

One-loop potential with scale invariance and effective operators

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We study quantum corrections to the scalar potential in classically scale invariant theories, using a manifestly scale invariant regularization. The subtraction scale μ of the dimensional regularization is generated after spontaneous scale symmetry breaking, from a subtraction *function* of the fields $\mu(\phi, \sigma)$ that we determine from general principles. The result is a scale invariant one-loop potential U for a higgs field ϕ and dilaton σ that contains an additional *finite* quantum correction $\Delta U(\phi, \sigma)$, beyond the Coleman Weinberg term. ΔU contains non-polynomial effective operators like ϕ^6/σ^2 whose quantum origin is explained, while U respects the Callan-Symanzik equation. A flat direction is maintained at the quantum level, the model has vanishing vacuum energy and the one-loop correction to the mass of ϕ remains small without additional tuning beyond the initial, classical tuning that enforces a hierarchy $\langle\phi\rangle \ll \langle\sigma\rangle$. The approach is useful to models that investigate scale symmetry at the quantum level.

*Proceedings of the
Corfu Summer Institute 2015 "School and Workshops on Elementary Particle Physics and Gravity"
1-27 September 2015, Corfu, Greece*

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1. General considerations and motivation

Scale invariant theories [1] are an interesting alternative to supersymmetry to address the mass hierarchy problem. Since such theories forbid the presence of dimensionful parameters in the Lagrangian, scale symmetry be it classical or quantum, must be broken in the real world. This breaking can be explicit or spontaneous. We investigate the latter case, since this preserves the UV behaviour of the initial scale invariant theory. In the spontaneous breaking we have the dilaton σ as the Goldstone mode of this symmetry, with non-vanishing vacuum expectation value (vev) $\langle\sigma\rangle\neq 0$. In a broader setup that includes gravity, $\langle\sigma\rangle$ may be related to the Planck scale. We do not detail how σ acquires a vev, but simply search for solutions with $\langle\sigma\rangle\neq 0$. All scales of the theory are then related to $\langle\sigma\rangle$. A hierarchy of scales which are vev's of the different fields present, can then be generated either by one initial (classical) tuning of the couplings to small values [2] or as in [3].

The purpose of this talk based on [4] is to consider a classically scale invariant theory and to show how to compute the quantum corrections to the scalar potential in a manifestly scale invariant way. Such approach is important since it preserves at the quantum level the initial symmetry of the theory and its UV properties, relevant for the Higgs physics. Investigating scale invariant theories at quantum level is non-trivial because the regularization of their quantum corrections breaks the scale symmetry *explicitly*¹. Indeed, in its traditional form, the regularization, be it dimensional regularization (DR) or some other scheme, introduces a subtraction scale² μ . Its presence ruins exactly the symmetry that we want to study at the quantum level. To avoid this situation one should generate the subtraction scale in a dynamical way. Consider then replacing μ of the DR scheme by a field-dependent subtraction *function* $\mu(\sigma)$ [17], see also more recent [18]. Having couplings or masses that are field-dependent is something familiar in string theory. After spontaneous breaking of scale symmetry when $\langle\sigma\rangle\neq 0$, the subtraction scale is generated as $\mu(\langle\sigma\rangle)$. Doing so has implications at the quantum level, presented below.

Preserving the scale symmetry of the action during the UV regularization of the quantum correction is actually required in theories which are non-renormalizable, to avoid regularization artefacts. Since some of these theories may be non-renormalizable [19, 20], it is then worth exploring the consequence of a such regularization. This is also important from the naturalness problem point of view, as argued long ago by Bardeen [21]. The Standard Model with the higgs mass $m_h = 0$ has an extra symmetry, classical scale invariance; therefore, if one uses schemes that break this symmetry, which is what happens in general, quadratic divergences can be regarded as artefacts of this regularization and can thus be ignored³. This gives a phenomenological motivation to study a scale invariant regularization of quantum corrections, with spontaneous breaking of scale symmetry. This talk (based on [4]) continues previous similar studies [18, 19, 22], with some

¹As a result of such explicit breaking the initial flat direction of the classically scale invariant theory is lifted; there is extensive model building in this direction, see for example more recent [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16].

