Conformal transformations of leading twist operators in QCD: nonsinglet operators

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QCD evolution equations in minimal subtraction schemes have a hidden symmetry: the evolution kernel commutes with three operators that obey the commutation relations of the $SL(2)$ algebra. The explicit form of these operators can be found by analysis of QCD at the critical point in non-integer $d = 4 - 2\varepsilon$ space-time dimensions. Quantum corrections to symmetry generators in $d = 4 - 2\varepsilon$ correspond to the conformal symmetry breaking in the physical ($d = 4$) theory. The $SL(2)$ commutation relations lead to nontrivial constraints on the evolution kernels. We present the explicit two loop expressions for the generators of the $SL(2)$ algebra. This result allows one to restore the nonforward evolution kernels for the nonsinglet operators from the known NNLO anomalous dimensions.

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1. Introduction

Recent progress in accelerator and detector technologies has made it possible the study of hard exclusive reactions with identified particles in the final state. The relevant nonperturbative input in such processes involves operator matrix elements between states with different momenta, dubbed generalized parton distributions (GPDs), or vacuum-to-hadron matrix elements — the distribution amplitudes (DAs). The scale dependence of these matrix elements is governed by the renormalization group equations (RGE). The different momenta in the initial and the final state complicates the RG analysis since in this case one has to keep under control the mixing with operators involving total derivatives. The matrix of anomalous dimensions (for a given moment) is triangular: the diagonal entries correspond to the anomalous dimensions that are known to NNLO accuracy [1, 2], but the nondiagonal contributions require a dedicated calculation.

A direct calculation of nondiagonal matrix elements in higher orders is quite challenging. However, it has been known for some time [3] that conformal symmetry of the QCD Lagrangian allows one to restore nondiagonal entries in the mixing matrix at given order of perturbation theory performing an additional calculation at one order less. This result was used to compute two-loop evolution kernels for the twist-two operators in QCD [4, 5, 6, 7, 8, 9].

An alternative technique was suggested in [10]. It employs the fact that the majority of QFT models in $d = 4 - 2\varepsilon$ dimensions possess a nontrivial Wilson-Fisher fixed point. At this (*critical*) point a theory enjoys, as a rule, *exact* scale and conformal invariance (see Ref. [11] for a review). Therefore one can expect that the RG kernels will commute with the generators of the conformal group. Of course, the generators differ from their canonical expression due to quantum corrections. Finally, taking into account that in the MS-like scheme the RG kernels (anomalous dimensions matrices) do not depend explicitly on the space-time dimension one concludes that the RG kernels are essentially the same in $d = 4$ theory and $d = 4 - 2\varepsilon$ theory at the critical point [1].

The utility of this approach was illustrated in [10] on several examples to two- and three-loop accuracy for scalar theories, and in [12] on the example of the two-loop evolution equation for flavor-nonsinglet operators in QCD. The two–loop expression for the generator of special conformal transformations was derived in Ref. [13]. Below we discuss the details of calculation and present the results.

2. Conformal QCD

QCD considered in $d = 4 - 2\varepsilon$ dimension possesses a nontrivial critical point for a large number of quark flavors. Indeed, the β-function

$$\beta(a) = M\frac{da}{dM} = 2a\big(-\varepsilon - \gamma_g\big),\qquad(2.1)$$

where $a = \alpha_s/4\pi$ and

$$\gamma_g = \beta_0\, a + \beta_1 a^2 + \mathcal{O}(a^3),\qquad(2.2)$$

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[1]At least for the models with one coupling constant.

with

$$\beta_0 = \frac{11}{3}N_c - \frac{2}{3}N_f, \qquad\qquad \beta_1 = \frac{2}{3}\left[17N_c^2 - 5N_cN_f - 3C_FN_f\right] \qquad (2.3)$$

vanishes ($\beta(a_*) = 0$) for

$$a_*(\varepsilon) = -\frac{\varepsilon}{\beta_0} - \left(\frac{\varepsilon}{\beta_0}\right)^2\frac{\beta_1}{\beta_0} + O(\varepsilon^3). \qquad (2.4)$$

The vanishing of the β-function implies that the correlators of local gauge-invariant operators transform in a proper way under scale and conformal transformations.

