resonant corrections are included. The solid red line includes all known corrections. On the right, the cross section is normalized to the full cross section at a scale of 80 GeV.

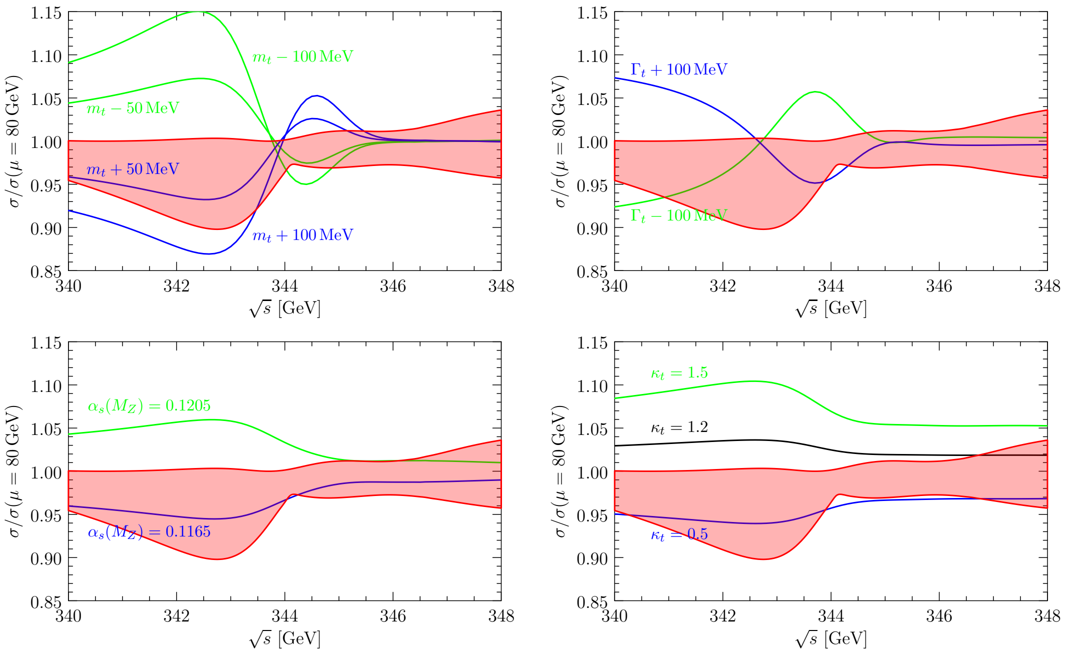

Figure 5: Change of the $t\bar{t}$ cross section upon variation of the top quark mass (upper left), width (upper right), strong coupling (lower left), and top Yukawa coupling y_t with $\kappa_t = y_t/y_t^{\mathrm{SM}}$ (lower right). The shaded band corresponds to the normalised full cross section shown in figure 4.

4. Conclusion

For the determination of the properties of the top quark at a future electron-positron collider a precise theory prediction is vital. Effects beyond QCD can be as big as 10% and exceed the uncertainty of the N^{3}LO QCD prediction. The top-quark mass can be extracted from the cross section with an uncertainty of presumably less than 100 MeV and an equally accurate measurement of the top-quark width seems also feasible. The sensitivity to the top Yukawa coupling is small and its determination will require a very precise knowledge of the value of the strong coupling constant.

The code employed for the presented analysis has been made public [23] and can be downloaded from https://www.hepforge.org/downloads/qqbarthreshold/.

Acknowledgements

A.M. is supported by a European Union COFUND/Durham Junior Research Fellowship under EU grant agreement number 267209. This work was supported by the DFG Sonderforschungsbereich/Transregio 9 “Computergestützte Theoretische Teilchenphysik”, the Gottfried Wilhelm Leibniz programme of the Deutsche Forschungsgemeinschaft (DFG) and the DFG cluster of excellence “Origin and Structure of the Universe”.

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