²In DR the scale μ relates the dimensionless (renormalized) coupling $\lambda^{(r)}$ to the dimensionful one λ once the $d = 4$ theory is continued analytically to $d = 4 - 2\epsilon$; for a quartic higgs coupling: $\lambda = \mu^{2\epsilon} (\lambda^{(r)} + \sum_n a_n / \epsilon^n)$.

³ m_h^2 remains quadratic in the scale (of “new physics”) generated by spontaneous scale symmetry breaking.

notable differences and with new results shown below.

We consider a classical, scale invariant theory of a Higgs-like scalar ϕ and the dilaton σ , with

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - V(\phi, \sigma) \quad (1.1)$$

where

$$V = \frac{\lambda_\phi}{4} \phi^4 + \frac{\lambda_m}{2} \phi^2 \sigma^2 + \frac{\lambda_\sigma}{4} \sigma^4 \quad (1.2)$$

There exists a non-trivial solution with spontaneous breaking of scale symmetry $\langle \sigma \rangle \neq 0$, (also $\langle \phi \rangle \neq 0$) provided that two conditions are met: firstly, $\lambda_m^2 = \lambda_\phi \lambda_\sigma$ with $\lambda_m < 0$ and secondly,

$$\frac{\langle \phi \rangle^2}{\langle \sigma \rangle^2} = -\frac{\lambda_m}{\lambda_\phi}, \quad \Rightarrow \quad V = \frac{\lambda_\phi}{4} \left(\phi^2 + \frac{\lambda_m}{\lambda_\phi} \sigma^2 \right)^2 \quad (1.3)$$

Then spontaneous scale symmetry breaking implies electroweak symmetry breaking at tree-level, with $V_{min} = 0$ i.e. vacuum energy is vanishing. There is a flat direction corresponding to a massless dilaton σ while the mass of ϕ is $m_\phi^2 = -2\lambda_m(1 - \lambda_m/\lambda_\phi) \langle \sigma \rangle^2$. Although we are not interested in the exact value of $\langle \sigma \rangle$, since scale invariance is expected to be broken by Planck physics it is likely that $\langle \sigma \rangle \sim M_{Planck}$. To ensure a hierarchy $\langle \phi \rangle \ll \langle \sigma \rangle$ and that the mass of the higgs-like ϕ is near the electroweak scale, one can tune once the couplings (at the classical level) to enforce a relation $\lambda_\sigma \ll |\lambda_m| \ll \lambda_\phi$ [2, 3]. Such hierarchy of couplings is stable under the quantum corrections of the renormalization group flow [2]. One would like to explore these issues at the quantum level in a scale invariant approach.

More generally, in theories with scale symmetry, the potential for ϕ and σ has the form $V = \sigma^4 W(\phi/\sigma)$. Assuming spontaneous breaking of this symmetry $\langle \sigma \rangle \neq 0$, the two minimum conditions $\partial V/\partial \phi = \partial V/\partial \sigma = 0$ become

$$W'(x_0) = W(x_0) = 0, \quad x_0 \equiv \frac{\langle \phi \rangle}{\langle \sigma \rangle}; \quad \langle \sigma \rangle, \langle \phi \rangle \neq 0. \quad (1.4)$$

One minimum condition fixes the ratio $\langle \phi \rangle/\langle \sigma \rangle$ while the second one gives a relation among the couplings of the theory such as $W(x_0) = 0$. If these two equations for W have a solution x_0 , then $\langle \phi \rangle \propto \langle \sigma \rangle$. A flat direction exists in the plane (ϕ, σ) with $\phi/\sigma = x_0$ and the vacuum energy vanishes $V(\langle \phi \rangle, \langle \sigma \rangle) = 0$. These results can remain valid at the *quantum level* as well, provided that the calculation of the quantum corrections is manifestly scale invariant, since then the potential remains of the form $V \sim \sigma^4 \tilde{W}(\phi/\sigma)$ where \tilde{W} is the quantum corrected W . Also, the flat direction corresponding to the Goldstone mode (dilaton) remains flat at the quantum level (to all orders) if the calculation preserves the scale symmetry which is broken only spontaneously.

