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Neutrino Theory: Mass, Interactions, Connections

Ernset Ma[*†]
University of California, Riverside
E-mail: ma@phyun8.ucr.edu

I review neutrino theory in the context of a number of topics, including the many possible mass-generating mechanisms and their associated interactions. I begin with a perspective of the development of the standard model of quarks and leptons. The understanding of how they acquire mass in the context of $SU(2)_L \times U(1)_Y$ gauge symmetry predicted the Z boson as well as the Higgs boson h. Neutrinos are special because they could be Majorana fermions, and many mechanisms are possible for them to acquire mass. I review a variety of such mechanisms and what they imply in terms of new particles and new interactions. I also touch upon leptogenesis and cofactor zeros.

The presence of neutrino mass may be connected with other phenomena in particle physics beyond the standard model, such as new gauge interactions, dark matter, and strong CP. The family structure of neutrinos may be indicative of some underlying flavor symmetry, such as the non-Abelian discrete A_4 symmetry of the tetrahedron. The notion that neutrino masses are generated radiatively in one loop through dark matter is also an active area of research in recent years. This is the so-called scotogenic mechanism, from the Greek 'scotos' meaning darkness. Finally the possible unification of dark matter with matter is briefly mentioned.

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1. Neutrino Mass in Perspective

To understand neutrino mass, let us think about the other fermion masses that we know about. Consider the original 1967 Weinberg model of leptons. The gauge group is $SU(2)_L \times U(1)_Y$ with one Higgs doublet $(\phi^+, \phi^0) \sim (2, 1/2)$ and the lepton content is

$$(\nu_e, e)_L \sim (2, -1/2), \quad e_R \sim (1, -1), \tag{1.1}$$

$$(\nu_\mu, \mu)_L \sim (2, -1/2), \quad \mu_R \sim (1, -1). \tag{1.2}$$

Recall that in 1967, we only knew about the two families of leptons. As ϕ^0 acquires a nonzero vacuum expectation value v, the charged leptons obtain masses through $\bar{e}_L e_R \phi^0$ and $\bar{\mu}_L \mu_R \phi^0$. Neutrinos are massless and their interactions are limited to

$$\mathscr{L}_\nu = \frac{g}{\sqrt{2}} \bar{\nu}_L \gamma^\mu l_L W^+_\mu + H.c. + \frac{g}{2\cos\theta_W} \bar{\nu}_L \gamma^\mu \nu_L Z_\mu. \tag{1.3}$$

The new particles are $W^\pm$, Z, and the one physical Higgs boson $h = \sqrt{2}[Re(\phi^0) - v]$. The $W^\pm$ bosons are the mediators of the known charged-current weak interactions responsible for nuclear β decay, so we know that they should be there. On the other hand, the neutral Z boson is associated with neutral-current interactions. They are not so easy to see in charged-particle interactions because they hide under the much more dominant electromagnetic interactions. However, the neutrino interactions predicted by Eq. (1.3) could be searched for. They were indeed observed in 1973, thus making the Weinberg model so much more convincing, and excluding for example the Georgi-Glashow model where there is no Z. The physical $W^\pm$ and Z bosons were then discovered in 1983. As for the Higgs boson h, that had to wait until 2012.

The 1967 Weinberg model did not deal with quarks. Since there were only three known quarks (u, d, s) at the time, and the charged current was supposed to be given by

$$J^\mu_u = \bar{u}_L \gamma^\mu (\cos\theta_C d_L + \sin\theta_C s_L), \tag{1.4}$$

where θ_C is the Cabibbo angle, the Z boson would then couple to $\bar{d}_L \gamma^\mu s_L$, which is called a flavor-changing neutral current, and known to be highly suppressed experimentally. To solve this, the charm quark c was invented with

$$J^\mu_c = \bar{c}_L \gamma^\mu (-\sin\theta_C d_L + \cos\theta_C s_L), \tag{1.5}$$

which makes the Z couplings diagonal in flavor and be in good agreement with experiment. This is the Glashow-Iliopoulos-Maiani mechanism. In 1974, the J/ψ particle was discovered, which proved to be a $\bar{c}c$ bound state. The lesson we learn is that every theoretical invention has experimental consequences. It is only accepted after being proven correct. The standard model (SM) of quarks and leptons was born.

I now come to the topic of neutrino mass. Neutrinos are massless in the standard model because ν_L is part of a doublet and there is no corresponding singlet. Suppose physics beyond the SM occurs at mass scales much greater than the electroweak breaking scale v, then Weinberg showed in 1979 that there is a unique dimension-five operator for Majorana neutrino mass [1]:

$$\mathscr{L}_5 = \frac{f_{\alpha\beta}}{2\Lambda} (\nu_\alpha \phi^0 - l_\alpha \phi^+)(\nu_\beta \phi^0 - l_\beta \phi^+) + H.c. \tag{1.6}$$

Hence $\mathscr{M}_\nu = f_{\alpha\beta} v^2/\Lambda$. This shows that a seesaw is always obtained for $v << \Lambda$.

There are of course important exceptions. If new physics occurs at scales lower than v, then other possibilties exist. For example, if there are singlet neutral fermions N_R, often called right-handed neutrinos or sterile neutrinos, then the interaction $f\bar{N}_R \nu_L \phi^0 + H.c.$ is allowed and ν_L pairs up with N_R to form a Dirac fermion of mass $fv << v$. This assumes that the $N_R N_R$ Majorana mass term is forbidden. To do so, a global $U(1)$ lepton number symmetry L may be imposed. If these mass terms are nonzero, then L is broken to $(-1)^L$. If they are also much smaller than v, then a possible 6×6 neutrino mass matrix exists, with many phenomenological implications. In these lectures, I will take the conventional view that only the known three neutrinos are light.

2. Early Specific ideas

If N_R is added with m_N very large, then

$$\begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix} \Rightarrow m_\nu \simeq \frac{-m_D^2}{m_N}. \tag{2.1}$$

This is the famous original seesaw mechanism [2] (coined by Yanagida) proposed in 1979. It is the most minimal addition to the SM, implying only the interaction $f\bar{N}_R \nu_L \phi^0$ which generates $m_D = fv$, but even that has important consequences: see leptogenesis later.

Even without N_R, it is possible to make ν_L massive. If a Higgs triplet (ξ^{++}, ξ^+, ξ^0) is added with $\langle \xi^0 \rangle = u$, then the coupling $(f/2)\xi^0 \nu\nu$ implies $m_\nu = fu$. This scalar triplet mechanism [3], first proposed in 1980, requires new scalar particles with gauge interactions as well as

$$\begin{aligned} V &= m^2 \Phi^\dagger \Phi + M^2 \xi^\dagger \xi + \frac{1}{2}\lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2}\lambda_2 (\xi^\dagger \xi)^2 + \lambda_3 |2\xi^{++}\xi^0 - \xi^+\xi^+|^2 \\ &+ \lambda_4 (\Phi^\dagger \Phi)(\xi^\dagger \xi) + \frac{1}{2}\lambda_5 [|\sqrt{2}\xi^{++}\xi^- + \xi^+ \bar{\phi}^0|^2 + |\xi^+ \phi^- + \sqrt{2}\xi^0 \bar{\phi}^0|^2] \\ &+ \mu(\bar{\xi}^0 \phi^0 \phi^0 + \sqrt{2}\xi^- \phi^0 \phi^- + \xi^{--}\phi^+\phi^+) + H.c. \end{aligned} \tag{2.2}$$

If $\mu = 0$, then ξ may be assigned lepton number $L = -2$, and $u \neq 0$ breaks it spontaneously, resulting in a massless Goldstone boson (triplet majoron) which was ruled out soon after the discovery of Z from its invisible decay, because it would add to it the equivalent of two light neutrino species. The minimum of V is given by

$$0 = m^2 + \lambda_1 v^2 + (\lambda_4 + \lambda_5) u^2 + 2\mu u, \tag{2.3}$$

$$0 = u[M^2 + \lambda_2 u^2 + (\lambda_4 + \lambda_5) v^2] + \mu v^2. \tag{2.4}$$

For $\mu \neq 0$, another solution exists [4], i.e. $u \simeq -\mu v^2/[M^2 + (\lambda_4 + \lambda_5)v^2]$, where $v^2 \simeq -m^2/\lambda_1$. Note that this is also a seesaw formula for $v^2 << M^2$.

