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The numerical approach to quantum field theory in a non-commutative space

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Numerical simulation is an important non-perturbative tool to study quantum field theories defined in non-commutative spaces. In this contribution, a selection of results from Monte Carlo calculations for non-commutative models is presented, and their implications are reviewed. In addition, we also discuss how related numerical techniques have been recently applied in computer simulations of dimensionally reduced supersymmetric theories.

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1. Introduction

The idea that the geometry of spacetime becomes non-commutative at the Planck scale is a central feature of quantum gravity theories [1]: it arises naturally in a string theory context [2] and entails far-reaching consequences for quantum field theory (QFT), including a variety of surprising phenomenological implications [3, 4]; a well-known example of the latter is the mixing of ultraviolet and infrared (UV/IR) degrees of freedom [5].

Typically, analytical studies of QFT in non-commutative spaces (NC QFT) involve some form of approximation—be it a truncated perturbative expansion, or some large-N limit, et c. In order to test the robustness of results obtained under these approximations, it is desirable to check them against some other *ab initio* formulation of the theory. For a generic NC QFT, a possible way to do this is by numerical evaluation of correlation functions, in a Feynman path-integral formulation of the theory [6]. During the past fifteen years, this approach has been successfully pursued in several works, in which various types of field theories in non-commutative spaces of different dimensions have been mapped to appropriately defined finite-dimensional matrix models, and investigated by Monte Carlo integration. More recently, the numerical technology developed in these works has also found applications in computer simulations of dimensionally reduced supersymmetric theories, paving the way to the non-perturbative study of many open theoretical issues—including, in particular, problems related to the gauge/gravity duality [7, 8, 9].

2. Implementation

In the Feynman path-integral formalism, expectation values in a generic QFT (defined in ordinary, commutative Minkowski spacetime) are given by

$$\langle \mathscr{O}_1^{(\mathrm{M})} \dots \mathscr{O}_n^{(\mathrm{M})} \rangle = \frac{\int \mathscr{D}\phi \; \mathscr{O}_1^{(\mathrm{M})} \dots \mathscr{O}_n^{(\mathrm{M})} \exp\left(iS^{(\mathrm{M})}\right)}{\int \mathscr{D}\phi \; \exp\left(iS^{(\mathrm{M})}\right)}. \tag{2.1}$$

In a Euclidean formulation, the previous expression is replaced by

$$\langle \mathscr{O}_1 \dots \mathscr{O}_n \rangle = \frac{\int \mathscr{D}\phi \; \mathscr{O}_1 \dots \mathscr{O}_n \exp\left(-S\right)}{\int \mathscr{D}\phi \; \exp\left(-S\right)}, \tag{2.2}$$

where S denotes the Euclidean action, a functional of the fields ϕ. The similarity between eq. (2.2) and the expression for correlation functions in a statistical system suggests that the functional integrals appearing on the r.h.s. of eq. (2.2) could be evaluated using techniques analogous to the computational tools of statistical mechanics, including low- or high-temperature expansions, or numerical integration by Monte Carlo methods.

The latter approach requires the definition of a measure for the fields, that is both (*i*) mathematically well-defined, and (*ii*) suitable for numerical calculations. In practice, this means that the fields in the original theory have to be traded for a finite number of degrees of freedom, often defined in terms of matrix variables. This procedure is straightforward for bosonic fields, which are represented by ordinary, commuting c-numbers in the Feynman path-integral formalism. To enforce the correct statistics for fermionic fields, on the other hand, they have to be represented by Graßmann numbers; even though a direct computer evaluation of "ensemble averages" of products of Graßmann variables is not possible, the integration over these quantities can be carried out exactly for a Euclidean action which depends bilinearly on the fermionic fields. This fermionic Gaußian integral results into the determinant of the large, but finite-dimensional, matrix (a discretized version of the Dirac operator in the original theory), while fermionic operators appearing in a generic observable $\mathscr{O}$ are associated with elements of the inverse of such matrix [10, 11]. Note that, in general, both the determinant and the inverse matrix elements are highly non-local, yet completely well-defined functions of the bosonic degrees of freedom to which the fermionic fields are coupled. Thus, as long as the purely bosonic contribution to the Euclidean action is a real quantity bounded from below, and the fermionic matrix determinant is positive (a requirement that holds under certain conditions), each possible set of values of the bosonic degrees of freedom (a *configuration*) is associated with a properly normalized, real positive Boltzmann weight, and the "discretized" version of eq. (2.2) describes a completely well-defined statistical-mechanics model, with a finite number of degrees of freedom. Typically, the Boltzmann weight is a strongly peaked function in configuration space, so the high-dimensional integrals defining the correlation functions can be evaluated numerically, by Monte Carlo sampling. Note that the "discretization" of the original theory to a finite number of degrees of freedom introduces an intrinsic cutoff; the original theory is then recovered in the limit in which the "matrix size" (or, more generally, the number of degrees of freedom of the discretized version of the system) is taken to infinity. More precisely, in this limit the original theory arises as *a good low-energy effective description* of the discretized model—where "low-energy" means "at scales that are well-separated from the cutoff scale".

