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On non-abelian next-to-leading-power threshold logarithms

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We present recent results in an ongoing program to construct a resummation formalism for threshold logarithms at next-to-leading power (NLP) in the threshold expansion. We discuss a factorised expression for the Drell-Yan scattering amplitude, valid at NLP, which arises from the Low-Burnett-Kroll-Del Duca theorem, and correctly reproduces all relevant logarithms at two loops for the abelian-like color structure. We then discuss methods to generalise this formalism to the full non-abelian theory, and propose a simple ansatz which succeeds in generating all non-abelian NLP logarithms at two loops, in terms of a small set of universal soft and collinear functions.

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1. Introduction

Threshold resummation is a well-known tool, widely used to improve the precision and extend the range of applicability of perturbative QCD calculations for high-energy hadronic cross sections [1]. It is based on the universality of singular contributions to scattering amplitudes arising from long-distance dynamics: for sufficiently inclusive observables, singularities cancel at the level of the cross section, and the remaining finite logarithmic contributions can be organised in a factorised form. Gauge invariance and the renormalisation group can then be used to compute to all perturbative orders towers of logarithms that give significant contributions to many phenomenologically relevant observables. Interestingly, this well-understood paradigm based on factorisation has recently been extended using partly numerical methods to a class of non-factorising event-shape observables in electron-positron annihilation [2, 3].

When discussing 'singular contributions' to scattering amplitudes and cross sections, one normally refers to terms that individually diverge when integrated over loop momenta, or over phase space regions corresponding to undetected radiation. In a renormalisable theory, such divergences are always logarithmic, and arise from the leading power in the Laurent expansion of the integrand in powers of the relevant momentum components. It is, however, well known since the early days of quantum field theory that the universal behaviour of long-distance contributions to scattering amplitudes and cross sections extends to next-to-leading power in the Laurent expansion, at least for soft radiation [4, 5, 6]. At next-to-leading power (NLP), universality still means that the radiative amplitude can be expressed in terms of the non-radiative amplitude: at this level, however, the resulting expression is not a product, rather it is given by the action of a differential operator on the Born matrix element. The original insight by Low [4] was later generalised by Burnett and Kroll to full QED [5], and, importantly, by Del Duca [6] to the case of massless particles, where the presence of collinear enhancements requires the introduction of a new universal function. We refer to these results collectively as the LBKD theorem, and we will review them, in an updated form, in the next section. We note in passing that, to the best of our knowledge, no general results of this kind are known for real radiation that is hard, but evaluated at next-to-leading power in the collinear expansion[1].

To be more precise about our ultimate goal, we note that the class of observables that we are interested in is characterised by the measurement of a threshold variable ξ, which vanishes at Born level. Real radiation which is soft or collinear changes the value of ξ by a small amount, but with a very large amplitude. The divergent contribution is removed by virtual corrections, which do not change the value of ξ. The resulting distribution then takes the general form

$$\frac{d\sigma}{d\xi} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi} \right)^n \sum_{m=0}^{2n-1} \left[c_{nm}^{(-1)} \left(\frac{\log^m \xi}{\xi} \right)_+ + c_{nm}^{(\delta)} \, \delta(\xi) + c_{nm}^{(0)} \log^m \xi + \ldots \right] , \tag{1.1}$$

Leading-power threshold logarithms are governed by the coefficients $c_{nm}^{(-1)}$ which are closely connected to infrared and collinear divergences of scattering amplitudes. Terms with support only at the threshold, with coefficients $c_{nm}^{(\delta)}$, come from combining finite virtual corrections with finite terms in the expansion of the phase space measure for real radiation: in simple cases, they can

[1] See, however, [7].

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also be controlled to all orders in perturbation theory [8, 9, 10, 11]. Our current work focuses on next-to-leading-power (NLP) threshold logarithms, determined by the coefficients $c_{nm}^{(0)}$ in Eq. (1.1). These terms can give significant contributions to the observable when the predominant value of ξ, determined by kinematics, is small.

A number of techniques have been applied in recent years to study NLP logarithms, providing strong evidence that they can be organised to all orders in perturbation theory. Here, we will first briefly review some of these results, focusing in particular on Refs. [12, 13][2]. The main result of Ref. [13], a factorised expression for the Drell-Yan scattering amplitude obtained building on the construction of [6], is technically limited to abelian-like contributions to the cross section, characterised in QCD at two loops by the color factor C_F^2. In the last part of this contributions, we will briefly discuss the challenges of including purely non abelian contributions as well, and we will present a simple ansatz for the expected factorisation, which in principle applies, with suitable adaptations, to all processes involving parton annihilation into electroweak final states.