If a charged singlet scalar χ^+ and a second scalar doublet (ϕ_2^+, ϕ_2^0) are added, then a one-loop m_ν is generated, as shown in Fig. 1. This important radiative mechanism [5] for neutrino mass was proposed in 1980 and implies new gauge and other interactions for these new scalar particles.

In 1989, it was proposed that a fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^-)_R$ be added [6] instead of N_R, then $\bar{\Sigma}_R^0 \nu_L \phi^0$ also implies m_D, and $m_\nu \simeq -m_D^2/m_\Sigma$. There are now new particles $\Sigma^\pm$ with gauge interactions.

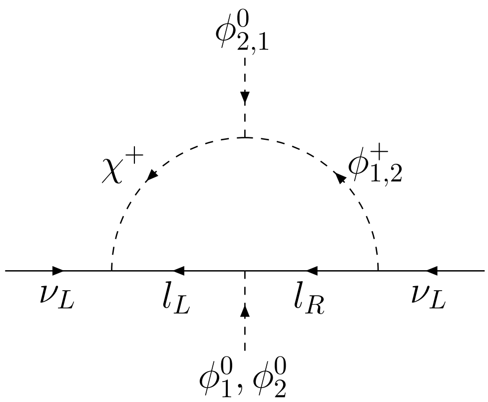

Figure 1: One-loop generation of neutrino mass with a charged scalar singlet.

Activity in neutrino theory research picked up in 1998. The known three tree-level mechanisms were collected together within the same context and shown to be the only ones, and the three generic one-loop mechanisms were also first obtained [7]. The Weinberg operator $\mathscr{L}_5$ of Eq. (1.6) was shown to have three and only three tree-level realizations as shown in Fig. 2.

- (I) fermion singlet N_R (1979),
- (II) scalar triplet (ξ^{++}, ξ^+, ξ^0) (1980),
- (III) fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^-)$ (1989).

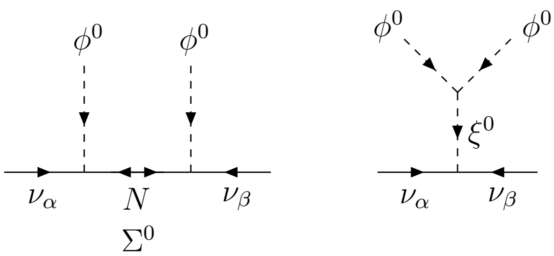

Figure 2: Three tree-level realizations of the Weinberg operator.

The nomenclature of Type I,II,III seesaw [7] was then established. In each case, $\mathscr{L}_5$ is the same:

- (I) $(\phi^0 \nu_i - \phi^+ l_i)(\phi^0 \nu_j - \phi^+ l_j)$,
- (II) $\phi^0 \phi^0 \nu_i \nu_j - \phi^+ \phi^0 (\nu_i l_j + l_i \nu_j) + \phi^+ \phi^+ l_i l_j$,
- (III) $(\phi^0 \nu_i + \phi^+ l_i)(\phi^0 \nu_j + \phi^+ l_j) - 2\phi^+ \nu_i \phi^0 l_j - 2\phi^0 l_i \phi^+ \nu_j$.

It was also shown that there are three generic one-loop realizations as shown in Figs. 3 to 5.

- (R1) one external Higgs line each attached to the internal scalar (fermion) line [5],
- (R2) two external Higgs lines attached to the internal scalar line [8],
- (R3) two external Higgs lines attached to the internal fermion line [9].

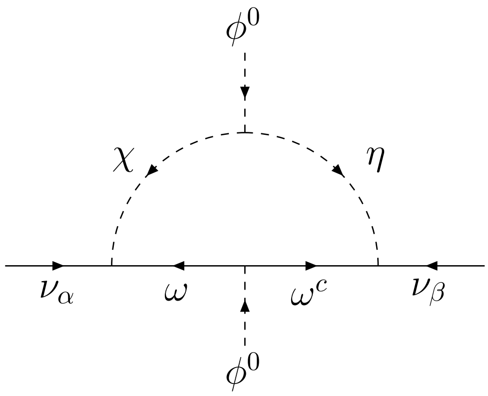

Figure 3: One-loop mechanism (R1) for generating neutrino mass.

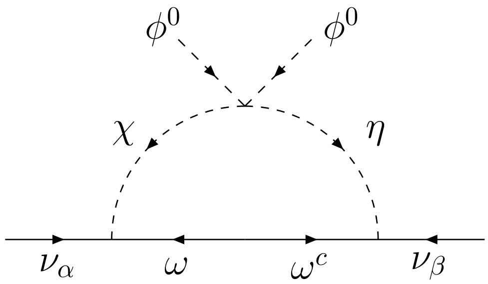

Figure 4: One-loop mechanism (R2) for generating neutrino mass.

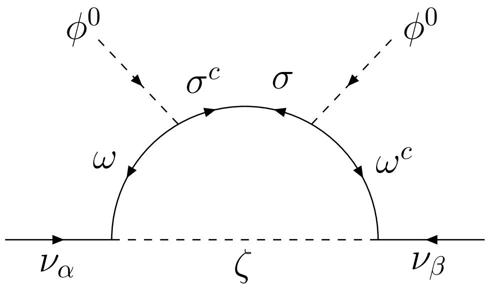

Figure 5: One-loop mechanism (R3) for generating neutrino mass.

Later in June 1998, SuperK announced at the Neutrino 98 Conference in Japan that atmospheric neutrino oscillations were firmly established. Headline news around the world appeared:

“NEUTRINOS HAVE MASS !!″

In 2002, SNO established solar neutrino oscillations. With two different mass-squared differences needed to explain the two kinds of oscillations, we know that at least two neutrinos must have nonzero masses. Absent of other information, how neutrinos get their mass is still unknown. In fact, we still do not know if its mass is Dirac or Majorana (assumed in these lectures), without a positive signal from neutrinoless double beta decay.

3. Seesaw Variants

Neutrino mass literature used to be almost exclusively dominated by the Type I seesaw, but after 1998 and more so after 2006, other ideas are being discussed with greater frequency. Nevertheless, Type I seesaw is the simplest idea, but basically impossible to prove. On the other hand, there are variants which may offer some hope of experimental verification. Neutrino mass generation may also be connected with other phenomena, such as dark matter, flavor, and the strong CP problem.

With 1 doublet neutrino ν and 1 singlet neutrino N, their 2×2 mass matrix is the well-known

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix}, \tag{3.1}$$

resulting in the famous seesaw formula $m_\nu \simeq -m_D^2/m_N$. Hence $\nu - N$ mixing is given by $m_D/m_N \simeq \sqrt{|m_\nu/m_N|} < 10^{-6}$, for $m_\nu < 1$ eV and $m_N > 1$ TeV. Since N has no other interaction, the only way that it can be produced is through its mixing with ν. It is basically hopeless in the case of Type I seesaw.