Let us now see how this can be done for path integrals in NC QFT. In general, the details of the regularization in terms of finite matrices depend on the model under consideration: the simplest examples are provided by NC scalar field theory in two dimensions. If one defines this theory on a fuzzy sphere [12], then the scalar field can be directly mapped to a Hermitian matrix Φ of finite size N, and the Euclidean action takes the form

$$S = \frac{4\pi}{N} \mathrm{Tr} \left(\Phi [L_i, [L_i, \Phi]] + rR^2\Phi^2 + \lambda R^2 \Phi^4 \right), \tag{2.3}$$

where R is the sphere radius, L_i (with $i = 1, 2, 3$) denotes a generator of the $su(2)$ algebra, in the representation of spin $j = (N-1)/2$, while r and λ are the rescaled coefficients of the quadratic and quartic terms in the potential, respectively. The computation of correlation functions is then expressed in terms of integrals over the entries of the Φ matrix

$$\langle \mathscr{O}_1 \dots \mathscr{O}_n \rangle = \frac{\int \prod_{i,j=1}^N \mathrm{d}\Phi_{ij} \; \mathscr{O}_1 \dots \mathscr{O}_n e^{-S}}{\int \prod_{i,j=1}^N \mathrm{d}\Phi_{ij} e^{-S}}. \tag{2.4}$$

This type of calculation has been carried out by Monte Carlo methods in a number of works [13, 14, 15, 16, 17] (including a very recent study of the entanglement entropy [18]).

A different way to map NC scalar field theory in two dimensions to a finite-dimensional matrix model was introduced in ref. [19], following an approach which is related to large-N volume reduction in the twisted Eguchi-Kawai model [20, 21] (see also ref. [22, subsection 4.7] for a detailed discussion). In this case, the Euclidean action of the discretized model reads

$$S = \mathrm{Tr} \left[\frac{1}{2} \sum_{\mu=1}^{2} \left(\Gamma_\mu \Phi \Gamma_\mu^\dagger - \Phi \right)^2 + \frac{\bar{m}^2}{2} \Phi^2 + \frac{\bar{\lambda}}{4} \Phi^4 \right], \tag{2.5}$$

where the kinetic term involves the "twist eaters" Γ_μ, which are $N \times N$ unitary matrices satisfying the 't Hooft-Weyl algebra, $\Gamma_\mu \Gamma_\nu = z_{\nu\mu} \Gamma_\nu \Gamma_\mu$, with $z_{12} = z_{21}^\star = -e^{i\pi/N}$. Also this formulation of NC scalar field theory in two dimensions has been studied numerically in various works [23, 24, 25].

Before moving on to a selection of results from numerical studies of NC QFT, we conclude this section comparing these types of discretization with the regularization of QFT on a spacetime grid that is performed in lattice field theory [26]. As it was suggested in the literature [27, 28], the discretization of NC QFT in terms of matrix models could provide an interesting and viable alternative to computations based on the lattice regularization (even for theories defined in ordinary, commutative spaces, if the non-commutativity parameter can be made sufficiently small, so that results can be eventually extrapolated to the commutative limit): it is therefore important to assess similarities and differences between the two approaches.

- Perhaps the most striking difference with the lattice regularization is that matrix-model discretizations of NC field theories can often be formulated in a way that preserves the spacetime symmetries exactly *at every value of the cutoff*. A very clear example is provided by the action for the scalar theory on the fuzzy sphere defined in eq. (2.3), which is explicitly invariant under continuum SO(3) rotations at every value of N. By contrast, the lattice breaks the group of Euclidean rotations down to a discrete subgroup—typically, the dihedral (in $D = 2$ spacetime dimensions), octahedral (in $D = 3$) or hyperoctahedral (in $D = 4$) group, if the lattice is a regular square, cubic or hypercubic grid. Similarly, the lattice also breaks the group of continuum translations down to the subgroup of translations by integer multiples of the lattice spacing a in each direction. This implies that, on the lattice, the spacetime symmetries of the continuum theory are explicitly broken by discretization artifacts, and get restored *only in the continuum limit* $a \to 0$.