2. Quantum scale invariance of the potential and effective operators

Let us then calculate the one-loop potential in a scale invariant regularization. To enforce dimensionless couplings in the usual DR scheme, one replaces the quartic couplings $\lambda \rightarrow \lambda \mu^{4-d}$, with μ the subtraction scale. This breaks classical scale symmetry. To avoid this, replace μ by an unknown function of the fields, $\mu(\phi, \sigma)$, whose vacuum expectation value generates dynamically the subtraction scale⁴ $\mu(\langle\phi\rangle, \langle\sigma\rangle)$. This function is determined later, but is assumed to be continuous and differentiable at all fields values. Then, in $d = 4 - 2\varepsilon$ dimensions, the potential is

$$\tilde{V}(\phi, \sigma) = \mu(\phi, \sigma)^{4-d} V(\phi, \sigma) \quad (2.1)$$

To be general, we also allowed a ϕ -dependence of the subtraction function. There are now “evanescent” interactions between say σ in $\mu(\phi, \sigma)$ and $V(\phi, \sigma)$, that are absent in the limit $d = 4$. They are due to the scale symmetry in $d = 4 - 2\varepsilon$. The one-loop potential is then computed as usual, but with \tilde{V} instead of original V :

$$U = \tilde{V}(\phi, \sigma) - \frac{i}{2} \int \frac{d^d p}{(2\pi)^d} \text{Tr} \ln [p^2 - \tilde{V}_{\alpha\beta} + i\varepsilon] \quad (2.2)$$

Here $\tilde{V}_{\alpha\beta} = \partial\tilde{V}/\partial\alpha\partial\beta$, with $\alpha, \beta = \phi, \sigma$. Up to $\mathcal{O}(\varepsilon^2)$ terms

$$\tilde{V}_{\alpha\beta} = \mu^{2\varepsilon} [V_{\alpha\beta} + 2\varepsilon \mu^{-2} N_{\alpha\beta}], \quad (2.3)$$

$$N_{\alpha\beta} \equiv \mu (\mu_\alpha V_\beta + \mu_\beta V_\alpha) + (\mu \mu_{\alpha\beta} - \mu_\alpha \mu_\beta) V, \quad (2.4)$$

where $\mu_\alpha = \partial\mu/\partial\alpha$, $\mu_{\alpha\beta} = \partial^2\mu/\partial\alpha\partial\beta$, $V_\alpha = \partial V/\partial\alpha$, and $V_{\alpha\beta} = \partial^2 V/\partial\alpha\partial\beta$, are field dependent quantities. Denote by M_s^2 the eigenvalues of the matrix $V_{\alpha\beta}$ and⁵ by $\kappa \equiv 4\pi e^{3/2-\gamma_E}$, then

$$U = \mu(\phi, \sigma)^{2\varepsilon} \left\{ V - \frac{1}{64\pi^2} \left[\sum_{s=\phi, \sigma} M_s^4 \left(\frac{1}{\varepsilon} - \ln \frac{M_s^2(\phi, \sigma)}{\kappa \mu^2(\phi, \sigma)} \right) + \frac{4(V_{\alpha\beta} N_{\beta\alpha})}{\mu^2(\phi, \sigma)} \right] \right\} \quad (2.5)$$

In the last term a summation over repeated indices (fields) is understood. The counterterms are

$$\delta U_{c.t.} \equiv \mu(\phi, \sigma)^{2\varepsilon} \left\{ \frac{1}{4} \delta Z_{\lambda_\phi} \lambda_\phi \phi^4 + \frac{1}{2} \delta Z_{\lambda_m} \lambda_m \phi^2 \sigma^2 + \frac{1}{4} \delta Z_{\lambda_\sigma} \lambda_\sigma \sigma^4 \right\} \quad (2.6)$$

from which one easily finds the coefficients $\delta Z_{\lambda_j} \equiv Z_{\lambda_j} - 1$; they have values identical to those if the theory were regularized with $\mu=\text{constant}$ (the same is true about the beta functions of λ_j). For

⁴after spontaneous scale symmetry breaking.

