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Asymptotically Safe $R+R^2$ gravity

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In this work a class of $R+R^2$ inflationary models emerging in the context of the *Asymptotic Safety* approach is discussed. In particular, an extended comparison between the theoretical results and the latest *Planck* data is reported.

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1. Introduction

The search of a theory of quantum gravity is one of the most important unsolved problem in modern Theoretical Physics. As Einstein Gravity is a non-renormalizable theory at the perturbative level, different proposal have tried to address the issue of non-perturbative renormalizability. According to the *Asymptotic Safety* idea [47] gravity can be a fundamental theory and still be predictive if the high energy behavior is governed by the scaling around a *Non Gaussian Fixed Point* (NGFP), that is a fixed point defined at non-zero values of the (dimensionless) coupling constants. In this framework, the Einstein-Hilbert theory results in a predictive theory as the NGFP's critical manifold has finite dimensionality [2–6, 8, 11, 15, 16, 20–24, 28–34, 37–40, 44].

The Asymptotic Safety Theory found applications in different fields of Physics, such as in Astrophysics and Cosmology. As pointed by Weinberg in [46], it is possible to use the *Asymptotic Safe theory of gravity* to study inflation. The main idea is to study the natural evolution of the RG trajectories, and see if at the inflationary era the resulting effective theory can describe a period of exponential expansion which lasts long enough to solve the problems of standard cosmology. In this way it is possible to relate the quantum gravity effect at the Planck scale to the observed temperature fluctuations in the CMB [35], and compare the theoretical results with the observations.

In this work we use this idea to extend the analysis of [9] (see also [7]). In particular, we will study all the possible inflationary scenarios generated by the quadratic UV Lagrangian:

$$\mathcal{L}_{UV} = \frac{1}{16\pi G}(R - 2\Lambda) + \beta R^2 \tag{1.1}$$

The reasons of this choice is that in the UV limit the leading contribution in the $f(R)$ expansion is given by a quadratic term in the Ricci scalar [1]. Other contributions come from the linearization around the NGFP. So it is natural to consider, as Planck scale Lagrangian, an $f(R)$ expansion that contains both the main contribution R^2 together with the Einstein-Hilbert terms, that describe the behavior of gravity at lower energy scale. In this article we want to extend the analysis of [9] to see in which cases a quantum quadratic Lagrangian can provide a finite period of nearly exponential expansion, and compare the theoretical results with the latest Plank data on the CMB power spectrum (see also [12, 13, 17, 41–43] for other examples of quantum quadratic gravity inflationary models).

2. Derivation of the inflationary potential

The angular power spectrum of the CMB temperature fluctuations can be described by the well known spectral index n_s and tensor-to-scalar ratio r.

In the usual theoretical analysis these parameters are determined by using the slow roll approximation: a scalar field $\phi(t)$ moves along its potential $V(\phi)$, and the inflation occurs if it rolls very slowly compared to the expansion of the universe. This condition -

slow-roll condition - is mathematically verified when the slow-roll parameters:

$$\epsilon(\phi) = \frac{1}{2\kappa^2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \qquad \eta(\phi) = \frac{1}{\kappa^2}\left(\frac{V''(\phi)}{V(\phi)}\right) \tag{2.1}$$

are very small, compared to the unity. Usually it is assumed that the inflation ends when the function $\epsilon(\phi)$ assumes unitary value, $\epsilon(\phi_f) = 1$. This condition identifies the value $\phi_f = \phi(t_f)$ of the scalar field ϕ when the exponential expansion ends. The initial value of the field $\phi_i = \phi(t_i)$ is implicitly defined by the number of e-folds $N(\phi_i)$, that is given by the following integral function:

$$N(\phi_i) = \int_{\phi_f}^{\phi_i} \frac{V(\phi)}{V'(\phi)}\, \mathrm{d}\phi \tag{2.2}$$

For a fixed number of e-folds N, this relation allows us to determine ϕ_i, and so to calculate the spectral index and the tensor-to-scalar ratio in a simple way:

$$n_s = 1 + 2\,\eta(\phi_i) - 6\,\epsilon(\phi_i) \tag{2.3}$$

$$r = 16\,\epsilon(\phi_i) \tag{2.4}$$

For a given model - i.e. for a given function $V(\phi)$ - these theoretical values can be computed and then compared to the corresponding values obtained from the Planck data.