- The treatment of fermionic fields (especially as it concerns chirality and anomalies) is somewhat simpler in matrix-model formulations NC field theories [29, 30, 31, 32, 33], whereas all ultralocal, chirally symmetric lattice formulations of the Dirac operator lead to unphysical doubler modes [34, 35], which can be removed at the cost of sacrificing either exact chiral symmetry at finite lattice spacing [36] or the ultralocality of the Dirac operator [37, 38, 39].

- Also the construction of supersymmetric models is usually simpler [40, 41], while a "direct" implementation of supersymmetry on the lattice requires fine-tuning [42], and more sophisticated formulations of lattice supersymmetry, which involve twisted formulations or orbifold constructions, are actually closer to matrix-model formulations of NC field theory [43].

- The typical computational costs of present NC field theory simulations are not prohibitive. Most state-of-the-art lattice QCD calculations, instead, require supercomputing resources.

- The reason for the latter difference is not in intrinsic limitations of the lattice formulation, but rather in the fact that the primary focus of numerical studies of NC models is on *qualitative* features of theories beyond the Standard Model, typically at energy scales far from the reach of present experiments, whereas lattice calculations are now in a precision era, and aim at accurate *quantitative* results for QCD phenomenology [44]—while topics beyond QCD (e.g. large-N gauge theories [45, 46], strongly coupled gauge theories for dynamical electro-weak

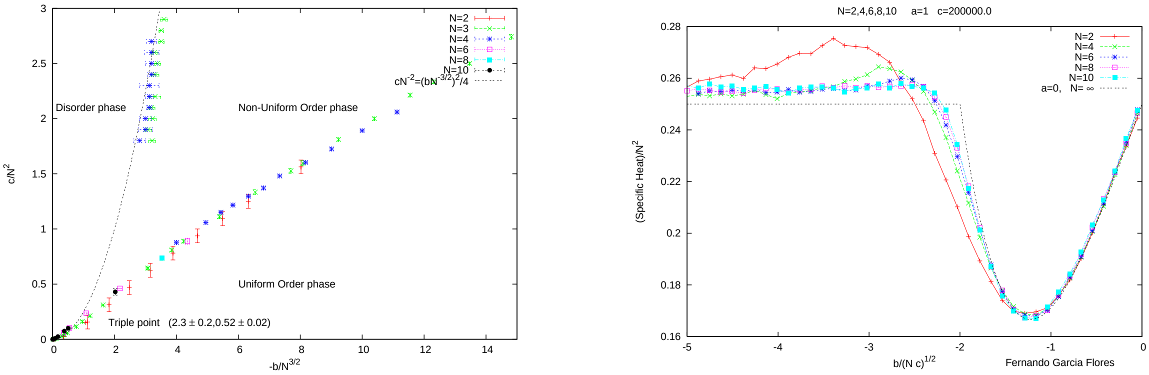

Figure 1: The phase structure (left panel) and the scaling of the specific heat at the disorder/non-uniform order transition with the matrix size N, and its comparison with analytical expectations in the large-N limit (right panel) of NC ϕ^4 theory in two dimensions, from the simulations on a fuzzy sphere reported in ref. [16].

symmetry breaking [47, 48], et c.) are covered in a sizable, but much smaller, fraction of the lattice literature.

3. Examples of results

In this section, we review a (limited) selection of results from recent numerical studies of NC models. We start from the results obtained in simulations of scalar field theories in subsection 3.1, before moving to those for non-supersymmetric gauge theories in subsection 3.2, and finally discussing those for supersymmetric models in subsection 3.3.

3.1 Results for NC scalar field theory

Several works [13, 14, 15, 16, 25, 49] studied the *phase structure* of NC ϕ^4 theory in two dimensions, finding numerical evidence for a *striped phase* [50] characterized by non-uniform order. As an example, fig. 1 shows the phases and the specific heat obtained in ref. [16].

Another recent numerical study of this theory was presented in ref. [25], in which the persistence of the striped phase in the continuum limit was confirmed by accurate extrapolations; as shown in fig. 2, taken from that article, this exotic phase is true to its name, with typical configurations exhibiting a characteristic striped pattern.