⁵The eigenvalues M_s^2 are the roots (q) of $q^2 - q(V_{\phi\phi} + V_{\sigma\sigma}) + (V_{\phi\phi} V_{\sigma\sigma} - V_{\phi\sigma}^2) = 0$.

example $\delta Z_{\lambda_\phi} = 1/(16\pi^2\varepsilon)(9\lambda_\phi + \lambda_m^2/\lambda_\phi)$, etc. The renormalized potential is then

$$\begin{aligned}
U(\phi, \sigma) &= V(\phi, \sigma) + \frac{1}{64\pi^2} \left\{ \sum_{s=\phi, \sigma} M_s^4(\phi, \sigma) \left(\ln \frac{M_s^2(\phi, \sigma)}{\mu^2(\phi, \sigma)} - \frac{3}{2} \right) + \Delta U(\phi, \sigma) \right\} \\
\Delta U &= \frac{-4}{\mu^2} \left\{ V [(\mu\mu_{\phi\phi} - \mu_\phi^2)V_{\phi\phi} + 2(\mu\mu_{\phi\sigma} - \mu_\phi\mu_\sigma)V_{\phi\sigma} + (\mu\mu_{\sigma\sigma} - \mu_\sigma^2)V_{\sigma\sigma}] \right. \\
&\quad \left. + 2\mu(\mu_\phi V_{\phi\phi} + \mu_\sigma V_{\phi\sigma})V_\phi + 2\mu(\mu_\phi V_{\phi\sigma} + \mu_\sigma V_{\sigma\sigma})V_\sigma \right\} \quad (2.7)
\end{aligned}$$

We recovered the usual Coleman-Weinberg term [23, 24] in a modified, scale invariant form. We also found a new, additional contribution ΔU , not considered in [18], which is a *finite* one-loop correction to the potential. ΔU emerges from the correction $\propto \varepsilon$ to the field dependent masses (that are derivatives of \tilde{V} , see eq.(2.3)) of the states that “run” in the loop; when this correction $\propto \varepsilon$ multiplies the pole $1/\varepsilon$ of the momentum integral, one obtains a finite term (last term in (2.5)) that gives ΔU . Also ΔU vanishes on the tree level ground state when V and its first derivatives vanish.

The problem with the result in eq.(2.7) is that it depends on the unknown function $\mu(\phi, \sigma)$, which generates the subtraction scale after spontaneous scale symmetry breaking. Obviously, physical observables cannot depend on the regularization done with this function. We should then determine this function from some general principles; then, the potential must respect the Callan-Symanzik equation, to enable us to make physical predictions.

To this purpose, consider first a particular example of $\mu(\phi, \sigma)$ used in previous models [18, 22]

$$\mu(\phi, \sigma) = z(\xi_\phi \phi^2 + \xi_\sigma \sigma^2)^{1/2} \quad (2.8)$$

With this, one computes the expression of ΔU which in the particular limit $\lambda_m \rightarrow 0$ becomes

$$\begin{aligned}
\Delta U \Big|_{\lambda_m=0} &= -3 \left[\xi_\phi \xi_\sigma [\lambda_\phi (9\lambda_\phi + \lambda_\sigma) \phi^6 \sigma^2 + \lambda_\sigma (\lambda_\phi + 9\lambda_\sigma) \phi^2 \sigma^6] \right. \\
&\quad \left. + 7(\lambda_\phi^2 \xi_\phi^2 \phi^8 + \lambda_\sigma^2 \xi_\sigma^2 \sigma^8) - (\xi_\phi^2 + \xi_\sigma^2) \lambda_\phi \lambda_\sigma \phi^4 \sigma^4 \right] (\xi_\phi \phi^2 + \xi_\sigma \sigma^2)^{-2} \quad (2.9)
\end{aligned}$$