In our case the main task is to find an analytical expression for the inflationary potential by starting from the quantum gravity theory described by the Planck scale Lagrangian in eq. 1.1. In order to obtain an effective Lagrangian at the inflationary scale, it is necessary to know the scale dependence of the dimensionless running coupling constants $g(k)$, $\lambda(k)$ and $\beta(k)$. Such coupling constants are supposed to converge to the NGFP in the UV limit [28, 36]. The analytical calculation of the inflationary potential is however possible only if the (dimensionless) running cosmological constant $\lambda(k)$ is approximated with its tree level scaling, as discussed in [9, 11, 16].
The energy-scale dependence of the dimensionless coupling constants in eq. 1.1 is [9, 11]:

$$g(k) = \frac{(6\,\pi\,k^2)\,g(\mu)}{6\,\pi\mu^2 + 23\,g(\mu)\,(k^2 - \mu^2)} \tag{2.5}$$

$$\beta(k) = \beta_* + b_0\left(\frac{k^2}{\mu^2}\right)^{-\frac{\theta_3}{2}} \tag{2.6}$$

$$\lambda(k) \sim c_0\, k^{-2} \tag{2.7}$$

Where μ is an infrared renormalization scale and θ_3 is the critical exponent relative to $\beta(k)$ (see [11, 28, 36]). The free parameters $(b_0, c_0, g(\mu))$ define a family of renormalization group trajectories, and their number reflect the fact that the critical manifold is characterized by only three relevant directions.

As pointed in [7, 18, 25] an effective Lagrangian $\mathcal{L}_k$ at the scale k is obtained by substituting in the UV Lagrangian the scale-depending coupling constants:

$$\mathcal{L}_k = \frac{k^2}{16\pi g(k)} \left\{ R - 2\,k^2\,\lambda(k) \right\} + \beta(k)\,R^2 \tag{2.8}$$

Where, in our case, $g(k)$, $\lambda(k)$ and $\beta(k)$ are the dimensionless running coupling constants defined through the expressions 2.5, 2.6 and 2.7.

In order to find an effective action at the inflationary era, it is necessary to identify the energy scale k [27, 45] with some physical quantity that, in this specific case, can describe the natural evolution of the RG flow.
A possibility is to relate k to the cosmological time through the relation $k \sim t^{-1}$ but, as discussed in [10], this choice breaks the diffeomorphism invariance. An other possibility comes from the following observation: if the scale factor $a(t)$ is a power law then the Ricci scalar is $R \sim t^{-2}$. The idea is thus to perform the following cutoff identification [26]:

$$k^2 \equiv \xi\, R \tag{2.9}$$

Following this argument, the RG effective action is therefore:

$$S = \frac{1}{2\kappa^2} \int \sqrt{-g}\, \left\{ R + \alpha\, R^{2-\frac{\theta_3}{2}} + \frac{R^2}{6\,m^2} - \Lambda \right\} \tag{2.10}$$

Where α and Λ are effective coupling constants:

$$\alpha = -\frac{2\,b_0\,\mu^{\theta_3}}{M_{\rm pl}^2} \tag{2.11}$$

$$\Lambda = \frac{(6\,\pi - 23\,g(\mu))\,c_0\,\mu^2}{6\,\pi^2 \mu^2 - 23\,(\mu^2 + 2\,\xi\,c_0)\,g(\mu)} \tag{2.12}$$