2. Towards NLP resummation

After the initial suggestion provided by the LBKD theorem, a number of partial results have contributed to the evidence that NLP logarithms can be understood to all orders in perturbation theory, and several different approaches have been applied to achieve increasingly refined results in this direction.

A first phenomenological analysis of the effect of NLP logarithms on a collider observable was performed in [16], in the case of the total cross section for Higgs boson production via gluon fusion. A somewhat more systematic approach was pursued, several years later, in Ref. [17]. As noted there, a set of simple modifications of the standard resummation formula for LP threshold logarithms for processes which are electroweak at tree level can account for an important subset of NLP contributions. More specifically, the suggestion of [17] is to incorporate in the standard resummation some natural refinements of the phase space analysis near threshold, including the choice of the scale of the running coupling. Furthermore, Ref. [17] implements a modification of the perturbative splitting functions which was originally suggested in Ref. [18]. Using as an example the Drell-Yan cross section, the proposal can be summarised by the following expression for the logarithm of the Mellin transform of the partonic cross section, $\widehat{\omega}(N,Q^2)$.

$$\ln\left[\widehat{\omega}(N,Q^2)\right] = F_{\rm DY}\left(\alpha_s\right) + \int_0^1 dz\, z^{N-1} \left\{ \frac{1}{1-z}\, D\left[\alpha_s\left(\frac{(1-z)^2 Q^2}{z}\right)\right] \right. \\ \left. + 2\int_{Q^2}^{(1-z)^2Q^2/z} \frac{dq^2}{q^2}\, P_s\left[z,\alpha_s(q^2)\right] \right\}_+ . \tag{2.1}$$

Here, the function $F_{\rm DY}\left(\alpha_s\right)$ is responsible for the resummation of N-independent terms, and can be determined order by order, following Ref. [10]. The function $D(\alpha_s)$ is the standard soft function of threshold resummation, and $P_s(z,\alpha_s)$ is the modified splitting function determined according to to [18]. As shown in [17], Eq. (2.1) is quite successful in reproducing NLP logarithms at higher

[2] See also [14, 15].

orders, based on lower-order perturbative calculations. It still however falls short of a complete resummation, as expected. Similar ideas were implemented also in a more recent analysis of Higgs production [19, 20], where the prediction was further refined by including all-order small-x information, corresponding to small N for the Mellin transform $\widehat{\omega}(N,Q^2)$. Significant results have also been obtained using the renormalization group invariance of physical kernels [21], a method which has been extended and applied also to Higgs boson production in [22]. Finally, preliminary results have been obtained in [23, 24] using soft-collinear effective theory, which appears to be a promising approach for a systematic treatment.

From our viewpoint, the problem of organising NLP logarithms involves a series of ingredients. First of all, one must push the eikonal approximation beyond leading power, accounting for next-to-soft emissions. This was achieved in Refs. [25, 26], at first using a first-quantized path-integral formalism, and then employing purely diagrammatic techniques. This methods lead to a set of corrections to the soft-collinear factorisation of scattering amplitudes described in [27], which can be shown to formally exponentiate. Next, one must include non-factorisable corrections, which, for massive QED, are described by Low's theorem. In the path-integral formalism of Ref. [25], these corrections show up as small translations of the Wilson lines orginating in the hard scattering and describing soft emissions at long distances. Finally, one must include the enhancements due to collinear singularities in the massless limit, which were first studied by Del Duca in [6]. At the level of abelian-like diagrams, the result of these manipulations is an expression for the radiative amplitude in terms of the non-radiative amplitude and a set of universal factors describing soft and collinear dynamics. It can be written as

$$\mathscr{A}^{\mu}(p_j,k) = \sum_{i=1}^{2} \Bigg\{ q_i \left(\frac{(2p_i - k)^{\mu}}{2p_i \cdot k - k^2} + G_i^{\nu\mu} \frac{\partial}{\partial p_i^{\nu}} \right) + G_i^{\nu\mu} \left[\frac{J_{\nu}(p_i,k,n_i)}{J(p_i,n_i)} - q_i \frac{\partial}{\partial p_i^{\nu}} \Big(\ln J(p_i,n_i) \Big) \right] \Bigg\} \mathscr{A}(p_i;p_j) . \tag{2.2}$$