This simplifies further if $\lambda_m^2 = \lambda_\phi \lambda_\sigma$ which ensures spontaneous breaking of scale symmetry; however the term $\propto \xi_\phi \xi_\sigma \phi^6$ remains even in this case, unless $\xi_\phi \xi_\sigma = 0$ when μ depends on one field only. Now, when $\lambda_m \rightarrow 0$, the “visible” sector of higgs-like ϕ is classically decoupled from the “hidden” sector of the dilaton σ . Nevertheless, we see that in this limit the two sectors still interact at the quantum level, which is unacceptable. This situation is more general and applies when the subtraction function depends on both fields. The reason for this is related to how ΔU is generated, from “evanescent” interactions introduced by scale invariance of the action in $d = 4 - 2\varepsilon$, see $\tilde{V}(\phi, \sigma) = \mu^{2\varepsilon} V(\phi, \sigma)$. Since $\mu(\phi, \sigma)$ contains both fields, it brings interactions with any term in $V(\phi, \sigma)$, not only with that proportional to λ_m . This explains the presence in (2.9) of non-decoupling interactions terms proportional to λ_ϕ .

Similar considerations apply for a general subtraction function, which, up to a relabeling of the fields, can be written as $\mu(\phi, \sigma) = z\sigma \exp(g(\phi/\sigma))$, with g an arbitrary continuous, differentiable function. Then one can show that ΔU vanishes in the decoupling limit ($\lambda_m = 0$) only if g is a constant⁶. We conclude that the subtraction function must depend on the dilaton only, with $\mu(\sigma) = z\sigma$. Here z is some arbitrary dimensionless parameter, whose role will be clarified shortly⁷.

With $\mu(\sigma) = z\sigma$ uniquely identified and with V of eq.(1.2) one obtains

$$\Delta U = \frac{\lambda_\phi \lambda_m \phi^6}{\sigma^2} - (16\lambda_\phi \lambda_m + 6\lambda_m^2 - 3\lambda_\phi \lambda_\sigma) \phi^4 - (16\lambda_m + 25\lambda_\sigma) \lambda_m \phi^2 \sigma^2 - 21\lambda_\sigma^2 \sigma^4 \quad (2.10)$$

This simplifies further for our case with $\lambda_m^2 = \lambda_\phi \lambda_\sigma$ of spontaneous symmetry breaking that generates a subtraction scale $z\langle\sigma\rangle$. In the decoupling limit ($\lambda_m \rightarrow 0$) there are no quantum interactions left between ϕ and σ , since $\Delta U \rightarrow 0$. With ΔU of eq.(2.10) the one-loop potential becomes

$$U(\phi, \sigma) = V(\phi, \sigma) + \frac{1}{64\pi^2} \left[\sum_{s=\phi, \sigma} M_s^4(\phi, \sigma) \left(\ln \frac{M_s^2(\phi, \sigma)}{z^2 \sigma^2} - \frac{3}{2} \right) + \Delta U(\phi, \sigma) \right] \quad (2.11)$$

This quantum expression is scale invariant. It has a form and properties similar to those discussed in section 1 (text around eq.(1.4)).

3. More about quantum corrections

One notes the presence in ΔU of eq.(2.10) of higher dimensional *non-polynomial* operators such as ϕ^6/σ^2 , in addition to other finite quantum results induced by manifest scale invariance. It is expected that more such operators be generated at higher loop orders. Needless to say, the correction ΔU is missed in calculations that are not scale invariant such as the usual DR scheme, since the result depends on derivatives of μ wrt σ which vanish if $\mu=\text{constant}$. Finally, after a Taylor expansion, the above operator can be re-written as a series of standard effective polynomial operators in fluctuations $\tilde{\sigma}$, where $\sigma = \langle\sigma\rangle + \tilde{\sigma}$

$$\frac{\phi^6}{\sigma^2} = \frac{\phi^6}{\langle\sigma\rangle^2} \left(1 - \frac{2\tilde{\sigma}}{\langle\sigma\rangle} + 3\frac{\tilde{\sigma}^2}{\langle\sigma\rangle^2} + \dots \right) \quad (3.1)$$

The logarithm $\ln(z^2\sigma^2)$ of the Coleman-Weinberg (CW) term can also be expanded about $\langle\sigma\rangle$ to recover the usual CW term obtained for $\mu=\text{constant}$ ($=z\langle\sigma\rangle$), plus additional corrections. In conclusion, U contains new quantum corrections that can be re-written as series of polynomial terms in $\tilde{\sigma}$, suppressed by $\langle\sigma\rangle$.

