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Precision Measurement of $\sin^2\theta_w$ at MESA *

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A forthcoming experiment of low-energy elastic electron proton scattering at the new MESA facility in Mainz is planned to provide a high-precision measurement of the parity-violating polarisation asymmetry. This experiment is expected to lead to a precision determination of the weak mixing angle, competitive with Z-pole data. We discuss the challenges for theory to derive predictions with the required accuracy.

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1. The running weak mixing angle

One of the central parameters in the theory of the electroweak interaction is the weak mixing angle. In the Standard Model (SM) it can be defined as a scale-dependent quantity. Many measurements have confirmed the SM prediction for its running, as shown in Fig. 1. The most precise single measurements at the Z pole from LEP1 and SLD are only marginally consistent with each other and additional data with similarly high accuracy are required to improve the strength of SM tests as well as limits from searches for new physics.

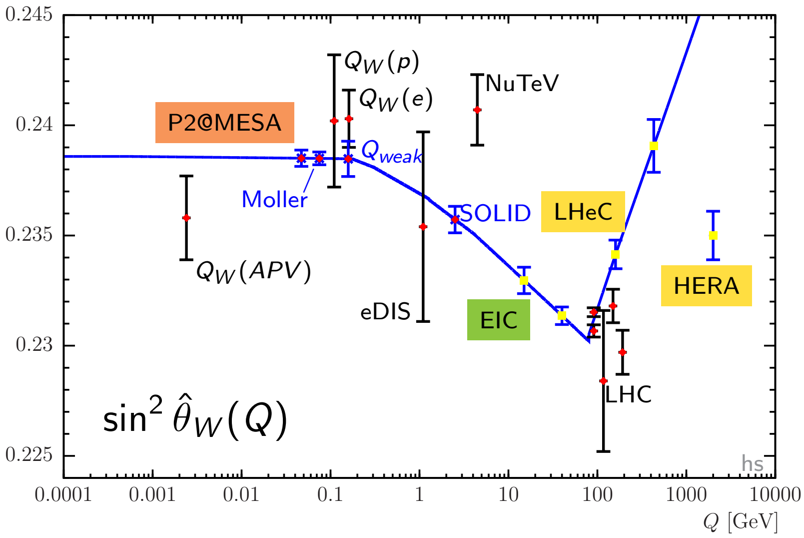

Figure 1: Running weak mixing angle compared with data. Existing measurements are shown in black for atomic parity violation in Cesium ($Q_W(APV)$), parity-violating electron scattering off protons ($Q_W(p)$ and eDIS), and electrons ($Q_W(e)$), neutrino nucleon scattering (NuTeV). Data points at the Z pole are from LEP1, SLD, Tevatron, CMS and ATLAS. Blue symbols show results of possible future experiments: P2@MESA, the Moller, Qweak and SOLID experiments at JLAB, an electron-ion collider (EIC), the LHeC and an estimate for the analysis of existing data from the HERA experiments. See also Ref. [1]. The scale dependence of the weak mixing angle was calculated using the program described in [2].

2. The P2 experiment at MESA

The new accelerator MESA (Mainz Energy-Recovery Superconducting Accelerator) being built at Mainz University will provide an electron beam with an energy up to $E = 155$ MeV and with a high degree of longitudinal polarization (above 85 %). The P2 experiment will measure the parity-violating asymmetry between left- and right-handed electrons,

$$A_{LR} = \frac{\sigma(e_\downarrow) - \sigma(e_\uparrow)}{\sigma(e_\downarrow) + \sigma(e_\uparrow)} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left(Q_W(p) - F(Q^2)\right) \tag{2.1}$$

at scattering angles in the range from 25 to 45 degrees, corresponding to an averaged squared momentum transfer Q^2 of about 0.0045 GeV^2. A_{LR} is determined by the weak charge of the proton, given in the SM at leading order by the weak mixing angle $\sin^2\theta_W$,

$$Q_W(p) = 1 - 4\sin^2\theta_W\,. \tag{2.2}$$

The experimental set-up aims to measure A_{LR} with a total uncertainty of 1.5 % which, by error propagation, leads to a 0.13 % measurement of $\sin^2\theta_W$.