Here the G tensor is part of a decomposition of the Minkowski metric according to

$$K_i^{\mu\nu} = \frac{(2p_i - k)^{\nu}}{2p_i \cdot k - k^2} k^{\mu} ; \qquad G_i^{\mu\nu} = \eta^{\mu\nu} - K_i^{\mu\nu} , \tag{2.3}$$

and it plays the role of a projector avoiding double counting between the first and subsequent terms in Eq. (2.2). The function J is the conventional jet of soft-collinear factorization, described and computed in [27]. For quark fields, it is defined by

$$J(p,n) u(p) = \langle 0 | \Phi_n(0,\infty) \, \psi(0) | p \rangle . \tag{2.4}$$

The radiative jet function J_μ, first introduced in [6], is defined for quarks as

$$J_{\mu}(p,n,k) \, u(p) = \int d^d y \, \mathrm{e}^{-\mathrm{i}(p-k)\cdot y} \left\langle 0 \middle| T \Big[\Phi_n(y,\infty) \, \psi(y) \, j_{\mu}(0) \Big] \middle| p \right\rangle , \tag{2.5}$$

where $j_\mu = \overline{\psi}\gamma_\mu \psi$ is the abelian quark current. The radiative jet is responsible for NLP logarithms arising from soft emissions inside collinear loops. In both Eq. (2.4) and Eq. (2.5), Φ_n is a Wilson line in direction n^μ, introduced to preserve gauge invariance. For general factorisation proofs, it is appropriate to take $n^2 \neq 0$ in order to avoid spurious collinear singularities originating from the Wilson line. In the case of Eq. (2.2), however, it is instrumental to consider the case $n^2 = 0$. Indeed, in this case one can take advantage of the renormalisation group invariance of the jet factor on the second line of Eq. (2.2), together with fact that for $n^2 = 0$ all radiative corrections vanish in dimensional regularisation for the bare non-radiative jet J, which thus equals unity to all orders. Eq. (2.2) then drastically simplifies to

$$\mathscr{A}^{\mu}(p_j,k) = \sum_{i=1}^{2}\left(q_i\,\frac{(2p_i-k)^{\mu}}{2p_i\cdot k-k^2}+q_i\,G_i^{\nu\mu}\,\frac{\partial}{\partial p_i^{\nu}}+G_i^{\nu\mu}J_{\nu}(p_i,k)\right)\mathscr{A}(p_i;p_j)\,, \tag{2.6}$$

Eq. (2.6) forms the basis for a study of NLP logarithms for any process not involving hard collinear final-state radiation, which must be treated separately. The radiative jet function is easily evaluated at tree level, yielding

$$J^{\nu(0)}(p,n,k) = -\frac{p^{\nu}}{p\cdot k}+\frac{k^{\nu}}{2p\cdot k}-\frac{\mathrm{i}\,k_{\alpha}\Sigma^{\alpha\mu}}{p\cdot k}\quad , \tag{2.7}$$

where Σ is a Lorentz generator in the quark spinor representation; at one-loop, and for $n^2 = 0$, it was evaluated in Ref. [13] wih the result

$$\begin{aligned} J^{\nu(1)}(p,n,k;\varepsilon) = (2p\cdot k)^{-\varepsilon}\Bigg[&\left(\frac{2}{\varepsilon}+4+8\varepsilon\right)\left(\frac{n\cdot k}{p\cdot k}\frac{p^{\nu}}{p\cdot n}-\frac{n^{\nu}}{p\cdot n}\right)-(1+2\varepsilon)\,\frac{\mathrm{i}\,k_{\alpha}\Sigma^{\alpha\nu}}{p\cdot k} \\ &+\left(\frac{1}{\varepsilon}-\frac{1}{2}-3\varepsilon\right)\frac{k^{\nu}}{p\cdot k}+(1+3\varepsilon)\left(\frac{\gamma^{\nu}\,\not{n}}{p\cdot n}-\frac{p^{\nu}\,\not{k}\,\not{n}}{p\cdot k\,p\cdot n}\right)\Bigg]+\mathscr{O}(\varepsilon^2,k)\,. \end{aligned}$$

As a powerful test of this framework, in Ref. [13] we reproduced all LP and NLP threshold logarithms arising in the NNLO Drell-Yan inclusive cross section (for the relevant C_F^2 color structure), using only one-loop information. Interestingly, even non-logarithmic terms at $\mathscr{O}(1/N)$ can be computed exactly, suggesting that the formalism is properly set up in the sense of a $1/N$ expansion. We note, however, that the definition of the radiative jet, Eq. (2.5), is specific to the abelian limit, since the current j_{μ} appearing in the matrix element does not include non-abelian corrections. This makes the task of extending our results to the full theory non-trivial, as we discuss in the next section.