Consider now 1 ν and 2 singlets $N_{1,2}$. Their 3×3 mass matrix is then

$$\mathcal{M}_{\nu n} = \begin{pmatrix} 0 & m_D & 0 \\ m_D & m_1 & m_N \\ 0 & m_N & m_2 \end{pmatrix}, \tag{3.2}$$

resulting in $m_\nu \simeq m_D^2 m_2/(m_N^2 - m_1 m_2)$. Since the limit $m_1 = 0$ and $m_2 = 0$ corresponds to lepton number conservation ($L = 1$ for ν and N_2, $L = -1$ for N_1), their smallness is "natural". This is called the inverse seesaw [10], first proposed in 1986. Here $\nu - N_1$ mixing is $\sim m_\nu/m_D$, whereas $\nu - N_2$ mixing is $\sim m_D/m_N$, and they are not constrained to be the same as was in the canonical seesaw. For example, let $m_D \sim 10$ GeV, $m_N \sim 1$ TeV, $m_2 \sim 10$ keV, then $m_\nu \sim 1$ eV, and $\nu - N_1$ mixing $\sim 10^{-10}$ is very small, but $\nu - N_2$ mixing $\sim 10^{-2}$ is large enough to be observable [11]. Note that the geometric mean of the two mixings is again 10^{-6} as before.

Another variant is the linear seesaw [12] proposed in 2004:

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & m_D & m'_D \\ m_D & 0 & m_N \\ m'_D & m_N & 0 \end{pmatrix}, \tag{3.3}$$

resulting in $m_\nu \simeq -2m_D m'_D/m_N$, which is only linear in m_D. However, for the linear seesaw to work, m'_D must be very small. In the limit it is zero, $\mathcal{M}_{\nu N}$ is the same as that of the inverse seesaw in the conserved lepton number limit, so they must have the same origin. To prove this [11], let $m'_D/m_D = \tan\theta$, then rotate the (N_1, N_2) basis by θ, and we get

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & m_D/c & 0 \\ m_D/c & m_N s_2 & m_N c_2 \\ 0 & m_N c_2 & -m_N s_2 \end{pmatrix}, \tag{3.4}$$

where $c = \cos\theta$, $c_2 = \cos 2\theta$, and $s_2 = \sin 2\theta$. For small θ, this is just the inverse seesaw, with

$$m_\nu \simeq \frac{-2m_N m'_D}{m_D}\left(\frac{m_D^2}{m_N^2}\right) = -2m_D m'_D/m_N. \tag{3.5}$$

If the inverse or linear seesaw mechanisms are invoked to get large $\nu - N$ mixing for each family, we will need 2 singlet neutrinos for each doublet neutrino. Is that really necessary? Consider two families with just $\nu_{1,2}$ and $N_{1,2}$. Let $\mathcal{M}_N = diag(M'_1, M'_2)$, and the 2×2 matrix $\mathcal{M}_D$ linking $\nu_{1,2}$ to $N_{1,2}$ be of the special form

$$\mathcal{M}_D = \begin{pmatrix} a_1 b_1 & a_1 b_2 \\ a_2 b_1 & a_2 b_2 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} (b_1 \; b_2). \tag{3.6}$$

One neutrino mass is zero because the determinant of $\mathcal{M}_D$ is zero by construction. If we now also impose the arbitrary condition [13]

$$\frac{b_1^2}{M'_1} + \frac{b_2^2}{M'_2} = 0, \tag{3.7}$$

then the other neutrino mass is zero as well. However, $\nu - N$ mixing may be large, because $a_i b_j$ need not be small. In other words, zero mixing is now not the limit of zero neutrino mass as in the previous seesaw formulas. If small deviations from this texture are present, small neutrino mass will appear, but the large $\nu - N$ mixing will remain. This is fine tuning, but a lot of phenomenological studies have been done in this context.

The addition of heavy singlet fermions N_i to the SM induces both $\mathcal{L}_5$ which causes $\nu - N$ mass mixing, and a dimension-six operator [14]

$$\mathcal{L}_6 = \Lambda^{-2} f_{\alpha\beta} (\Phi^\dagger \bar{L}_\alpha) i\partial^\mu \gamma_\mu (L_\beta \Phi), \tag{3.8}$$

which causes kinetic mixing. Both operators will probe m_D/m_N. Let the neutrino mixing matrix be $(1+\eta)U$, then the deviations from unitarity are bounded roughly by [15]

$$|\eta| < \begin{pmatrix} 2.0\times 10^{-3} & 6.0\times 10^{-5} & 1.6\times 10^{-3} \\ \sim & 8.0\times 10^{-4} & 1.1\times 10^{-3} \\ \sim & \sim & 2.7\times 10^{-3} \end{pmatrix}. \tag{3.9}$$

For the texture hypothesis for three families, i.e.

$$\mathcal{M}_D = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} (b_1 \; b_2 \; b_3), \tag{3.10}$$

with the condition

$$\frac{b_1^2}{M'_1} + \frac{b_2^2}{M'_2} + \frac{b_3^2}{M'_3} = 0, \tag{3.11}$$

the parameters $\eta_{e\tau}$ and $\eta_{\mu\tau}$ are then correlated [16]:

$$\left|\frac{\eta_{e\tau}}{1.6\times 10^{-3}}\right|^2 + \left|\frac{\eta_{\mu\tau}}{1.1\times 10^{-3}}\right|^2 < 1. \tag{3.12}$$

The texture hypothesis actually has a symmetry origin [17], first shown in 2009. The 4×4 $\nu - N$ mass matrix may be rotated to become

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & 0 & m_1 & m_2 \\ 0 & 0 & 0 & m_2 \\ m_1 & 0 & M_1 & M_3 \\ 0 & m_2 & M_3 & M_2 \end{pmatrix}. \tag{3.13}$$

In other words, the 2×2 $\mathcal{M}_D$ is now diagonal, whereas $\mathcal{M}_N$ is arbitrary. Note that $m_1 = 0$ and $M_1 = 0$ here would result in two massless neutrinos. These are in fact the two equivalent conditions expressed by Eqs. (3.6) and (3.7). In other words, the obscure origin of the two zeros are now made manifest. The two massless neutrinos in this basis are ν_1, and

$$\nu_2' = \frac{M_3\nu_2 - m_2 N_1}{\sqrt{M_3^2 + m_2^2}}, \tag{3.14}$$

showing explicitly how the large mixing occurs between ν_2 and N_1. If the usual lepton number conservation is used to try to obtain this form, it would not only forbid M_1 but also M_2, which is arbitrary here. So what is the symmetry which maintains this texture?

The answer is generalized lepton number L under which $\nu_{1,2} \sim 1$, but $N_1 \sim 3$ and $N_2 \sim -1$. In addition to the usual Higgs doublet (ϕ^+, ϕ^0) with $L = 0$, we now need a Higgs singlet χ_2 with $L = 2$. Then m_2 comes from $\langle \phi^0 \rangle$, M_2 comes from $\langle \chi_2 \rangle$, and M_3 from $\langle \chi_2^* \rangle$. This symmetry renders $m_1 = 0$ at tree level, because there is no Higgs doublet with $L = -4$, and also $M_1 = 0$ at tree level, because there is no Higgs singlet with $L = \pm 6$. In one loop, M_1 will be induced, thus giving ν_2' an inverse seesaw mass = $M_1 m_2^2/M_3^2$, as shown in Fig. 6. Once ν_2' is massive, ν_1 also gets a two-loop

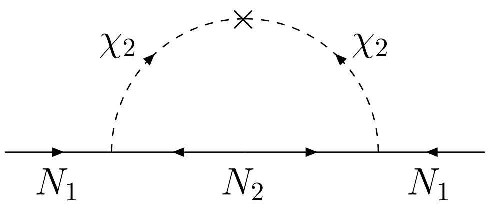

Figure 6: One-loop generation of M_1.

radiative mass [18] from the exchange of two Ws, as shown in Fig. 7.