Due to operator mixing the RGEs for a set of the leading twist operators of a given dimension $\{\mathcal{O}_i, i = 1, \dots, n\}$, take the matrix form

$$\left(\left(M\partial_M + \beta(a)\partial_a\right)\delta_{ik} + \gamma_{ik}(a)\right)\mathcal{O}_i = 0, \qquad (2.5)$$

where γ_{ik} is the anomalous dimension matrix. The form of the matrix depends strongly on the choice of the basis operators, $\mathcal{O}_i$, and the symmetry properties of the problem are not transparent in this formulation. It proves more convenient to consider the generating function for the local-operators, the so-called light-ray operator [14]. For the twist–two nonsinglet operators we are interested in it takes the form

$$[\mathcal{O}](x; z_1, z_2) = Z\mathcal{O}(x; z_1, z_2) = Z\bar{q}(x + z_1 n)\slashed{n}q(x + z_2 n) = \sum_{m,k}\frac{z_1^m z_2^k}{m!k!}\left[\bar{q}(x)\overleftarrow{D}_+^m\slashed{n}\overrightarrow{D}_+^k q(x)\right], \quad (2.6)$$

where $D_+ = n^\mu D_\mu$ and the Wilson line is implied between the quark fields on the light-cone. The square brackets denote the renormalized operator in the $\overline{\text{MS}}$ scheme and the renormalization factor Z is an integral operator acting on the light-cone coordinates of the fields z_1, z_2

$$Z = 1 + \sum_{k=0}^{\infty}\frac{1}{\varepsilon^k}Z_k(a), \qquad\qquad Z_k(a) = \sum_{\ell=k}^{\infty}a^\ell Z_k^{(\ell)}. \qquad (2.7)$$

The RGE for the light-ray operator $[\mathcal{O}]$ takes the form

$$\left(M\partial_M + \beta(a)\partial_a + \mathbb{H}(a)\right)[\mathcal{O}](x; z_1, z_2) = 0, \qquad (2.8)$$

where $\mathbb{H}$ is an integral operator related to the renormalization factor (2.7) as follows

$$\mathbb{H}(a) = 2\gamma_q(a) - M\frac{d}{dM}ZZ^{-1} = 2\gamma_q(a) + 2\sum_{\ell=1}^{\infty}\ell a^\ell Z_1^{(\ell)} = \sum_{k\geq 1}a^k\mathbb{H}^{(k)}. \qquad (2.9)$$

γ_q is the quark anomalous dimension. The evolution kernel can be written as [14]

$$\mathbb{H}(a)[\mathcal{O}](z_1, z_2) = \int_0^1 d\alpha\int_0^1 d\beta\, h(\alpha, \beta)[\mathcal{O}](z_{12}^\alpha, z_{21}^\beta), \qquad (2.10)$$

where $z_{12}^\alpha = z_1\bar{\alpha} + z_2\alpha$ and $\bar{\alpha} = 1 - \alpha$. The weight function $h(\alpha, \beta)$ is given by a series in the coupling constant

$$h(\alpha, \beta) = a\, h^{(1)}(\alpha, \beta) + a^2 h^{(2)}(\alpha, \beta) + \dots \qquad (2.11)$$

Let us stress here that the fixed-order kernels $h^{(k)}(\alpha, \beta)$ in the $\overline{\text{MS}}$ scheme do not depend on the space-time dimension by construction. It means that the evolution kernel $\mathbb{H}$ at the critical point depends on the space-time dimension d only through the value of the critical coupling, $a_* = a_*(\varepsilon)$,

$$\mathbb{H}(a_*) = a_* \mathbb{H}^{(1)} + a_*^2 \mathbb{H}^{(2)} + \ldots$$

Thus the evolution kernel in the four dimensional theory inherits all symmetries of the kernel in the conformal d dimensional theory.