While κ^2 and m^2 result from the following identifications:

$$\kappa^2 \equiv 8\,\pi\,G_N = \frac{48\,\pi^2\,g(\mu)}{6\,\pi^2 \mu^2 - 23\,(\mu^2 + 2\,\xi\,c_0)\,g(\mu)} \tag{2.13}$$

$$m^2 = \frac{M_{\rm pl}^2}{12\,(23\,\xi\,(96\,\pi^2 - \beta_*)^{-1})} \tag{2.14}$$

Following the analysis of [9, 16], because of $\theta_3 \sim 1$ it is useful to set $\theta_3 = 1$: in this way, it is possible to perform a Weyl transformation from the Jordan's to the Einstein frame [14, 19], and to obtain an analytical expression for the inflationary potential. In particular, it

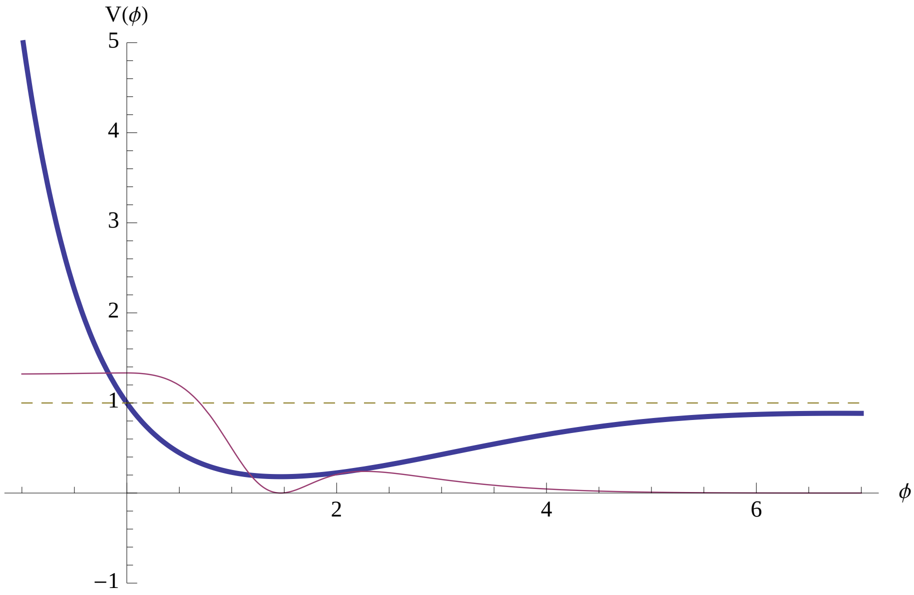

Figure 1: Inflationary potential $V_{-}(\phi)$ for $\alpha = -10$ and $\Lambda = 2$ (blue line) and the slow-roll function $\epsilon(\phi)$ (red line).

was found that there are two possible branches for $V(\phi)$:

$$V_{\pm}(\phi) = \frac{m^2 \mathrm{e}^{-2\sqrt{\frac{2}{3}}\kappa\phi}}{256\kappa^2}\Big\{192\left(\mathrm{e}^{\sqrt{\frac{2}{3}}\kappa\phi}-1\right)^2 - 3\alpha^4 + 128\Lambda - 3\alpha^2\left(\alpha^2+16\,\mathrm{e}^{\sqrt{\frac{2}{3}}\kappa\phi}-16\right)$$
$$\mp 6\alpha^3\sqrt{\alpha^2+16\,\mathrm{e}^{\sqrt{\frac{2}{3}}\kappa\phi}-16} - \sqrt{32}\alpha\Big[\left(\alpha^2+8\mathrm{e}^{\sqrt{\frac{2}{3}}\kappa\phi}-8\right)\pm\alpha\sqrt{\alpha^2+16\,\mathrm{e}^{\sqrt{\frac{2}{3}}\kappa\phi}-16}\Big]^{\frac{3}{2}}\Big\} \tag{2.15}$$

In which α and Λ are dimensionless effective coupling constants, obtained by rescaling $\alpha \to \alpha/3\sqrt{3}m$ and $\Lambda \to \Lambda m^2$.