At non-zero Q^2, A_{LR} is affected by form factor contributions $F(Q^2)$ due to the fact that the proton is not a point-like particle. $F(Q^2)$ and consequently A_{LR} can be decomposed as [3]

$$\begin{aligned} F(Q^2) &= F_{\text{EMFF}}(Q^2) + F_{\text{Axial}}(Q^2) + F_{\text{Strangeness}}(Q^2)\,, \\ A_{LR} &= A_{\text{Q}_\text{W}} + A_{\text{EMFF}} + A_{\text{Axial}} + A_{\text{Strangeness}} \end{aligned} \tag{2.3}$$

into contributions determined by electric and magnetic form factors of the proton and neutron, $G_E^{p,n}$, $G_M^{p,n}$

$$F_{\text{EMFF}}(Q^2) = \frac{\varepsilon G_E^p G_E^n + \tau G_M^p G_M^n}{\varepsilon (G_E^p)^2 + \tau (G_M^p)^2}\,, \tag{2.4}$$

the axial proton form factor G_A^p

$$F_{\text{Axial}}(Q^2) = \frac{(1-4\sin^2\theta_W)\sqrt{1-\varepsilon^2}\sqrt{\tau(1+\tau)}G_M^p G_A^p}{\varepsilon (G_E^p)^2 + \tau (G_M^p)^2}\,, \tag{2.5}$$

and a part containing the strangeness form factors G_E^s, G_M^s

$$F_{\text{Strangeness}}(Q^2) = \frac{\varepsilon G_E^p G_E^s + \tau G_M^p G_M^s}{\varepsilon (G_E^p)^2 + \tau (G_M^p)^2}\,. \tag{2.6}$$

Here we have used the usual kinematic variables

$$\varepsilon = [1 + 2(1+\tau)\tan^2(\theta/2)]^{-1}\,, \qquad \tau = Q^2/4m_p^2 \tag{2.7}$$

and m_p is the proton mass.

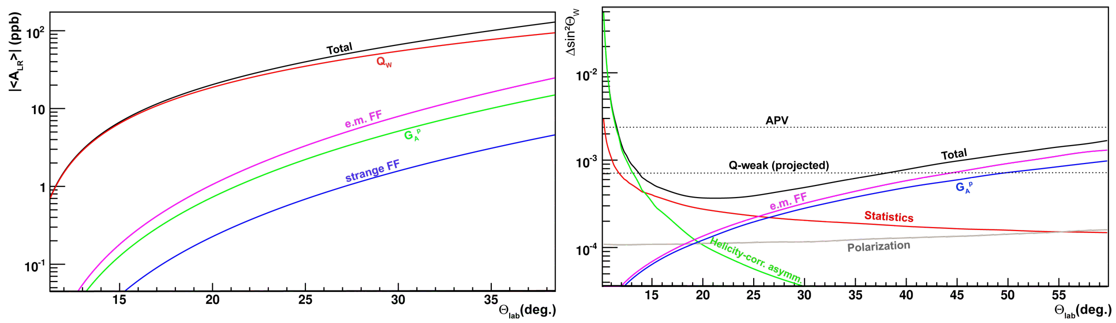

Figure 2: Left: proton weak charge and form factor contributions to A_{LR} as a function of the electron scattering angle for the beam energy $E = 200$ MeV. Right: estimated uncertainties for $\sin^2\theta_W$ at $E = 200$ MeV due to form factors, statistics, polarization measurement and helicity-correlated beam asymmetries. Figures taken from [4, 5].