3. Symmetries

It is natural to expect that the evolution kernel $\mathbb{H}(a_*)$ commutes with the generators of the conformal group. Most of the generators, however, act trivially, $[G_\alpha, \mathscr{O}] = 0$, on the twist-two operators considered here leaving us with the so-called *soft-collinear* $SL(2,R)$ subgroup of the conformal group. At the tree level the generators, $S_\alpha = \{S_+, S_0, S_-\}$ are given by well known expressions

$$S_+^{(0)} = z_1^2 \partial_{z_1} + z_2^2 \partial_{z_2} + 2(z_1 + z_2), \qquad S_0^{(0)} = z_1 \partial_{z_1} + z_2 \partial_{z_2} + 2, \qquad S_-^{(0)} = -\partial_{z_1} - \partial_{z_2}. \tag{3.1}$$

Beyond the leading order the generators are defined as follows. Let us expand the light-ray operators over the set of local conformal operators $\mathscr{O}_N$ and their descendants $\partial_+^k \mathscr{O}_N$,

$$[\mathscr{O}](x, z_1, z_2) = \sum_{N \geq 0} \sum_{k \geq 0} \Psi_{Nk}(z_1, z_2) \partial_+^k \mathscr{O}_N(x). \tag{3.2}$$

The expansion coefficients (coefficient functions) $\Psi_{Nk}(z_1, z_2)$ are homogeneous polynomials in z_1, z_2 of degree $N + k$ (of course they depend on the coupling constant a_* or the space-time dimension d). The transformation properties of the conformal operator $\mathscr{O}_N(x)$ is determined entirely by its scaling dimension $\Delta_N^* = d + N - 1 + \gamma_N(a*)$ and spin $s_N = N + 1$. Namely,

$$i[\mathbf{D}, [\mathscr{O}_N](x)] = (x\partial_x + \Delta_N^*)[\mathscr{O}_N](x). \tag{3.3}$$

for the scale and

$$i[\mathbf{K}^\mu, [\mathscr{O}_N](x)] = \left[2x^\mu(x\partial) - x^2 \partial^\mu + 2\Delta_N^* x^\mu + 2x^\nu \left(n^\mu \frac{\partial}{\partial n^\nu} - n_\nu \frac{\partial}{\partial n_\mu} \right) \right] [\mathscr{O}_N](x) \tag{3.4}$$

for the special conformal transformations. The variation $S_+[\mathscr{O}]$ of the light-ray operator under the special conformal transformation $\mathbf{L}_- = \frac{i}{2} \bar{n}^\mu \mathbf{K}_\mu$ is defined as

$$\begin{aligned}
[\mathbf{L}_-, [\mathscr{O}](x, z_1, z_2)] &= \sum_{N \geq 0} \sum_{k \geq 0} \Psi_{Nk}(z_1, z_2) \partial_+^k [\mathbf{L}_-, \mathscr{O}_N(x)] \\
&= \sum_{N \geq 0} \sum_{k \geq 0} \Psi'_{Nk}(z_1, z_2) \partial_+^k \mathscr{O}_N(x) = S_+[\mathscr{O}](x, z_1, z_2).
\end{aligned} \tag{3.5}$$

Going from the first to the second line we take into account that the conformal operators and their descendants form a basis in the space of local operators so that the expression $\partial_+^k [\mathbf{L}_-, \mathscr{O}_N(x)]$ can be re-expanded over the basis operators with a modified coefficient functions Ψ'_{Nk}. The other

generators S_-, S_0 are defined in the same way. Such a definition of the generators $S_{\pm,0}$ is not explicit as it involves the functions Ψ_{Nk} which are not known. Nevertheless it guarantees that the generators S_α satisfy the $SL(2,R)$ commutation relations,

$$[S_+, S_-] = 2S_0 \qquad\qquad [S_0, S_\pm] = \pm S_\pm \,. \qquad (3.6)$$

The commutation relations result in the following relations

$$S_-\Psi_{Nk} = -\Psi_{Nk-1}\,, \qquad S_0\Psi_{Nk} = (j_N + k)\Psi_{Nk}\,, \qquad S_+\Psi_{Nk} = (k+1)(2j_N + k)\Psi_{Nk+1}\,, \qquad (3.7)$$

where the conformal spin $j_N = (\Delta_N^* + s_N)/2$. Since the generator S_- corresponds to the translations it does not receive corrections and is given by its classical expression, $S_- = S_-^{(0)}$. It follows from the RG equation that the coefficient functions Ψ_{Nk} are the eigenfunctions of the evolution kernel, $\mathbb{H}(a_*)\Psi_{Nk} = \gamma_N(a_*)\Psi_{Nk}$. This leads immediately to the following expression for the generator S_0:

$$S_0 = S_0^{(0)} + \gamma_g(a_*) + \frac{1}{2}\mathbb{H}(a_*)\,. \qquad (3.8)$$

The generator S_+ can be presented as

$$S_+ = S_+^{(0)} + (z_1 + z_2)\left(\gamma_g(a_*) + \frac{1}{2}\mathbb{H}(a_*)\right) + (z_1 - z_2)\Delta_+(a_*)\,, \qquad (3.9)$$