3. Inflationary scenario and results

We are now ready to do a detailed analysis of all possible inflationary dynamics generated by the potential in eq. 2.15, extending the analysis of [9].

The scalar potential $V_{\pm}(\phi)$ in eq. 2.15 is characterized by a plateau for $\phi \to +\infty$ and, depending on the values of α and Λ, it can have the following behaviors:

- $V_{\pm}(\phi)$ has a minimum and $\lim_{\phi\to-\infty} V_{\pm}(\phi) = +\infty$
- $V_{\pm}(\phi)$ has no stationary points and $\lim_{\phi\to-\infty} V_{\pm}(\phi) = -\infty$

In particular, if $V_{\pm}(\phi)$ has a minimum then its value $V_{\pm}(\phi_{\mathrm{min}})$ is always positive for $V_{-}(\phi)$, while it can be either positive or negative for $V_{+}(\phi)$. In [9] we did a detailed analysis of

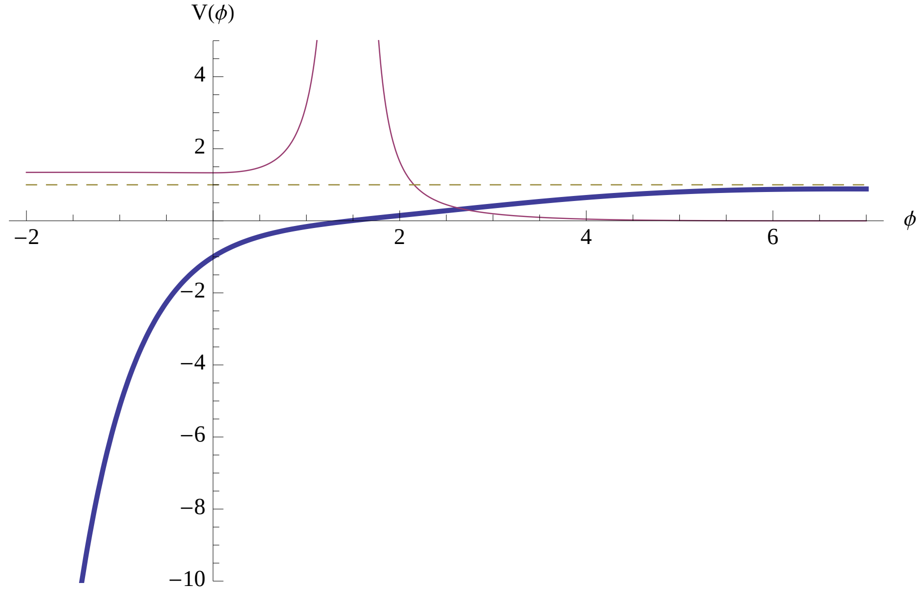

Figure 2: Inflationary potential $V_{-}(\phi)$ for $\alpha = -10$ and $\Lambda = -2$ (blue line) and the slow-roll function $\epsilon(\phi)$ (red line).

the inflationary dynamics for the case $V_{+}(\phi_{\min}) < 0$, in this paper we want to discuss the dynamics in the other two cases.

It is clear that a pre-heating phase due to inflaton's oscillations is possible if and only if $V_{\pm}(\phi)$ has a minimum. However, in the case in which $\exists\, \phi_{\min}$ s.t. $V'(\phi_{\min}) = 0$, inflation can have a natural end by violation of the slow-roll condition $\epsilon(\phi_f) = 1$ if and only if the minimum value of the potential is negative. The situation is the following: the "slow-rolling field" starts from the plateau region and moves towards the minimum, but if $V_{\pm}(\phi_{\min}) \geq 0$ then the value $\epsilon = 1$ is reached for $\phi = \phi_f < \phi_{\min}$. Then, ϕ get back towards the minimum and the oscillatory phase starts, but this means that these oscillations occur within a range of ϕ for which $\epsilon(\phi) < 1$ (see fig. 1): this class of potentials gives rise to an *eternal inflationary model.*

On the contrary, the class of potentials that diverges negatively is characterized by a well defined exit from inflation, as it is clear in the picture 2, but the reheating phase cannot be explained through inflaton's parametric oscillations. The inflationary dynamic for these two cases is depicted in fig. 3.