As an illustration we show the asymmetry and the uncertainty for the weak mixing angle as a function of the scattering angle at a beam energy of $E = 200$ MeV in Fig. 2 [4, 5]. Results for lower energies are similar. From the left panel of Fig. 2 we can see that A_{LR} is indeed dominated by the proton weak charge; form factor contributions are suppressed by about an order of magnitude in the range of scatterings angles relevant for the P2 experiment. In the right panel of Fig. 2, a break-down of the uncertainties for $\sin^2\theta_W$ is shown. At low Q^2, the statistical uncertainty and uncertainties due to helicity-correlated beam fluctuations are dominating. Form factor uncertainties are increasing with the scattering angle, but a minimum of the total uncertainty can be found at scattering angles corresponding to $Q^2 = 0.0045$ GeV^2. Details for the accelerator and detector systems and the polarimetry are described in [6, 7, 8].

3. Theory challenges

The prediction for the left-right asymmetry is affected by higher-order corrections,

$$A_{LR} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left(Q_W(p)(1+\delta_1) - \widetilde{F}(Q^2)\right) \tag{3.1}$$

where δ_1 comprises universal correction factors $\rho_{\rm NC}$ and κ as well as process-specific corrections due to vertex (Δ_e, Δ'_e) and box graph contributions ($\delta_{\rm Box}$),

$$Q_W(p)(1+\delta_1) = (\rho_{\rm NC}+\Delta_e)(1-4\kappa\sin^2\theta_W+\Delta'_e)+\delta_{\rm Box}\,. \tag{3.2}$$

In particular, photon-Z boson mixing contributes to corrections that can be absorbed into a scale-dependent weak mixing angle,

$$\sin^2\theta_{\rm eff}(\mu^2) = \kappa(\mu^2)\sin^2\theta_W\,. \tag{3.3}$$

Depending on the renormalization scheme, κ can also contain some non-universal loop corrections. At one-loop order, these corrections are well-known to a high precision [9, 10]. The high accuracy aimed for at P2@MESA will, however, require to evaluate also two-loop corrections. For Møller scattering, first steps towards a complete two-loop calculation have been made [11, 12, 13] and show that their effect on the measured asymmetry may be larger than naively expected.

3.1 QED corrections

Electromagnetic corrections are parity-conserving and do not affect A_{LR} directly. However, the momentum transfer has to be known with high precision in order to extract the weak charge from the measured asymmetry. Therefore, bremsstrahlung effects have to be calculated as well since they lead to a shift of the momentum transfer measured from the scattering angle of the electron relative to the true momentum transferred to the proton. In Fig. 3 we show the results of a calculation including one-photon bremsstrahlung. The Q^2-shift depends strongly on the beam energy and the scattering angle, as well as on a possible cutoff of the energy of photons radiated into the final state. It is obvious from these results that also the dominating two-photon bremsstrahlung contributions will have to be evaluated in order to reach the high-precision goal of P2@MESA.

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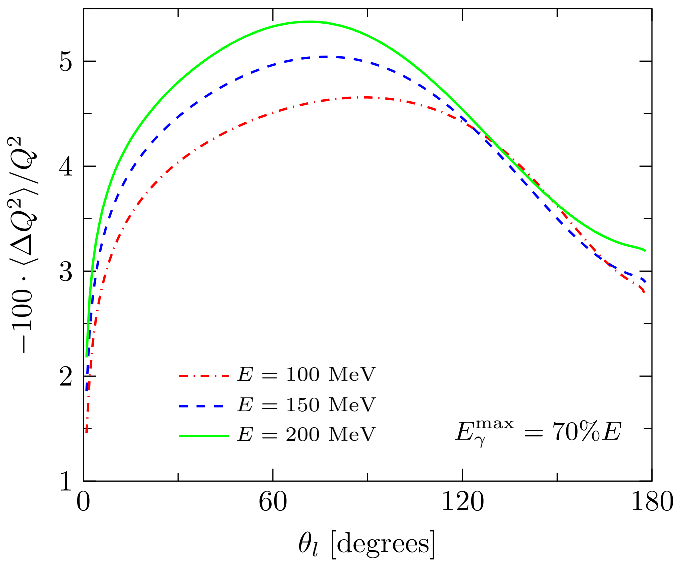

Figure 3: Shift of the momentum transfer due to photon bremsstrahlung.