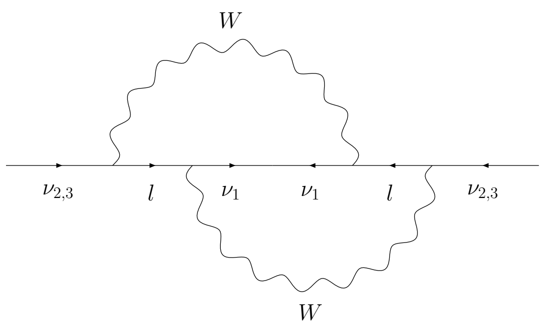

Figure 7: Two-loop generation of neutrino mass through W exchange.

With small m_1 and M_1, the 4×4 neutrino mass matrix of Eq. (3.13) is reduced to the 2×2

$$\mathcal{M}_\nu \simeq \begin{pmatrix} m_1^2 M_2/M_3^2 & -m_1 m_2/M_3 \\ -m_1 m_2/M_3 & M_1 m_2^2/M_3^2 \end{pmatrix}. \tag{3.15}$$

Since $M_2 \sim M_3$ in this hypothesis, the (1,1) entry is a canonical seesaw, whereas the (2,2) entry is an inverse seesaw, and the (1,2) or (2,1) entry is a linear seesaw. In this basis, only ν_2 has possible large mixing with N. Similarly, if 3 families are considered with 3 N, only one linear combination of the 3 ν may mix significantly with N.

For three families, let

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & 0 & 0 & m_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_3 \\ m_1 & 0 & 0 & M_1 & M_4 & M_5 \\ 0 & m_2 & 0 & M_4 & M_2 & M_6 \\ 0 & 0 & m_3 & M_5 & M_6 & M_3 \end{pmatrix}. \tag{3.16}$$

The texture hypothesis is equivalent to $m_1 = m_2 = 0$ and $M_1 = M_4 = 0$. To enforce this patterm at tree level, use $L = 1$ for $\nu_{1,2,3}$ as usual, but $L = 3, -2, -1$ for $N_{1,2,3}$. Add one Higgs doublet with $L = 0$ and three Higgs singlets with $L = 2, 3, 4$. Then $M_1 = M_4 = 0$ at tree level, but will become nonzero in one loop. Now $m_1 = m_2 = 0$ to all orders. To obtain nonzero $m_{1,2}$, a second Higgs doublet with $L = -4$ or $L = 1$ may be added, them ν_1 or ν_2 becomes massive in one loop and ν_2 or ν_1 in two loops. Lepton number has been used as a global $U(1)$ symmetry, but a discrete version (e.g. Z_7) also works. The global $U(1)$ may be gauged, using either $U(1)_{B-L}$ or $U(1)_\chi$ from E_6 where $Q_\chi = 5(B-L) - 4Y$. Many possible signatures of these extensions may be searched for at the Large Hadron Collider (LHC). For example, with $U(1)_\chi$, it is possible to produce N in pairs through $q\bar{q} \to Z' \to N\bar{N}$, if kinematically allowed. The final states to be analyzed are $l^\pm l^\mp W^\pm W^\mp$ and $l^\pm l^\pm W^\mp W^\mp$. Without new physics, N is not likely to be observable.

4. Leptogenesis

In these lectures, I assume m_ν is Majorana and $(-)^L$ is conserved. What is the mass scale of the new physics using $\mathcal{L}_5$?

$$\begin{aligned} m_\nu &\sim \frac{f^2 v^2}{\Lambda} \sim 1 \text{ eV}, \\ \Rightarrow \frac{\Lambda}{f^2} &\sim \frac{(100 \text{ GeV})^2}{1 \text{ eV}} \sim 10^{13} \text{ GeV}. \end{aligned} \tag{4.1}$$

If $f \sim 1$, then $\Lambda \sim 10^{13}$ GeV. This high scale is suitable for leptogenesis. For every seesaw mechanism, there is a leptogenesis scenario for the baryon asymmetry of the Universe.

- (I) [Fukugita,Yanagida (1986)] $N_1 \to l^\pm \phi^\mp$ at tree level interfering with one-loop (vertex and self-energy) amplitudes involving N_2 with CP violation.

- (II) [Ma,Sarkar (1998)] $\xi_1^{\pm\pm} \to l^\pm l^\pm, \phi^\pm \phi^\pm$ at tree level interferinmg with a one-loop (self-energy) amplitude involving ξ_2 with CP violation.

The lepton asymmetry generated is converted by sphalerons during the electroweak phase transition to the present observed baryon asymmetry of the Universe.

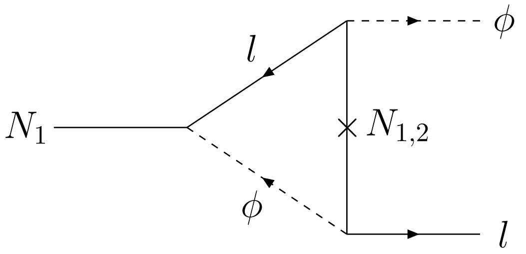

Figure 8: Type I leptogenesis through N decay.

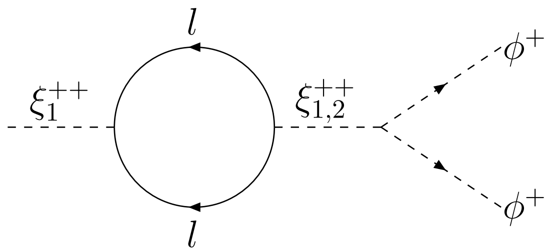

Figure 9: Type II leptogenesis through $\xi^{\pm\pm}$ decay.

5. Cofactor Zeros

If m_N is indeed very heavy, is there any way to know that Type I seesaw is the trye origin of neutrino mass? A possible answer is the existence of cofactor zeros in the observed neutrino mass matrix $\mathcal{M}_\nu$, which would be the consequence of zeros in $\mathcal{M}_N$ provided that $\mathcal{M}_D$ is diagonal [19], as poropsed in 2005. There is now enough experimental data to know that three patterns of cofactor zeros exist [20] which are consistent with best-fit neutrino-oscillation parameter values.

To put such an idea on firmer theoretical grounds, consider the anomaly-free family gauge symmetry $\sum_i x_i L - i$ with $\sum_i x_i = 0$, or $B - \sum_i y_i L_i$ with $\sum_i y_i = 3$. For example, let $x_i = (1,2,-3)$, then $\mathcal{M}_D$ is diagonal, and $\mathcal{M}_N$ has the $(x_i x_j)$ structure

$$\begin{pmatrix} 2 & 3 & -2 \\ 3 & 4 & -1 \\ -2 & -1 & -6 \end{pmatrix}. \tag{5.1}$$

If singlet scalars transforming as $(2,3,4)$ are used, then the (23),(32),(33) entries are zero, and two cofactor zeros will appear in $\mathcal{M}_\nu$.

Note that the special cases $L_e - L_\mu$, $L_e - L_\tau$, $L_\mu - L_\tau$, as well as $B - L_e - L_\mu - L_\tau$, $B - 3L_e$, $B - 3L_\mu$, $B - 3L_\tau$ have all been studied in the past. The $U(1)$ gauge boson Z' corresponding to $\sum_i x_i L_i$ or $B - \sum_i y_i L_i$ has good discovery reach at the LHC and is being studied. In fact, the Type I and III seesaw mechanisms would be easier to verify if N or $(\Sigma^+, \Sigma^0, \Sigma^-)$ couple to Z', i.e. $B - L$ or an unusual anomaly-free $U(1)$. An example of the latter is $e_R \sim -1$ and all others $\sim +1$.