3. A non-abelian diagram

The difficulties in generalising Eq. (2.2) to the full non-abelian theory are twofold. First of all, one must appropriately modify the definition of the radiative jet function, Eq. (2.5), in order to have a dynamical final state gluon: indeed, substituting the vector boson field with the fermion current in a matrix element is legitimate only in the abelian theory, where photons do not have self-interactions. Non-abelian NLP logarithms, however, receive contributions also from diagrams like the one displayed in Fig. 1, where the (next-to-)soft emitted gluon arises precisely from such a self-interaction. The diagram in Fig. 1 also illustrates the second difficulty: in fact, it is clear that this diagram has collinear enhancements in both the quark and the antiquark directions. Furthermore, power counting suggests that there will be a leading contribution to this diagram also from the soft momentum region. As shown by the analysis in Ref. [12], this did not happen for any of the

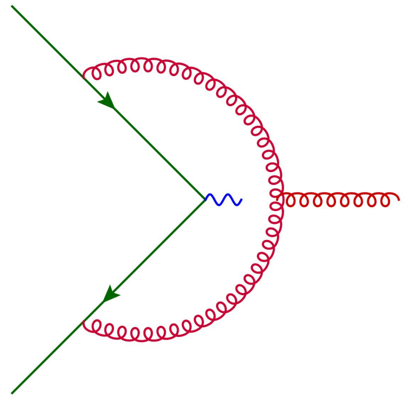

Figure 1: A non-abelian diagram contributing to all relevant momentum regions.

abelian diagrams discussed there. The presence of simultaneous soft and collinear enhancement will force us to introduce a new function, to avoid double counting, as was done in [27], and as briefly discussed below.

To be more precise, let us apply the method of regions [28] to the non-abelian diagram in Fig. 1, as was done for abelian-like diagrams in Ref. [12]. We are interested in momentum scalings corresponding to hard, soft, collinear and anti-collinear exchanges: picking the quark and antiquark momenta p_1 and p_2 along the $+$ and $-$ light-cone directions respectively, the relevant regions are characterized, in light-cone coordinates, as

$$\begin{aligned} &\text{Hard}: \quad l^\mu \sim \sqrt{\hat{s}}\,(1,1,1)\,, \qquad \text{Soft}: \quad l^\mu \sim \sqrt{\hat{s}}\left(\lambda^2,\lambda^2,\lambda^2\right)\,, \\ &\text{Collinear}: \quad l^\mu \sim \sqrt{\hat{s}}\left(1,\lambda,\lambda^2\right)\,, \quad \text{Anticollinear}: \quad l^\mu \sim \sqrt{\hat{s}}\left(\lambda^2,\lambda,1\right)\,, \end{aligned} \tag{3.1}$$

where λ is a dimensionless rescaling parameter. It is possible to evaluate directly the contribution of each momentum region from the diagram in Fig. 1 (and its complex conjugate) to the NNLO Drell-Yan K-factor

$$K^{(2)}(z) = \frac{1}{\sigma_0}\,\frac{d\sigma^{(2)}(z)}{dz}\,, \tag{3.2}$$

by simply contracting the diagram with the Born amplitude and dividing by the Born cross section σ_0. One verifies that this diagram, at this order, is the only diagram with a non-vanishing contribution from the soft momentum region at NLP level. The contribution is given by

$$\begin{aligned} K^{(2)}(z)\big|_{\rm soft} = \left(\frac{\alpha_s}{4\pi}\right)^2 C_A C_F \Bigg[&\frac{8\mathscr{D}_0(z)-8}{\varepsilon^3} - \frac{32\mathscr{D}_1(z)-32\log(1-z)+16}{\varepsilon^2} \\ &+ \frac{64\mathscr{D}_2(z)-64\log^2(1-z)+64\log(1-z)}{\varepsilon} \\ &- \frac{256}{3}\mathscr{D}_3(z) + \frac{256}{3}\log^3(1-z) - 128\log^2(1-z)\Big)\Bigg]\,, \end{aligned} \tag{3.3}$$

up to corrections $\mathscr{O}(\varepsilon,(1-z))$, and where we neglected transcendental constants for brevity. Any non-abelian radiative jet definition will contain soft contributions at NLP from non-abelian diagrams with the structure of Fig. 1; furthermore, the same diagram, with fermion lines replaced with

Wilson lines, will arise also from the radiative soft function which must be included in a complete factorisation, and which was reabsorbed in the hard function for the purpose of deriving Eq. (2.2). Along the lines of Ref. [27], it will then be necessary to introduce a Wilson-line version of the non-abelian radiative jet function to subtract the double counting of soft-collinear regions.