3.2 γZ box graphs

An important source of uncertainties in higher-order corrections is due to box graph contributions with the exchange of a photon and a Z boson. The presence of the massless photon in the loop makes these graphs sensitive to the low-scale hadronic structure of the proton.

The optical theorem can be used to relate the imaginary part of the box graphs to the γZ interference part of structure functions $F_k^{\gamma Z}$ for inelastic ep scattering (see Fig. 4),

$$\mathrm{Im}\Box_{\gamma Z}(E) = \frac{\alpha}{(s-m_p^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{max}} dQ^2 \frac{M_Z^2}{Q^2+M_Z^2} \left\{ F_1^{\gamma Z} + A F_2^{\gamma Z} + \frac{g_V^e}{g_A^e} B F_3^{\gamma Z} \right\} \tag{3.4}$$

where $s = 2m_pE + m_p^2$ is the squared center-of-mass energy, W the invariant mass of the hadronic intermediate state, ν the invariant energy transfer, $W^2 = m_p^2 + 2m_p\nu - Q^2$, A and B are kinematic factors, g_V^e, g_A^e the weak neutral-current vector and axial-vector coupling constants of the electron and the $F_k^{\gamma Z}$ are functions of ν and Q^2. The dispersion relations involve integrals over the full kinematic range, with a strong emphasis on the low-W, low-Q^2 range. The structure functions can,

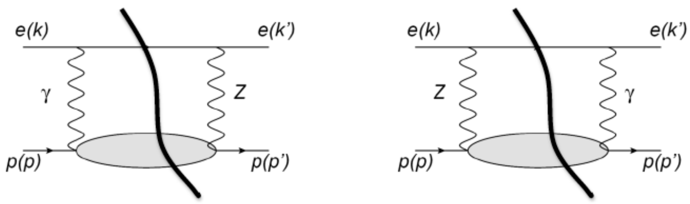

Figure 4: The optical theorem and dispersion relations allow one to relate the box graph corrections to elastic electron proton scattering with structure functions for inelastic scattering.

in principle, be measured in ep scattering. In practice, however, data are available only in a very restricted range of the kinematic variables. Missing information has to be modelled, for example by assuming dominance of low-lying resonances in the hadronic intermediate state of the box graph, or in baryon chiral perturbation theory.

The box graph contributions can be separated into vector and axial-vector parts of the proton current and the real parts that enter the corrections for A_{LR} are recovered by dispersion relations which read

$$\mathrm{Re}\Box^V_{\gamma Z}(E) = E\frac{2}{\pi}\int_{\nu_\pi}^{\infty}\frac{dE'}{E'^2-E^2}\mathrm{Im}\Box^V_{\gamma Z}(E')\,, \tag{3.5}$$

$$\mathrm{Re}\Box^A_{\gamma Z}(E) = \frac{2}{\pi}\int_{\nu_\pi}^{\infty}\frac{E'dE'}{E'^2-E^2}\mathrm{Im}\Box^A_{\gamma Z}(E')\,. \tag{3.6}$$

Invariance with respect to time reversal forces the vector part Eq. (3.5) to vanish for zero beam energy. This fact makes the measurement at P2@MESA ($E \leq 155$ MeV) much less sensitive to theoretical uncertainties than the competing experiment Qweak at the Jefferson Laboratory ($E =$ 1.165 GeV, $Q^2 = 0.026$ GeV^2) [14]. The energy-dependence of the vector γZ box graph correction from Ref. [15] is shown in Fig. 5.