6. Radiative Seesaw from Dark Matter

If a second scalar doublet (η^+, η^0) is added to the SM such that it is odd under a new exactly conserved Z_2 discrete symmetry [Deshpande, Ma (1978)], then η^0_R or η^0_I is absolutely stable. This simple idea for dark matter lay dormant for almost 30 years until it was used also to generate neutrino mass [Ma (2006)] by adding three neutral singlet fermions N which are also odd under Z_2. This is known as the scotogenic mechanism, from the Greek 'scotos' meaning darkness. Since N is odd, there is no $\nu N \phi^0$ coupling, but $h\nu N\eta^0$ as well as $(\lambda_5/2)(\Phi^\dagger\eta)^2 + H.c.$ are allowed, thus realizing the $R2$ radiative mechanism already known since 1998. The λ_5 term splits the mass of

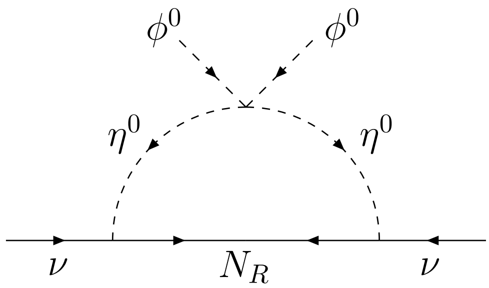

Figure 10: Radiative seesaw neutrino mass: the scotogenic mechanism.

$\eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$ so that $m_R^2 - m_I^2 = 2\lambda_5 v^2$. The one-loop neutrino mass is finite and given by

$$(\mathcal{M}_\nu)_{\alpha\beta} = \sum_i \frac{h_{\alpha i} h_{\beta i} M_i}{16\pi^2}[f(M_i^2/m_R^2) - f(M_i^2/m_I^2)], \tag{6.1}$$

where $f(x) = \ln x/(x-1)$. The splitting of $m_{R,I}$ makes this nonzero and finite. It also solves the problem of the direct detection of $\eta_{R,I}$ through Z excahnge with nuclei. Since Z_μ couples to $i(\eta_R\partial^\mu\eta_I - \eta_I\partial^\mu\eta_R)$, a mass gap of just a few hundred keV is enough to forbid its elastic scattering in underground dark-matter search experiments using nuclear recoil. However, the $h(\eta_R^2 + \eta_I^2)$ coupling remains. It will allow $\eta_{R,I}$ to be discovered in the next generation of detectors.

The linkage of neutrino mass to dark matter provides an important clue to the scale of new physics. It is a possible answer to the question: Is the new physics responsible for neutrino mass also responsible for some other phenomenon in particle physics and astrophysics? Here the answer is yes, and it is dark matter. Since dark matter is mostly assumed to be a weakly interacting massive particle (WIMP), its mass scale is reasonably set at 1 TeV. This is the crucial missing piece of information which allows us to expect observable new physics related to both dark matter and neutrino mass at the LHC.

7. Neutrino Connections

Any choice of a mechanism for neutrino mass is suggestive of new physics connections. In Type I seesaw, N is strongly indicative of $S0(10)$ because the spinorial representation $\underline{16}$ of $S0(10)$ contains exactly one family of quarks and leptons if N is included. However, N is also contained in the fermion $\underline{24}$ of $SU(5)$. In Type II seesaw, the Higgs triplet (ξ^{++}, ξ^+, ξ^0) is contained in the scalar $\underline{15}$ of $SU(5)$. In Type III seesaw, $(\Sigma^+, \Sigma^0, \Sigma^-)$ is contained in the fermion $\underline{24}$ of $SU(5)$.

The mass scale of neutrino mass generation may also be tied to other phenomena, such as new gauge interactions, dark matter, and strong *CP*. With three families of leptons and quarks, the theoretical framework for neutrino mass may also be connected to the flavor problem, allowing for flavor symmetries such as the non-Abelian discrete A_4 symmetry of the tetrahedron. Possible new neutrino interactions may also be suggestive of how matter and dark matter could be unified, under $SU(6)$ for eaxmple.

8. Strong *CP*

The anchor mass scale of the Type I seesaw is similar to that of the invisible axion for solving the strong *CP* problem. This is suggestive of a common origin, as proposed in a supersymmetric model [21]. Add to the minimal supersymmetric standard model (MSSM) 6 neutral singlet superfields $\hat{N}^c_{1,2,3}$ and $\hat{S}_{0,1,2}$. Under the anomalous Peccei-Quinn symmetry $U(1)_{PQ}$, they and the usual MSSM superfields transform according to

$$\hat{S}_0 \sim -2, \quad \hat{S}_1 \sim -1, \quad \hat{S}_2 \sim 2; \quad \hat{H}_u = (\hat{h}_u^+, \hat{h}_u^0), \ \hat{H}_d = (\hat{h}_d^0, \hat{h}_d^-) \sim -1; \tag{8.1}$$
$$\hat{Q} = (\hat{u}, \hat{d}), \ \hat{u}^c, \ \hat{d}^c, \ \hat{L} = (\hat{\nu}, \hat{e}), \ \hat{e}^c, \ \hat{N}^c \sim 1/2. \tag{8.2}$$

The superpotential is then given by

$$\begin{aligned} \hat{W} &= m_2 \hat{S}_2 \hat{S}_0 + f \hat{S}_2 \hat{S}_1 \hat{S}_1 + h_1 \hat{S}_1 \hat{N}^c \hat{N}^c + h_2 \hat{S}_2 \hat{H}_u \hat{H}_d \\ &+ h_N \hat{H}_u \hat{L} \hat{N}^c + h_e \hat{H}_d \hat{L} \hat{e}^c + h_u \hat{H}_u \hat{Q} \hat{u}^c + h_d \hat{H}_d \hat{Q} \hat{d}^c. \end{aligned} \tag{8.3}$$

Since $\hat{W}$ does not allow $\hat{L}$ and $\hat{H}_d$ to have the same *PQ* charge, *R* parity is automatically conserved. Now $\hat{W}$ has only one mass scale m_2, which sets the scale for $U(1)_{PQ}$ breaking as follows:

$$V_{SUSY} = |m_2 S_0 + f S_1^2|^2 + m_2^2 |S_2|^2 + 4 f^2 |S_1|^2 |S_2|^2, \tag{8.4}$$

so that $v_2 = 0$, $m_2 v_0 + f v_1^2 = 0$ is a supersymmetry preserving minimum. The linear combination $(v_1 \hat{S}_1 + 2 v_0 \hat{S}_0)/\sqrt{|v_1|^2 + 4|v_0|^2}$ is a massless superfield, and $v_{0,1} \sim m_2$. At the same time, $m_N = 2 h_1 v_1$ which means that the seesaw scale is of order the axion decay constant. At this stage, $U(1)_{PQ}$ has been broken at the scale m_2 without breaking the supersymmetry. Another mass scale M_{SUSY} must be added, i.e. soft supersymmetry breaking terms, so that

$$v_2 \sim M_{SUSY}, \quad m_2 v_0 + f v_1^2 \sim M_{SUSY}^2. \tag{8.5}$$

The μ term of the MSSM becomes $h_2 v_2 \sim M_{SUSY}$. With electroweak symmetry breaking, the axion is then contained in the phases of $S_{0,1,2}$ as well as $h^0_{u,d}$ as in the DFSZ model [22];

$$a = V^{-1} (v_1 \theta_1 + 2 v_0 \theta_0 - 2 v_2 \theta_2 + v_u \theta_u + v_d \theta_d). \tag{8.6}$$

The 4×4 neutralino mass matrix of the MSSM now becomes 7×7 with the inclusion of $\tilde{S}_{0,1,2}$ fermions. Note that $\tilde{S}_2$ and $(2 v_0 \tilde{S}_1 - v_1 \tilde{S}_0)/\sqrt{4|v_0|^2 + |v_1|^2}$ combine to form a Dirac fermion with mass $m_2 \sqrt{1 + 4|v_0^2/v_1^2|}$. The remaining Majorana fermion is the axino with mass $-2 f v_2/(1 + 4|v_0^2/v_1^2|)$ which couples very weakly ($\sim v_{u,d}/V$) to the MSSM neutralinos. Even if this axino is a significant component of dark matter, it will not be detected directly in underground experiments. The corresponding saxion also has mass $\sim M_{SUSY}$ and couples just as weakly.