where the kernel $\Delta_+(a_*)$ commutes with S_- and $S_0^{(0)}$ and is usually referred to as conformal anomaly. Let us stress here that in distinction to the scaling (anomalous) dimensions at the critical point, which are physical observables and therefore do not depend on scheme, the evolution kernel, $\mathbb{H}(a_*)$, conformal anomaly $\Delta_+(a_*)$ and the coefficient functions Ψ_{Nk} are scheme-dependent. Their explicit form cannot be fixed from the symmetry consideration only and requires a separate calculation. The kernel $\mathbb{H}(a_*)$ and the conformal anomaly $\Delta_+(a_*)$ are, nevertheless, not independent. It follows from the commutation relations (3.6) that, as was expected, the kernel $\mathbb{H}(a_*)$ commutes with the generators S_α. Two commutation relation, $[S_-, \mathbb{H}] = [S_0, \mathbb{H}] = 0$, are satisfied by construction, while the last one, $[S_+, \mathbb{H}(a_*)] = 0$, results in a nontrivial relation between $\mathbb{H}$ and Δ_+. Namely,

$$[S_+^{(0)}, \mathbb{H}(a_*)] = [\mathbb{H}(a_*), z_1 + z_2]\left(\gamma_g(a_*) + \frac{1}{2}\mathbb{H}(a_*)\right) + [\mathbb{H}(a_*), (z_1 - z_2)\Delta_+(a_*)]\,. \qquad (3.10)$$

Expanding Eq. (3.10) in a series in a_* one gets

$$[S_+^{(0)}, \mathbb{H}^{(1)}] = 0\,,$$
$$[S_+^{(0)}, \mathbb{H}^{(2)}] = [\mathbb{H}^{(1)}, z_1 + z_2]\left(\gamma_g^{(1)} + \frac{1}{2}\mathbb{H}^{(1)}\right) + [\mathbb{H}^{(1)}, (z_1 - z_2)\Delta_+^{(1)}]\,, \qquad (3.11)$$

and so on. Since the kernel $\mathbb{H}(a_*)$ commutes with $S_-^{(0)}$ and $S_0^{(0)}$ these equations demonstrate that the non-invariant part of the ℓ-loop kernel (with respect to the canonical conformal transformations) is completely fixed by lower order kernels $\mathbb{H}^{(k)}, \Delta_+^{(k)}, k = 1, \ldots, \ell - 1$.

4. Conformal Ward Identity

An explicit form of the generator of special conformal transformations S_+, or the conformal anomaly $\Delta_+(a_*)$ can be derived from analysis of the conformal Ward Identity (CWI) [7, 9, 13]. It follows from the definition (3.5) of the generator S_+ that the correlator of the two light-ray operators,

$$\mathscr{G}(x;z,w) = \langle [\mathscr{O}^{(n)}](0,z) [\mathscr{O}^{(\bar{n})}](x,w) \rangle = \mathscr{N} \int D\Phi\, e^{-S_R(\Phi)} [\mathscr{O}^{(n)}](0,z) [\mathscr{O}^{(\bar{n})}](x,w), \qquad (4.1)$$

satisfies the following relation

$$\frac{i}{2}\bar{n}^\mu \left(\langle [\mathbf{K}_\mu, [\mathscr{O}^{(n)}](0,z)] [\mathscr{O}^{(\bar{n})}](x,w) \rangle + \langle [\mathscr{O}^{(n)}](0,z) [\mathbf{K}_\mu, [\mathscr{O}^{(\bar{n})}](x,w)] \rangle \right) =$$
$$= \left[(n\bar{n})S_+^{(z)} - \frac{1}{2}x^2(\bar{n}\partial_x) \right] \mathscr{G}(x;z,w) = 0, \quad (4.2)$$

where $z = \{z_1, z_2\}$, $w = \{w_1, w_2\}$ and we assume that $(x\cdot n) = (x\cdot\bar{n}) = 0$. Let us stress here that (4.2) is a consequence of Eqs. (3.4) and (3.5). On the other hand one can derive Eq. (4.2) from invariance of the path integral in (4.1) under the change of variables:

$$\Phi \mapsto \Phi + \delta_K^\mu \Phi, \qquad \delta_K^\mu \Phi = \left(2x_\mu(x\partial) - x^2\partial_\mu + 2\Delta_\Phi x_\mu - 2x^\nu\Sigma_{\mu\nu} \right)\Phi(x), \qquad (4.3)$$

see e.g. Ref. [15]. $\Sigma_{\mu\nu}$ in (4.3) is the generator of spin rotations,

$$\Sigma_{\mu\nu}c = \Sigma_{\mu\nu}\bar{c} = 0, \qquad \Sigma_{\mu\nu}q = \frac{i}{2}\sigma_{\mu\nu}q, \qquad \Sigma_{\mu\nu}A_\alpha = g_{\nu\alpha}A_\mu - g_{\mu\alpha}A_\nu$$

and it is convenient to choose the dimensions of the QCD fundamental fields, Δ_Φ, as follows [9]:

$$\Delta_q = \frac{3}{2} - \varepsilon, \qquad \Delta_A = 1, \qquad \Delta_c = 0, \qquad \Delta_{\bar{c}} = 2 - \varepsilon. \qquad (4.4)$$

The CWI takes the form

$$\langle \delta[\mathscr{O}^{(n)}](0,z) [\mathscr{O}^{(\bar{n})}](x,w) \rangle + \langle [\mathscr{O}^{(n)}](0,z) \delta[\mathscr{O}^{(\bar{n})}](x,w) \rangle = \langle \delta S_R [\mathscr{O}^{(n)}](0,z) [\mathscr{O}^{(\bar{n})}](x,w) \rangle, \quad (4.5)$$

where $\delta = \bar{n}_\mu \delta_K^\mu$, δS_R is the corresponding variation of the QCD action (in Euclidean space)

$$\delta_K^\mu S_R = \int d^d x\, 2x^\mu \left(\mathscr{N}(x) - (d-2)\partial^\rho \mathscr{B}_\rho(x) \right), \qquad (4.6)$$

and

$$\mathscr{N}(x) = 2\varepsilon \mathscr{L}_R^{YM+gf} = 2\varepsilon \left(\frac{1}{4}Z_A^2 F^2 + \frac{1}{2\xi}(\partial A)^2 \right), \qquad \mathscr{B}_\rho(x) = Z_c^2 \bar{c}D^\rho c - \frac{1}{\xi}A^\rho(\partial A). \quad (4.7)$$

It should be noted that the variation (4.6) does not vanish even at $d = 4$. However, this non-vanishing term being the BRST variation does not contribute to the correlators of gauge invariant operators. This implies that a correlator of the nonlocal operator with two (anti)quark fields, $\langle \mathscr{O}q\bar{q} \rangle$, is not a convenient object for analysis in the case of gauge theories. The further analysis goes along

the lines of Ref. [16]. The operator $\mathcal{N}$ has to be written as the sum of renormalized (finite) operators,

$$\mathcal{N} = -\frac{\beta(a)}{a}\left[\mathcal{L}^{YM+gf}\right] - (\gamma_A + \gamma_g)\Omega_A - \sum_{\Phi \neq A}\gamma_\Phi\Omega_\Phi + \frac{\gamma_A}{\xi}[(\partial A)^2] + z_c\partial^\mu\Omega_\mu + z_b\partial_\mu[\mathcal{B}^\mu]. \quad (4.8)$$

Here Ω_Φ is an EOM operator, $\Omega_\Phi = \Phi(y)\left(\delta S_R/\delta\Phi(y)\right)$ and $\partial^\mu\Omega_\mu = \Omega_{\bar{c}} - \Omega_c$, γ_Φ are the field anomalous dimensions, and $z_c(g,\xi)$ and $z_b(g,\xi)$ are some finite coefficients. The last term being BRST variation does not contribute as well as the ghost EOM to the correlator (4.5). The other EOM terms together with the gauge fixing term result in the necessary modification of the scaling dimensions in the expression for the conformal generators. Nontrivial corrections to the S_+ come from the first term on the r.h.s of (4.8). The correlator

$$\left\langle \int d^d y\,\mathcal{N}(y)[\mathcal{O}^{(n)}](0,z)\,[\mathcal{O}^{(\bar{n})}](x,w)\right\rangle$$