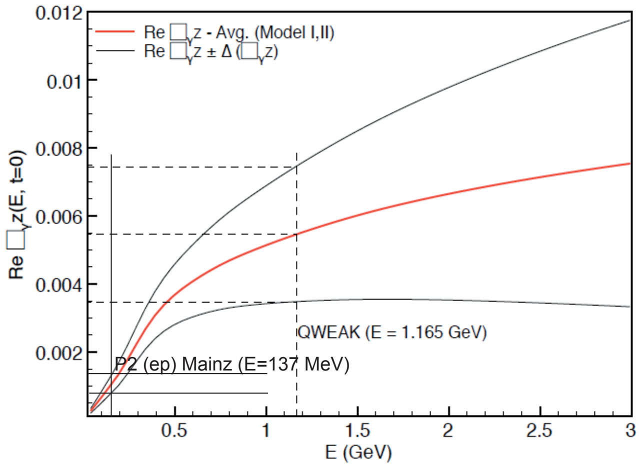

Figure 5: γZ box graph correction with its error estimate from Ref. [15]. The full and dashed lines indicate the beam energies and resulting uncertainties for the P2@MESA and Qweak experiments.

A recent update [16] has taken into account contributions from strange form factor contributions modelled in a unitarized partial wave analysis supplemented by a Regge theory inspired high-energy behaviour of the structure function input. The currently best estimate for the vector part of the γZ box graph correction is

$$\mathrm{Re}\,\Box^V_{\gamma Z}(E = 0.155\,\mathrm{GeV}) = (1.07 \pm 0.18) \times 10^{-3}\,, \tag{3.7}$$

to be compared with

$$\mathrm{Re}\,\Box^V_{\gamma Z}(E = 1.165\,\mathrm{GeV}) = (5.58 \pm 1.41) \times 10^{-3} \tag{3.8}$$

for the kinematics of the Qweak experiment. This numerical result, a 0.1 % correction for A_{LR} for P2@MESA, is well below the required accuracy and confirms the expectation that a measurement at lower beam energy is advantageous.

Parity violation in the hadronic system will also contribute through $\gamma\gamma$ box diagrams. Their study is presently underway [17].

The conventional formal definition of the nucleon's weak charge through the low-momentum limit of a measured parity-violating asymmetry is based on the assumption that form factor contributions vanish at $Q^2 = 0$ and can be written as $F(Q^2) = -Q^2 B(Q^2)$. With this assumption one could write

$$A_{exp} = A_0 \left(Q_W(p) + Q^2 B(Q^2)\right) \quad \rightarrow \quad Q_W(p) = \lim_{Q^2\to 0} \frac{A_{exp}}{A_0}\,. \tag{3.9}$$

However, this definition ignores the presence of box graph corrections which depend on both Q^2 and the beam energy, but do not vanish at Q^2. A revised definition of the weak charge, taking this observation into account, is

$$A_{exp} = A_0 \left(Q_W(p) + Q^2 B(Q^2) + \Box(E)\right) \quad \rightarrow \quad Q_W(p) = \lim_{Q^2,E\to 0} \frac{A_{exp}}{A_0} \tag{3.10}$$

where the zero-scale limit of both the momentum transfer and the center-of-mass energy has to be taken.

4. Conclusions

Civil construction for the extremely challenging high-precision experiments at MESA will start 2016 and we expect that a beam for the P2 experiment will be available before 2020. In the larger context of past, present and future *ep* scattering experiments, MESA will be the facility with the highest beam intensity at lowest electron energy, reaching an integrated luminosity of almost 10 ab^{-1} in 10,000 hours of data taking. The P2 experiment will measure the smallest PV-violating asymmetry, $A_{LR} \simeq 33$ ppb with highest precision $\Delta A_{LR} \simeq 0.44$ ppb.

The expected precision for $\sin^2\theta_W$ from this experiment will be competitive with the highest-precision results from Z-pole data, both from LEP1 and from expected LHC measurements. In searches for new physics, the precision will allow us to exclude 4-fermion contact interactions with mass scales up to 49 TeV, corresponding to the reach at the LHC with an integrated luminosity of 300 fb^{-1}. The planned experiment will be able to exclude models which change the running of the weak mixing angle, induced for example by new light gauge bosons [18], thus complementing other searches for so-called dark photons or Z-bosons.

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