The $U(1)_{PQ}$ symmetry in any given axion model actually also has an exactly conserved residual Z_2 symmetry which may be used to support a stable dark-matter candidate. In that case [23], the dark matter of the Universe has two possible components: axion and WIMP (weakly interacting massive particle). The $U(1)_{PQ}$ symmetry itself may also be used for radiative neutrino mass. To understand this in more detail, consider the QCD Lagrangian in the presence of instantons. It has an extra term

$$\mathscr{L} = \bar{\theta}\frac{g_s^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}, \tag{8.7}$$

which violates CP. The parameter $\bar{\theta}$ is given by $\theta_{QCD} + a/F_a$, where θ_{QCD} comes from the SM, and a is the axion field coming from the anomalous $U(1)_{PQ}$ with F_a its decay constant, i.e. vacuum expectation value divided by domain wall number. Using the fact that a is a dynamical field with a dynamical potential, Peccei and Quinn showed that it will adjust itself so that $\bar{\theta}$ relaxes to zero, thereby getting rid of this term entirely, thus explaining why the neutron electrric dipole moment is very suppressed and would be nonzero only from electroweak effects. The residual excitation (pointed out by Weinberg and Wilczek), i.e. the axion particle, is physical. Its mass is roughly given by $m_a \sim f_\pi m_\pi/F_a \sim 0.6\times 10^{-3}$ eV$/(F_a/10^{10}$ GeV$)$.

There are three generic axion models, according to what particles couple to $U(1)_{PQ}$. (A) KSVZ [24]: only new heavy singlet quarks; (B) DFSZ [22]: only the known quarks; (C) gluino [25]: only gluinos in supersymmetry. In each case, as $U(1)_{PQ}$ is spontaneously broken, an exactly conserved residual discrete Z_2 symmetry remains. In (B), it is $(-1)^{3B}$; in (C) it is R parity; and in (A) it is a new symmetry defined only on the new heavy singlet quarks. This latter Z_2 is tailor-made as a realization of the scotogenic mechanism.

Let the new heavy singlet quark of the KSVZ model be Q with charge $-1/3$. Under $U(1)_{PQ}$, $Q_L \sim 1/2$ and $Q_R \sim -1/2$, hence $f_Q\zeta^0\bar{Q}_LQ_R + H.c.$ (where $\zeta^0 \sim 1$ under $U(1)_{PQ}$) generates m_Q as well as a. To exploit the residual discrete Z_2 symmetry for dark matter, a neutral complex scalar $\eta^0 \sim 1/2$ is added. The Lagrangian involving $Q_{L,R}, \zeta^0$, and η^0 is then given by

$$\begin{aligned}\mathscr{L} &= \mu_\zeta^2|\zeta|^2 + \frac{1}{2}\lambda_\zeta|\zeta|^4 + \mu_\eta^2|\eta|^2 + \frac{1}{2}\lambda_\eta|\eta|^4 + \lambda'|\zeta|^2|\eta|^2 \\ &+ [f_Q\zeta\bar{Q}_LQ_R + f_d\eta\bar{Q}_Ld_R + \varepsilon_\eta\zeta^\dagger\eta^0\eta^0 + H.c.]\end{aligned} \tag{8.8}$$

Let $\zeta = (1/\sqrt{2})(v+\sigma)e^{ia/v}$, where $v = \sqrt{-2\mu_\zeta^2/\lambda_\zeta}$, then a is the axion and $v = F_a$. Further $m_Q = f_Qv/\sqrt{2}$ and for $\eta = (1/\sqrt{2})(\eta_1 + i\eta_2)$, $m^2_{1,2} = \mu_\eta^2 + \lambda'v^2/2 \pm \varepsilon_\eta v/\sqrt{2}$. Since $v > 4\times 10^8$ GeV from SN1987A, fine tuning is unavoidable for $m_{1,2} \sim$ TeV, just as in the case of the SM Higgs boson in any nonsupersymmetric axion model. On the other hand, $\varepsilon_\eta = 0$ corresponds to an extra U(1) symmetry, i.e. $\eta, Q_L, Q_R \sim 1$ independent of $U(1)_{PQ}$. For $F_a \sim 10^9$ GeV, the axion mass density of the Universe, i.e. $\Omega_a h^2 \sim 0.3(F_a/10^{12}\text{ GeV})^{7/6} \sim 10^{-4}$, is only 1 percent of dark matter. In that case, $m_Q \sim$ TeV if $f_Q \sim 10^{-5}$. Let $m_1 < m_2$, then the interaction $\eta_1^2(\Phi^\dagger\Phi)$ allows it to have the correct relic abundance and be consistent with direct-search bounds.

If three heavy neutral fermions $N_R \sim 1/2$ under $U(1)_{PQ}$ are added together with one scalar leptoquark $\xi^{-1/3} \sim 0$, there would be the additional couplings $\zeta^\dagger N_RN_R$, $\zeta^\dagger\eta^0\eta^0$, as well as $\xi^\dagger(\nu d - lu)$ and $\xi(\bar{Q}_LN_R$. As $U(1)_{PQ}$ breaks to Z_2, N_R gets a Majorana mass, and radiative neutrino masses are obtained in three loops. Now N may be dark matter instead of η_1. In fact, unsuppressed NN annihilation into $\xi^\dagger \xi$ is now possible. The predicted scalar leptoquark ξ will decay into ul or $d\nu$, enabling it to be discovered at the LHC.

9. Neutrino Flavor Symmetry

In 1978 (37 years ago), soon after the putative discovery of the third family of leptons and quarks, it was conjectured (without any data) independently by Cabibbo and Wolfenstein that the 3×3 unitary matrix linking charged leptons to neutrinos could be

$$U_{l\nu} = U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \tag{9.1}$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$. In the convention of the Particle Data Group (PDG), this implies $s_{23} = c_{23} = 1/\sqrt{2}$, $s_{12} = c_{12} = 1/\sqrt{2}$, $s_{13} = 1/\sqrt{3}$, $c_{13} = \sqrt{2/3}$, and $\delta = \pi/2$. If ω and ω^2 are interchanged, then $\delta = -\pi/2$. Note that the θ_{23} and δ predictions are consistent with present data, whereas θ_{12} and θ_{13} are not.