There are several ways to proceed: one possibility is to follow the original definition of the radiative jet, Eq. (2.5), introducing an appropriate non-abelian generalisation of the fermion current, as suggested in [13]. The definition of the non-abelian current is, however, not unique, and the gauge dependence of the result is more intricate, since such a current is covariantly conserved, so that the corresponding Ward identity contains non-linear corrections. An alternative, more physically motivated possibility is to define radiative jet functions, as well as radiative soft functions, in terms of matrix elements with a real final state gluon, avoiding the introduction of the current altogether. This has the advantage that such matrix elements can be directly understood as single-particle contributions to cross-section level quantities. A set of definitions along these lines is

$$\begin{aligned} \varepsilon^*_{(\lambda)}(k)\cdot J(p,k,n)u_{(s)}(p) &= \langle k,\lambda\,|\Phi_n(0,\infty)\,\psi(0)|\,p,s\rangle\,. \\ \varepsilon^*_{(\lambda)}(k)\cdot \mathscr{J}(\beta,k,n) &= \big\langle k,\lambda\,\big|\Phi_n(0,\infty)\,\Phi_\beta(\infty,0)\big|\,0\big\rangle\,, \\ \varepsilon^*_{(\lambda)}(k)\cdot W(\beta_1,\beta_2,k) &= \big\langle k,\lambda\,\big|\Phi_{\beta_1}(0,\infty)\,\Phi_{\beta_2}(\infty,0)\big|\,0\big\rangle\,. \end{aligned} \tag{3.4}$$

where β is the four-velocity vector associated with momentum p, according to $p^\mu = Q\beta^\mu$, $J(p,k,n)$ is the new, non-abelian, radiative jet function, $\mathscr{J}(\beta,k,n)$ is the eikonal counterpart of J, needed to subtract double countings, and $W(\beta_1,\beta_2,k)$ is a radiative soft function, describing the emission of a (next-to-)soft gluon from a pair of Wilson lines directed along the classical fermion trajectories. The functions defined in Eq. (3.4) are all obviously transverse. Furthermore, the Wilson lines along the physical directions β_1 and β_2 can easily be promoted to next-to-eikonal accuracy following Refs. [25, 26]. To check the viability of this option, we focussed once again on the case of light-like factorisation vectors n^μ, where a number of contributions vanish order by order for eikonal quantities. A simple and natural generalization of Eq. (2.6) is then given by

$$\mathscr{A}^{\mu a}(p_j,k) = \sum_{i=1}^{2}\left(\frac{1}{2}W^{\mu a} + \mathbf{T}_i^a G_i^{\nu\mu}\,\frac{\partial}{\partial p_i^\nu} + J^{\mu a}(p_i,k,n_i) - \tilde{\mathscr{J}}^{\mu a}(\beta_i,k,n_i)\right)\mathscr{A}(\{p_i\})\,. \tag{3.5}$$

At the two-loop level, for real-virtual contributions to the Drell-Yan cross section, we have verified that Eq. (3.5), with the definitions given by Eq. (3.4), does indeed reproduce all non-abelian NLP threshold logarithms that were missed by Eq. (2.6). This, however, still falls short of an all-order proof: indeed, the arguments leading to Eq. (2.2) do not readily generalise to the present definitions, since a transverse radiative jet is not related by a Ward identity to the non-radiative jet, as was the case for the quasi-abelian radiative jet defined in Eq. (2.5). A complete proof of Eq. (3.5), or rather of its generalisation to the generic $n^2 \neq 0$ case, must then rely upon the cancellation of the dependence on the factorisation vectors n^μ between the various contributions to the physical scattering amplitude. Work in this direction is in progress.

4. Perspective

We have described recent progress towards a complete factorisation of scattering amplitudes for parton annihilation into electroweak final states, identifying all sources of next-to-leading-power threshold logarithms for the corresponding cross sections. This is a necessary preliminary step to establish a complete resummation formalism for these logarithms, which will contribute to the construction of precise and controlled predictions for a number of processes of interest for LHC and future hadron colliders. We note in particular that the ansatz preliminarily described here for non-abelian effects in Drell-Yan production, if confirmed, would immediately lead to the development of an analogous formalism for Higgs production via gluon fusion: indeed, in that case, one would need to construct a radiative gluon jet function, describing collinear radiation off a hard gluon, which is intrinsically non-abelian already at Born level. The availability of three-loop results for the Higgs cross section, thanks to the remarkable results of Refs. [29, 30, 31], will provide a very significant test both of the correctness and of the impact of the results of this generalisation. A further extension of the formalism to processes with final state jets, such as DIS and electron-positron annihilation, and ultimately jet production in hadron-hadron scattering, must await a deeper understanding of power-suppressed collinear effects due to hard collinear radiation.

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