Related numerical studies have also been carried out on the fuzzy disc [51] (based on the construction presented in ref. [52]), in a tridimensional version of the model [53, 54], at finite temperature [55], in a multi-trace formulation [56, 57], et c.[1] The results of these numerical simulations can be compared with several analytical studies, including refs. [5, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76].

[1]A different type of study, in which Monte Carlo simulations were used to probe a space of random fuzzy geometries, was recently presented in ref. [58].

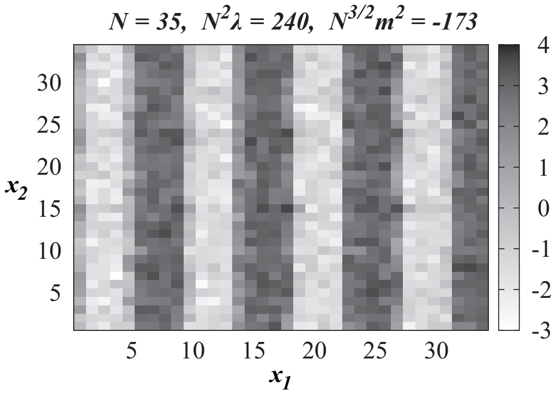

Figure 2: A typical configuration in the *striped phase* obtained in a numerical study of the NC ϕ^4 theory in two dimensions in ref. [25] using the mapping to a matrix model defined in ref. [19].

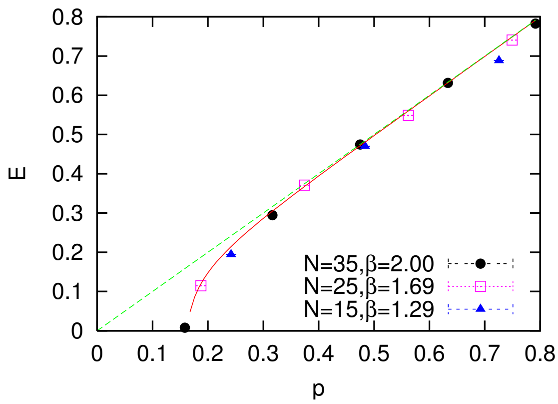

Figure 3: The photon dispersion relation obtained in numerical simulations of NC QED [104] reveals non-linear behavior at small momenta, in agreement with one-loop perturbative predictions [109, 110, 111].

3.2 Results for NC gauge theories

Analytical studies of gauge theories in NC spaces cover a *huge* body of literature: historically, the first example dates back to the first half of the past century [77]. During the past quarter-century, a major research line in these works has been the generalization of the Standard Model of particle physics to a NC framework—although this is only one of the motivations to study NC gauge theories. A very incomplete list of articles addressing problems in this research area includes refs. [78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99].

Numerical studies of gauge theories in NC spaces, on the other hand, have a relatively recent history [100, 101, 102, 103, 104, 105, 106, 107, 108]. An example of results of these studies is shown in fig. 3, taken from ref. [104], in which the photon dispersion relation obtained from numerical simulations of NC QED is displayed: the deviations from linear behavior (which are compatible with expectations derived analytically [109, 110, 111]) encode an interesting New Physics signature, which, as discussed in refs. [112, 113], could be potentially observable in ultra-high-energy cosmic rays.

3.3 Results for supersymmetric models

An interesting recent development in the research on NC theories regularized by means of finite matrices (and in numerical studies thereof) is based on the observation that closely related techniques can be applied for numerical simulations of dimensionally reduced $\mathcal{N}=4$ supersymmetric Yang-Mills theory [114], which describes the dynamics of N D0-branes in type IIA superstring theory [115]:

$$S = \frac{N}{2\lambda}\int_0^\beta \mathrm{d}\tau \,\mathrm{Tr}\left\{ (D_\tau X_i)^2 - \frac{1}{2}[X_i,X_j]^2 + \Psi_\alpha D_\tau \Psi_\alpha - \Psi_\alpha (\gamma_i)_{\alpha\beta}\left[X_i,\Psi_\beta\right] \right\}. \tag{3.1}$$

The extension to three or four spacetime dimensions can be achieved, by invoking arguments of large-N volume independence [116].