contains additional divergencies when the arguments of the renormalized operators coincide, i.e. when $y \sim 0$ or $y \sim x$ and gives nonzero contribution in spite of that it comes with vanishing coefficient $\beta(a_*) = 0$. It can be shown that the correction to the generator S_+ is related to the pair counterterms to the product of the operators $\mathcal{N}(y)[\mathcal{O}^{(n)}](0,z) \sim \delta(y)Z(\varepsilon)[\mathcal{O}^{(n)}](0,z)$. Technical details can be found in [13]. The one loop correction to the generator takes an extremely simple form [7, 12]

$$\Delta_+^{(1)}f(z_1,z_2) = 2C_F\int_0^1 d\alpha\int_0^1 du\,\frac{\bar{\alpha}}{\alpha}\left[f(z_{12}^{\alpha u},z_2) - f(z_1,z_{21}^{\alpha u})\right] \quad (4.9)$$

The two loop expression can be written [13] as

$$\Delta_+^{(2)}f(z_1,z_2) = \int_0^1 d\alpha\int_0^{\bar{\alpha}} d\beta\left[\omega(\alpha,\beta) + \omega^{\mathbb{P}}(\alpha,\beta)\mathbb{P}_{12}\right]\left[f(z_{12}^\alpha,z_{21}^\beta) - f(z_{12}^\beta,z_{21}^\alpha)\right]$$
$$+ \int_0^1 du\int_0^1 dt\,\varkappa(t)\left[f(z_{12}^{ut},z_2) - f(z_1,z_{21}^{ut})\right], \quad (4.10)$$

where $\mathbb{P}_{12}f(z_1,z_2) = f(z_2,z_1)$ is the permutation operator. The kernels $\varkappa(t)$, $\omega(\alpha,\beta)$, $\omega^{\mathbb{P}}(\alpha,\beta)$ receive contributions of three different color structures

$$\varkappa(t) = C_F^2\,\varkappa_{FF}(t) + C_FC_A\,\varkappa_{FA}(t) + C_F\beta_0\varkappa_{bF}(t),$$
$$\omega(\alpha,\beta) = C_F^2\,\omega_{FF}(\alpha,\beta) + C_FC_A\,\omega_{FA}(\alpha,\beta),$$
$$\omega^{\mathbb{P}}(\alpha,\beta) = C_F^2\,\omega_{FF}^{\mathbb{P}}(\alpha,\beta) + C_FC_A\,\omega_{FA}^{\mathbb{P}}(\alpha,\beta). \quad (4.11)$$

As an example, we give here the result for the C_FC_A color structure,

$$\varkappa_{FA}(t) = \frac{2\bar{t}}{t}\left\{(2+t)\left[\text{Li}_2(\bar{t}) - \text{Li}_2(t)\right] - (2-t)\left(\frac{t}{\bar{t}}\ln t + \ln\bar{t}\right) - \frac{\pi^2}{6}t - \frac{4}{3} - \frac{t}{2}\left(1 - \frac{t}{\bar{t}}\right)\right\},$$

$$\omega_{FA}(\alpha,\beta) = 2\left[\left(\frac{1}{\alpha} - \alpha\right)\left[\text{Li}_2\left(\frac{\beta}{\bar{\alpha}}\right) - \text{Li}_2(\beta) - 2\text{Li}_2(\alpha) - \ln\alpha\ln\bar{\alpha}\right] + \frac{\alpha}{\tau}\left(\tau\ln\tau + \bar{\tau}\ln\bar{\tau}\right)$$
$$- \bar{\beta}\ln\alpha - \frac{\bar{\alpha}}{\alpha}\ln\bar{\alpha}\right],$$

$$\omega_{FA}^{\mathbb{P}}(\alpha,\beta) = 2\left[\left(\bar{\alpha} - \frac{1}{\bar{\alpha}}\right)\left[\text{Li}_2\left(\frac{\alpha}{\bar{\beta}}\right) - \text{Li}_2(\alpha) - \ln\bar{\alpha}\ln\bar{\beta}\right] + \alpha\bar{\tau}\ln\bar{\tau} + \frac{\beta^2}{\bar{\beta}}\ln\bar{\alpha}\right]. \quad (4.12)$$

Here $\tau = \alpha\beta/\bar{\alpha}\bar{\beta}$. Explicit expressions for the other color structures can be found in Ref. [13].

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