In 2001 (14 years ago), without knowing about Cabibbo and Wolfenstein, U_ω was discovered by Ma and Rajasekaran [26] in the context of A_4. This non-Abelian discrete symmetry is that of the perfect tetrahedron. It has 12 elements and 4 irreducible representations: $\underline{1}, \underline{1}', \underline{1}'', \underline{3}$. Using

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}, \tag{9.2}$$

the following decompositions are obtained:

$$\underline{1} = 11 + 22 + 33, \tag{9.3}$$

$$\underline{1}' = 11 + \omega 22 + \omega^2 33, \tag{9.4}$$

$$\underline{1}'' = 11 + \omega^2 22 + \omega 33. \tag{9.5}$$

Let $(\nu, l)_i \sim \underline{3}$, $l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$, and $\Phi_i \sim \underline{3}$, then

$$\begin{aligned} \mathcal{M}_l &= \begin{pmatrix} f_e v_1^* & f_\mu v_1^* & f_\tau v_1^* \\ f_e v_2^* & f_\mu \omega v_2^* & f_\tau \omega^2 v_2^* \\ f_e v_3^* & f_\mu \omega^2 v_3^* & f_\tau \omega v_3^* \end{pmatrix} \\ &= \begin{pmatrix} v_1^* & 0 & 0 \\ 0 & v_2^* & 0 \\ 0 & 0 & v_3^* \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} f_e & 0 & 0 \\ 0 & f_\mu & 0 \\ 0 & 0 & f_\tau \end{pmatrix}. \end{aligned} \tag{9.6}$$

For $v_1 = v_2 = v_3$, a residual Z_3 symmetry exists with $U_\omega^\dagger$ as the link between $\mathcal{M}_l$ and $\mathcal{M}_\nu$. For many years, theoretical effort was focused on obtaining a specific form of $\mathcal{M}_\nu$ so that tribimaximal neutrino mixing is realized:

$$\begin{aligned} U_{TBM} &= \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \\ &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & i \end{pmatrix}. \end{aligned} \tag{9.7}$$

This means that [27, 28, 29]

$$\mathcal{M}_\nu = \begin{pmatrix} m_2 & 0 & 0 \\ 0 & (m_1-m_3)/2 & (m_1+m_3)/2 \\ 0 & (m_1+m_3)/2 & (m_1-m_3)/2 \end{pmatrix}. \tag{9.8}$$

This $\mathcal{M}_\nu$ is very hard to obtain in the context of a four-dimensional renormalizable field theory, because of the basic clash (or misalignment) of the residual symmetries: Z_3 for $\mathcal{M}_l$ and Z_2 for $\mathcal{M}_\nu$.

Whereas U_{TBM} was consistent with the data for many years, it all changed on March 8, 2012, when Daya Bay announced that θ_{13} had been measured at 8.8°. The 2014 PDG values for neutrino masses and mixing are:

$$\sin^2(2\theta_{12}) = 0.846 \pm 0.021,\ \Delta m^2_{21} = (7.53 \pm 0.18)\times 10^{-5}\ \mathrm{eV}^2, \tag{9.9}$$

$$\sin^2(2\theta_{32}) = 0.999(+0.001-0.018),\ \Delta m^2_{21} = (2.44 \pm 0.06)\times 10^{-3}\ \mathrm{eV}^2(\text{normal}), \tag{9.10}$$

$$\sin^2(2\theta_{32}) = 1.000(+0.000-0.017),\ \Delta m^2_{21} = (2.52 \pm 0.07)\times 10^{-3}\ \mathrm{eV}^2(\text{inverted}), \tag{9.11}$$

$$\sin^2(2\theta_{13}) = (9.3 \pm 0.8)\times 10^{-2}. \tag{9.12}$$

In retrospect, the $Z_3 - Z_2$ clash should have been a warning against tribimaximal mixing.

Since 2002, a special form [30, 31, 32] of $\mathcal{M}_\nu$ was known in the basis where the charged-lepton mass matrix is diagonal:

$$\mathcal{M}_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix}, \tag{9.13}$$

where A, B are real. This mass matrix has 6 parameters, but only 5 are observable, because there is arbitrariness in defining the phase of D versus C. It allows 3 arbitrary masses and 2 arbitrary angles, i.e. θ_{12} and θ_{13}. However, θ_{23} must be $\pi/4$ and $\delta_{CP} = \pm\pi/2$. Such a pattern may be called "cobimaximal" mixing. Note that $|U_{\mu i}| = |U_{\tau i}|$ is required, which is evocative of U_ω which has the equality of the absolute values of all its 9 elements. It is protected by a generalized CP symmetry under $\mu - \tau$ exchange, as first pointed out by Grimus and Lavoura [32]. Present T2K data with input from reactor data indicate indeed a preference for $\delta_{CP} = -\pi/2$.

The connection between Eq. (9.13) and U_ω is summarized by [33]

$$U_{l\nu} = U_\omega^\dagger \mathcal{O}, \tag{9.14}$$

where $\mathcal{O}$ is an orthogonal matrix, from which it is clear that $U^*_{\mu i} = U_{\tau i}$ for $i = 1,2,3$. Comparing this to the PDG form of $U_{l\nu}$, i.e.

$$U_{l\nu} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{9.15}$$

it is obvious that after rotating the phases of the third column and the second and third rows, the two matrices are identical if and only if $s_{23} = c_{23}$ and $\cos\delta = 0$, i.e. $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$.

Obviously $\mathcal{O}$ would come from diagonalizing a real mass matrix. So if $\mathcal{M}_\nu$ is somehow purely real in the A_4 basis, then

$$\mathcal{M}_\nu^{(e,\mu,\tau)} = U_\omega^\dagger \begin{pmatrix} a & c & e \\ c & d & b \\ e & b & f \end{pmatrix} U_\omega^* = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix}, \tag{9.16}$$

where

$$A = (a+2b+2c+d+2e+f)/3, \quad (9.17)$$
$$B = (a-b-c+d-e+f)/3, \quad (9.18)$$
$$C = (a-b-\omega c+\omega^2 d-\omega^2 e+\omega f)/3, \quad (9.19)$$
$$D = (a+2b+2\omega c+\omega^2 d+2\omega^2 e+\omega f)/3. \quad (9.20)$$

The special form of $\mathscr{M}_\nu$ is thus automatically obtained. The Majorana neutrino mass matrix is in general complex, so how does one guarantee it to be real? The answer was already there in a radiative inverse seesaw model of neutrino mass [9, 34], where the origin of the neutrino mass matrix is that of a real scalar mass-squared matrix. Actually the neutrino mass eigenvalues may pick up phases from the parameters involved in the loop calculation, but to obtain $|U_{\mu i}| = |U_{\tau i}|$, all that is required is for $\mathscr{M}_\nu$ to be diagonalized by $\mathscr{O}$.

10. Radiative Inverse Seesaw with A_4

Under A_4, let the three families of leptons transform as

$$(\nu_i, l_i)_L \sim \underline{3}, \quad l_{iR} \sim \underline{1}, \underline{1}', \underline{1}''. \quad (10.1)$$

Add the following new particles, all assumed odd under an exactly conserved discrete Z_2 (dark) symmetry, whereas all SM particles are even:

$$(E^0, E^-)_{L,R} \sim \underline{1}, \quad N_{L,R} \sim \underline{1}, \quad s_i \sim \underline{3}, \quad (10.2)$$

where (E^0, E^-) is a fermion doublet, N a neutral fermion singlet, and $s_{1,2,3}$ are real neutral scalar singlets. There is a 2×2 mass matrix linking (N_L, E_L^0) to (N_R, E_R^0), and there are Majorana mass terms for $N_L N_L$ and $N_R N_R$. Together they generate one-loop Majorana neutrino masses as shown. It is clear that $\mathscr{M}_\nu$ is determined by the real 3×3 mass-squared matrix of $s_{1,2,3}$, which is of course

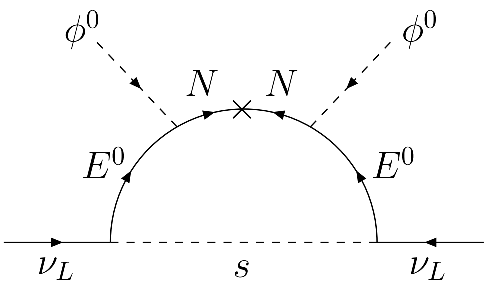

Figure 11: Radiative inverse seesaw neutrino mass with A_4.

proportional to the identity matrix if A_4 is unbroken. However, A_4 may be softly broken by arbitrary $s_i s_j$ terms, so that

$$\mathscr{M}_\nu = \mathscr{O} \begin{pmatrix} m_{\nu 1} & 0 & 0 \\ 0 & m_{\nu 2} & 0 \\ 0 & 0 & m_{\nu 3} \end{pmatrix} \mathscr{O}^T, \quad (10.3)$$

where $\mathscr{O}$ is an orthogonal matrix. Thus the desired form of $U_{l\nu}$ is obtained with $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$, once U_ω is applied [35].