This approach has been successfully pursued in a number of studies [117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130]. As an example of results, the left-hand-side panel of fig. 4 shows the dependence of the energy on the temperature T in dimensionally reduced, maximally supersymmetric Yang-Mills theory, obtained in the Monte Carlo study reported in ref. [122]: at high temperature, the numerical results are in agreement with the expectations from an analytical expansion presented in ref. [131], while at low temperatures they can be successfully compared with the classical-supergravity prediction derived in ref. [132]. The right-hand-side panel of fig. 4, instead, shows the numerical results, obtained in ref. [133] using the same formulation of the theory, for the logarithm of Wilson-Polyakov loop introduced in ref. [134]: this quantity is plotted against the temperature (to the power $-3/5$) and is compared with the high-temperature expansion derived in ref. [131] and with the classical-supergravity prediction, which is given by

$$\langle \ln W \rangle = \sqrt[5]{\frac{120\pi^2}{49}} \left(\frac{T}{\sqrt[3]{\lambda}} \right)^{-3/5} + \dots . \tag{3.2}$$

4. Summary and outlook

Numerical simulations provide a controlled and systematically improvable tool to study NC QFT from first principles. Not only can they be used to cross-check exact analytical results, but also (more importantly) to determine the validity range of analytical calculations that involve some form of approximation, or to provide guidance to understand problems which are harder to tackle analytically.

As we discussed above, typical Monte Carlo simulations of NC field theories can be successfully performed with modest computational resources, and the studies of this type that several groups have been carrying out during the past few years have led to many interesting results, for various NC theories.

Interestingly, some techniques devised for simulations of NC QFT can also be applied to field theories defined in ordinary, commutative spaces. For certain problems, the numerical implementation of matrix-model regularization methods inspired by NC field theories proves competitive with respect to more conventional (e.g. lattice) approaches: one striking example is provided by the investigation of dimensionally reduced supersymmetric gauge theories, that we briefly reviewed in subsection 3.3.

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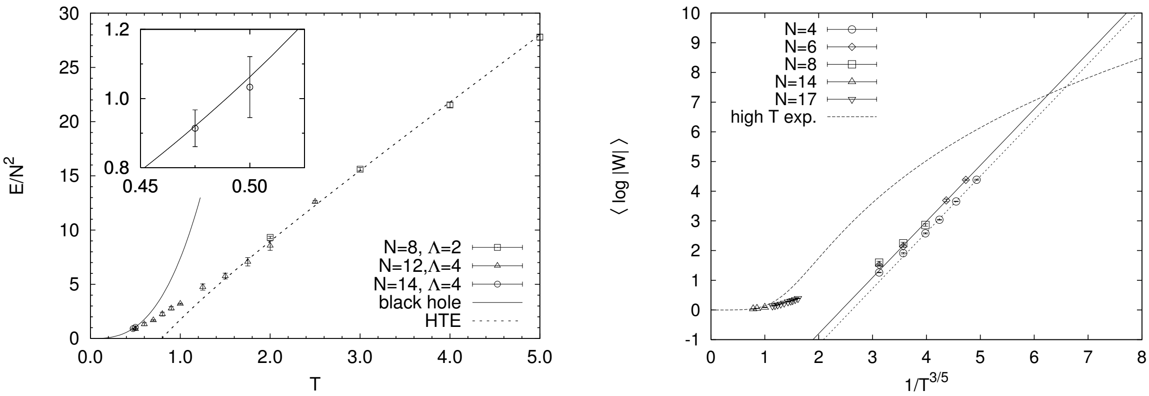

Figure 4: Examples of results from numerical simulations of supersymmetric matrix models. The left-hand-side panel shows the energy (normalized by N^2) in dimensionally-reduced $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, against the temperature T [122], in comparison with a high-temperature expansion [131] and with the classical supergravity prediction [132]. The right-hand-side panel shows the logarithm of Wilson-Polyakov loop [134] in dimensionally-reduced $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, against $T^{-3/5}$ [133] in comparison with analytical predictions from a high-temperature expansion [131] and with the expectation in the classical supergravity limit given by eq. (3.2).

Finally, it is worthwhile noting that some numerical challenges in Monte Carlo studies of NC field theories are common to lattice QCD, and any progress towards their solution in one research area could potentially lead to very fruitful developments in the other, too. For example, the authors of refs. [135, 136] devised a novel factorization method to cope with the numerical "sign problem" affecting their model: as is well-known, this notorious computational problem also hinders simulations of lattice QCD at finite baryon density [137] and of condensed-matter systems [138].

For all of these reasons, a closer interaction of the two communities would be highly desirable.

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