⁶We discard an extra solution for g and thus $\mu(\phi, \sigma)$ which is not continuous in $\phi = 0$ and also depends on 2 arbitrary constants rather than one (z), where the latter is taken care of by the Callan-Symanzik equation, (see later).

⁷To be exact, one actually has $\mu(\sigma) = z\sigma^{2/(d-2)}$, since the fields have mass dimension $(d-2)/2$ while μ has mass dimension 1. In our one-loop approximation and at this stage it is safe to take the limit $d \rightarrow 4$ in which case $\mu(\sigma) = z\sigma$.

At higher loops, the new operator ϕ^6/σ^2 can render the theory non-renormalizable. If the initial theory were regularized with $\mu=\text{constant}$, the scale symmetry is broken explicitly, there is no Goldstone mode, but such operator is never generated dynamically and the theory is renormalizable to all orders. Nevertheless such operators should be added “by hand” to the classical Lagrangian, since they respect its symmetries. The presence here of this non-polynomial operator is not too surprising; since it is not forbidden by the scale symmetry (preserved by the quantum calculation) such operator is expected to emerge at some loop-order. Its origin is due to loop corrections with “evanescent” interactions dictated by scale invariance in $d = 4 - 2\epsilon$.

The potential U still depends on the dimensionless subtraction parameter z , which apparently prevents one from making predictions for physical observables. However, its presence is understood by analogy to the subtraction *scale* dependence (in a given order) in the “ordinary” regularization with $\mu=\text{constant}$. The Callan-Symanzik equation should be respected by the potential and this will ensure that the physical observables do not depend on z (or on the subtraction scale $\mu(\langle\sigma\rangle) = z\langle\sigma\rangle$). In our case the Callan-Symanzik equation for U of eq.(2.11) is

$$\frac{dU(\lambda_j, z)}{d \ln z} = \left(\beta_{\lambda_j} \frac{\partial}{\partial \lambda_j} + \frac{\partial}{\partial z} \right) U(\lambda_j, z) = 0, \quad \text{sum over } j = \phi, m, \sigma; \quad \beta_{\lambda_j} \equiv \frac{d\alpha_j}{d \ln z} \quad (3.2)$$

This condition is easily verified, since the only explicit dependence on z is via the CW term and the (non-vanishing [25]) beta functions⁸ β_{λ_j} are found from the condition $(d/d \ln z)(\lambda_j Z_{\lambda_j}) = 0$ ($j = \phi, m, \sigma$, fixed). In conclusion the one-loop potential is independent of z and respects the Callan-Symanzik equation for theories with this symmetry [25].

Since U is scale invariant at the one-loop level, the necessary minimum conditions of vanishing first derivatives $U_\phi = U_\sigma = 0$, with $\langle\sigma\rangle \neq 0$, ensure that the ground state has vanishing vacuum energy $U_{min} = 0$ and that a flat direction exists (as discussed in Section 1). The spectrum consists of a massless (Goldstone) mode and the scalar ϕ receives quantum corrections. Its mass is then $m_\phi^2 = (U_{\phi\phi} + U_{\sigma\sigma})_{min}$. One can compute m_ϕ in some approximation, such as $\lambda_\sigma \ll |\lambda_m| \ll \lambda_\phi$, when minimising the potential is easier. It is possible to show that the quantum correction to the mass of ϕ due to the Coleman-Weinberg part of the potential does not require additional tuning of the couplings in order to keep it light [18], well below the scale $\langle\sigma\rangle$, where $\langle\sigma\rangle \gg \langle\phi\rangle$ (see also text after eq.(1.3)). This means that there are no quantum corrections to its mass of the type $\lambda_\phi^2 \langle\sigma\rangle^2$ or similar, that would require tuning the higgs quartic self-coupling λ_ϕ ⁹.