A more reasonable case is the one in which a "graceful exit" from inflation (i.e. by violation of the slow-roll condition) occurs, and a reheating phase due to inflaton oscillatory phase follows. This configuration is realized when $V_{\pm}(\phi_{\min}) \leq 0$ (see fig. 4, left panel), and the pre-heating phase is characterized by a limit cycle behavior [9], as shown in the right panel of fig. 4.

In this case the spectral index n_s is very similar to the value obtained by the *Planck* Collaboration, and the tensor-to-scalar ratio is compatible with their upper limit [35]. The results, for different values of α and Λ, are summarized in the following table [9]:

Cases		$N = 50$		$N = 55$		$N = 60$	
Λ	α	n_s	r	n_s	r	n_s	r
	1.0	0.965	0.0069	0.968	0.0058	0.971	0.0050
0	1.8	0.966	0.0074	0.969	0.0063	0.972	0.0055
	2.6	0.967	0.0076	0.969	0.0065	0.972	0.0056
	1.0	0.965	0.0070	0.968	0.0059	0.971	0.0051
1	1.8	0.966	0.0074	0.969	0.0063	0.972	0.0055
	2.6	0.967	0.0076	0.969	0.0065	0.972	0.0056

In the case in which the potential diverges negatively there is no reheating phase, but the well defined exit from inflation allow us to determine n_s and r with the usual slow-roll approximation. The way in which reheating occurs is beyond the purpose of our work. Our results for the spectral index and the tensor-to-scalar ratio are the following:

Cases		$N = 50$		$N = 55$		$N = 60$	
Λ	α	n_s	r	n_s	r	n_s	r
	-2.0	0.807	$1.5 \cdot 10^{-6}$	0.808	$6.0 \cdot 10^{-7}$	0.808	$2.3 \cdot 10^{-7}$
-5	$+1.0$	0.966	0.0066	0.969	0.0056	0.972	0.0048
	$+2.0$	0.966	0.0074	0.969	0.0064	0.972	0.0055
	-2.0	0.809	$1.6 \cdot 10^{-6}$	0.810	$6.1 \cdot 10^{-7}$	0.810	$2.4 \cdot 10^{-7}$
-10	$+1.0$	0.967	0.0063	0.970	0.0054	0.972	0.0047
	$+2.0$	0.967	0.0074	0.970	0.0063	0.972	0.0054

We note that if $\alpha > 0$ also these results are in agreement with the latest Planck data, but we have to remark that in this case no reheating phase occurs.

4. Conclusions

In this work we preformed a detailed analysis of all possible inflationary scenarios generated by a quantum quadratic gravity, extending the study of [9]. The renormalization group flow gives rise to a class of potentials whose behavior depends on two parameters. By spanning the parameter space, we found that there are three possible forms for the inflationary potential, each one corresponding to a particular inflationary dynamics. Potentials that diverge negatively for $\phi \to -\infty$ allow an exit from inflation in a natural way, but they are not able to produce the following reheating phase. In the case in which the potential has a minimum, an important role is played by its value at the minimum: if it is