Instead of using three Higgs doublets $\Phi_i \sim \underline{3}$ to obtain U_ω in the charged-lepton sector as in the original A_4 model of 2001, a radiative model of lepton mass is proposed [35]. Again the fermion doublet (E^0, E^-) and singlet N are used, but now in conjunction with two sets of charged scalars which are also odd under dark Z_2, i.e.

$$x_i^- \sim \underline{3}, \quad y_i^- \sim \underline{1}, \underline{1}', \underline{1}''. \tag{10.4}$$

To connect x with y, the soft scalar terms $x_i y_j^*$ are assumed to break A_4 to Z_3. Since m_l is radiative,

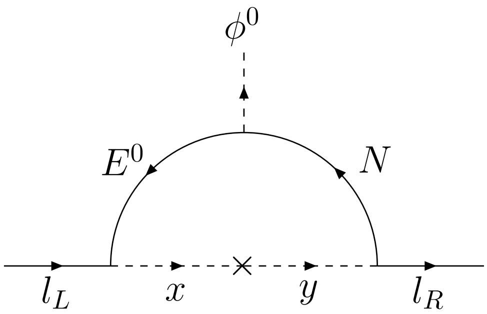

Figure 12: Radiative charged-lepton mass with A_4.

the corresponding Higgs coupling to $\bar{l}l$ now deviates from that of the SM [36] and is not supressed by the usual $16\pi^2$ factor of radiative corrections. This anomalous Higgs Yukawa coupling is potentially observable at the LHC.

The dark matter parity of this model is also derivable [37] from lepton parity. Under lepton parity, let the new particles $(E^0, E^-), N$ be even and s, x, y be odd, then the same Lagrangian is obtained. As a result, dark parity is simply given by $(-1)^{L+2j}$, which is odd for all the new particles and even for all the SM particles. Note that the tree-level Yukawa coupling $\bar{l}_L l_R \phi^0$ would be allowed by lepton parity alone, but is forbidden here because of the A_4 symmetry. The radiative lepton mass matrix is diagonal because of the Z_3 residual symmetry. This means that the muon anomalous magnetic moment Δa_μ gets a significant contribution from xy exchange, but not $\mu \to e\gamma$. Because Δa_μ is now of order m_μ^2/m_E^2 without the usual $16\pi^2$ suppression, a large $m_E \sim 1$ TeV is possible for the explanation of the experimental-theoretical discrepancy instead of the usual $m_E \sim 200$ GeV. As for $\mu \to e\gamma$, it will come from s exchange in the analog diagram to radiative neutrino mass. For $m_E \sim 1$ TeV, it will be suitably suppressed.

11. Unification of Matter with Dark Matter

The scotogenic mechanism of neutrino mass may be a clue to the possible unification of dark matter with matter [38], in the context of $SU(6)$ for example. The fundamental $\underline{5}_F^* = (d^c, d^c, d^c, e, \nu)$ of $SU(5)$ is extended to the $\underline{6}_F^* = (d^c, d^c, d^c, e, \nu, N)$ of $SU(6)$. The $\underline{10}_F$ of $SU(5)$ is extended by a heavy $\underline{5}_F$ to form a $\underline{15}_F$ of $SU(6)$. Together with the similar extensions of the scalar multiplets, the $SU(5)$ Yukawa terms

$$\underline{5}_F^* \times \underline{10}_F \times \underline{5}_S^*, \quad \underline{10}_F \times \underline{10}_F \times \underline{5}_S, \tag{11.1}$$

are extended to

$$\underline{6}^*_F \times \underline{15}_F \times \underline{6}^*_S, \quad \underline{15}_F \times \underline{15}_F \times \underline{15}_S. \tag{11.2}$$

Hence two different Higgs doublets are needed for quark and lepton masses. Whereas $\underline{5}^*_F + \underline{10}_F$ is anomaly-free in $SU(5)$, the corresponding combination in $SU(6)$ is $\underline{6}^*_F + \underline{6}^*_F + \underline{15}_F$. The $\underline{5}^*_F$ contained in the extra $\underline{6}^*_F$ is heavy and pairs up with $\underline{5}_F$ contained in the $\underline{15}_F$ through the $\underline{1}_S$ of $\underline{6}^*_S$. Consider now the $SU(6)$ scalar multiplet $\underline{21}_S$. It decomposes into $\underline{15}_S + \underline{5}_S + \underline{1}_S$ of $SU(5)$. It has then the second scalar doublet (η^+, η^0) and the interactions

$$\underline{6}^*_F \times \underline{6}^*_F \times \underline{21}_S, \quad \underline{15}^*_S \times \underline{15}^*_S \times \underline{21}_S \times \underline{21}_S. \tag{11.3}$$

Hence N gets a Majorana mass through the $\underline{1}_S$ of $\underline{21}_S$. The scotogenic interaction $(\nu\eta^0 - l\eta^+)N$ is now possible as well as the quartic $(\Phi^\dagger\eta)^2$ interaction. These three terms support a Z_2 symmetry as

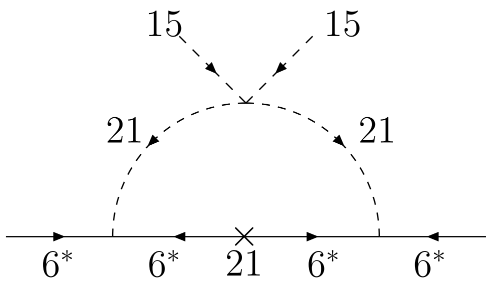

Figure 13: Scotogenic neutrino mass in $SU(6)$.

desired, but it is not absolute. Just as the proton is unstable at the scale of quark-lepton unification, dark matter is expected to be unstable at a similar scale. Matter and dark matter are thus unified through scotogenic neutrino mass.

The heavy color triplet gauge bosons contained in the adjoint $\underline{24}_V$ of $SU(5)$ mediate proton decay. The extra heavy color triplet gauge bosons contained in the adjoint $\underline{35}_V$ of $SU(6)$ will connect N to the quarks, resulting in $N \to p\pi^-, n\pi^0$. Another possibility is the mixing of the heavy scalar color triplet $\zeta^{-1/3}$ in $\underline{21}_S$ with its counterpart $\xi^{-1/3}$ in $\underline{15}_S$ through the adjoint $\underline{35}_S$ of $SU(6)$. The dark Z_2 is thus broken at a high scale, just as baryon number is supposed to be broken in grand unified theories.

12. Personal Remarks

Neutrino theory attempts to answer several fundamental questions, foremost is the scale of new physics responsible for neutrino mass and mixing. A possible hint is that they may also be connected to dark matter at the mass scale of 1 TeV. In the scotogenic framework, the A_4 transformation U_ω may be used to obtain a desirable form of $U_{l\nu}$, i.e. $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$, automatically if the origin of the neutrino mass matrix is a set of real scalars $s_i \sim \underline{3}$ under A_4. This scenario may be implemented with just one Higgs doublet if the charged-lepton masses are also radiative. The associated new particle should be observable at the LHC.

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