Further, the contribution to the (mass)² of ϕ due to the new correction ΔU that we found (not considered in [18]) is also under control. This correction is

$$\delta m_\phi^2 = \frac{1}{64\pi^2} (\Delta U_{\phi\phi} + \Delta U_{\sigma\sigma})_{min}$$

It can be shown [4] that δm_ϕ^2 contains only terms proportional to $\lambda_m^2 \langle\sigma\rangle$ or $\lambda_\sigma \langle\sigma\rangle$ (here $\lambda_\sigma =$

⁸These are $\beta_{\lambda_\phi} = 1/(8\pi^2)(9\lambda_\phi^2 + \lambda_m^2)$, $\beta_{\lambda_m} = 1/(8\pi^2)(3\lambda_\phi + 4\lambda_m + 3\lambda_\sigma)\lambda_m$ and $\beta_{\lambda_\sigma} = 1/(8\pi^2)(\lambda_m^2 + 9\lambda_\sigma^2)$.

⁹Such tuning of λ_ϕ would be the sign of re-introducing the hierarchy problem.

$\lambda_m^2/\lambda_\phi \ll \lambda_m$). Thus no tuning of λ_ϕ is required to keep δm_ϕ^2 much smaller than the UV scale $\langle\sigma\rangle$.

4. Final remarks and conclusions

Scale invariant theories can provide an alternative to supersymmetry in solving the mass hierarchy problem. The Standard Model classical Lagrangian has a scale symmetry in the limit of vanishing higgs mass. As emphasized long ago by Bardeen, the usual regularization of quantum corrections breaks this symmetry *explicitly* by the presence of the subtraction scale (via dimensional regularization, Pauli-Villars, etc) and introduces regularization artefacts. Obviously, in the real world scale symmetry is broken, but to preserve its ultraviolet properties, while generating all the mass scales of the theory (including the subtraction scale), it is recommended that this symmetry be broken spontaneously (i.e. *softly*). This means the spectrum of the theory will contain an additional, massless (Goldstone) mode of this symmetry (dilaton σ), whose vev generates the subtraction scale.

In this talk we presented the consequences of such an approach, with a regularization that preserves scale symmetry, to compute the one-loop corrections to the scalar potential of a classically scale invariant theory of a higgs-like ϕ and dilaton σ . One consequence is that the one-loop scalar potential contains additional *finite* quantum corrections (ΔU), beyond the familiar Coleman-Weinberg term, itself modified into a scale-invariant form (with $\mu(\sigma) = z\sigma$, where z is an arbitrary dimensionless parameter). The origin of ΔU is due to evanescent corrections (i.e. proportional to ϵ) to the field dependent masses that “run” in the one-loop diagram when these multiply the pole of its momentum integral, to give a finite ΔU . Also ΔU contains non-polynomial effective operators of the type ϕ^6/σ^2 , which can be Taylor expanded into a series of polynomial operators, suppressed by $\langle\sigma\rangle \gg \langle\phi\rangle$, but no dangerous operators of the opposite type σ^6/ϕ^2 can be generated. The quantum correction to the mass of ϕ (m_ϕ) due to ΔU remains thus under control, with no tuning needed of the higgs self-coupling to keep m_ϕ much smaller than the dilaton scale $\langle\sigma\rangle$. It would be interesting to check if this behaviour survives in higher loop orders.

It was also shown that the subtraction function cannot also depend on the higgs-like scalar ϕ . This is because in the classical decoupling limit $\lambda_m \rightarrow 0$ of the visible sector (of ϕ) from the hidden sector (of σ), there exists a non-decoupling quantum interaction between these sectors. As a result the subtraction function depends on σ only $\mu(\sigma) = z\sigma$, as considered above, and is unique. Since physical observables cannot depend on arbitrary parameters such as z (or the subtraction scale $\mu(\langle\sigma\rangle) = z\langle\sigma\rangle$), we checked that the Callan-Symanzik equation is respected by the potential. The above technical results can now be applied to the scale invariant version of the (classical) Standard Model Lagrangian to explore their phenomenological implications. The presence of the higher dimensional operators of the type found above can have particular implications for the stability of the SM ground state at the high scale.

Acknowledgements: This work was supported by a grant of the Romanian National Authority for Scientific Research CNCS-UEFISCDI, project number PN-II-ID-PCE-2011-3-0607.

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