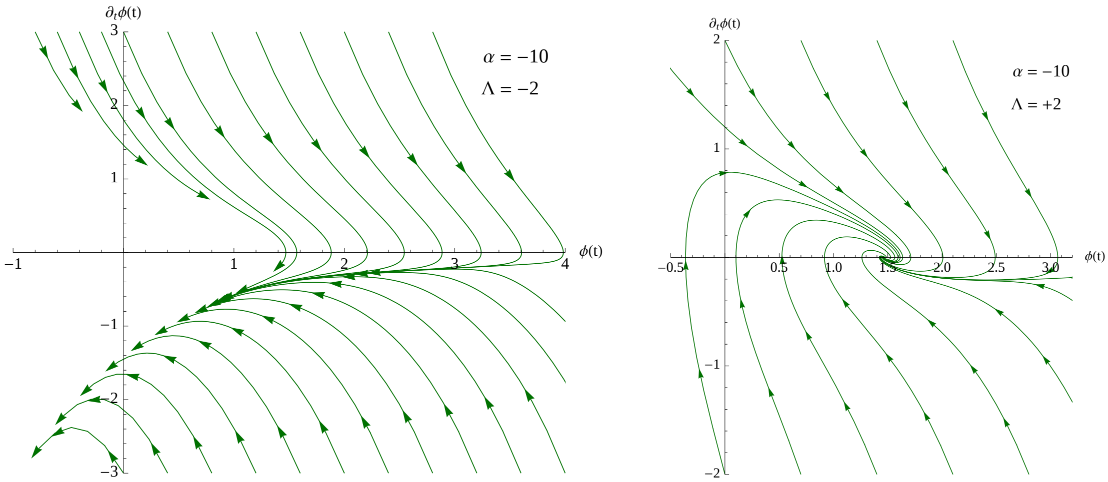

Figure 3: Phase diagrams in the $(\phi(t), \partial_t\phi(t))$ plane that describe the inflationary dynamics for the class of potentials with a negative divergence (left panel) and with $V_\pm(\phi_{\rm min}) < 0$ (right panel).

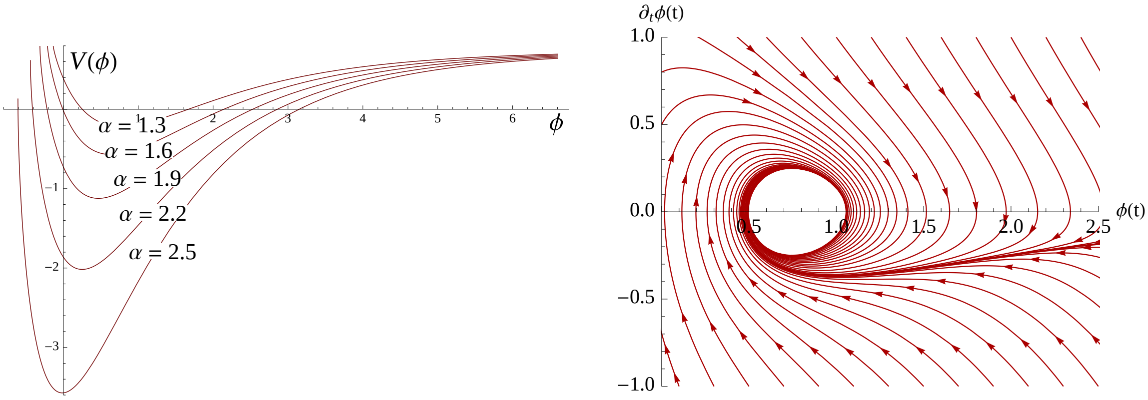

Figure 4: Inflationary potential $V_+(\phi)$ for $\Lambda = 1.4$ and different values of α. As proved in [9], in this case the post-inflationary dynamics is characterized by a limit cycle behavior.

positive the inflaton is constrained in the slow-roll region, and thus inflation can never end (eternal inflation dynamics). The only case in which both a natural exit from inflation and a reheating phase occur, is given by the class of potentials with a negative value of the minimum. As shown in [9], in this case the values of the spectral index and the tensor-to-scalar ratio are in agreement with the latest values obtained by the *Planck* Collaboration.

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