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Design of Non-overshooting State-Feedback Controller for the Fractional Derivative Multi-input And Multi-output System

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This paper investigates the non-overshooting tracking problem as to a class of fractional order linear time invariant (FO-LTI) and multi-input multi-output (MIMO) systems. A method is provided for designing the state feedback controller to asymptotically track the step references without non-overshooting. The state feedback matrix F proposed here can take from the combined eigenvalue and eigenvector placement methods given in Moore. The state feedback matrix F can control the system's overshooting. The decay rate of non-overshooting controller can be arbitrary and only be determined by the expected eigenvalues. Finally, a numerical example illustrates the effectiveness of the proposed method.

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1. Introduction

Many high precision control systems are applied in the manufacture and production process. Overshooting is one of the most dynamic time domain indexes about designing the control systems. In the complex process of manufacturing, like high speed lathe feeding controller, the aircraft climb and drop section and design of the self-controlled robot are all requiring the system to produce non-overshooting state-feedback as far as possible with step response[1][2]. Its significance aimed at reducing the damage to the productionby a wide margin. In recent decades, many scholars have researched the overshooting of the systems. Chen and Hong considered using the backward method to research the necessary and sufficient conditions of the non-overshooting control rate of system's order less than four about strictly proper integer order nonlinear system[3]. Gyurkovics and Takacs provided some conditions by formulated in some matris ineaualities to constraint the continuous and discrete timing systems, but their research was confined to nonlinear constraint elements and paid no attention to the linear elements[4]. Nguyen and Leonessa considered three components reling on the feedback lows: a predictor, a reference model and a controller. The designed of feedback control lows forecasted the output to the MIMO linear system[5]. Geromel and Souza figured out the nonuniform data rates and testified the stability of the performance to the closed loop system by designing a special teo points boundary value[6]. Kulkarni, Purwar and Sharma considered a controller consisted of non-linear and linear elements for TRMS and guaranteed minimum settling time with nonovershooting to response[7]. Formerly, scholars studied the state feedback controller which was limited to the low order real number system and the nonlinear system. In this article, we expand the domain of the controller to the fractional order.

The overshoot of step response relates to the zero pole of system. Accordingly, the feedback low designed by the combined eigenvalue and eigenvector placement methods given in Moore[8]. Let LMI system for continuous time obtain a non-overshoot reference input under any initial conditions by using linear state feedback controller. This article aims at designing a non-overshooting state-feedback controller for the fractional derivative MIMO system. The problem of non-overshooting tracking for a class of fractional order linear time invariant MIMO systems is considered. The state feedback controller based on this method can make the reference input without overshoot. The decadent rates of the non-overshooting controller can be arbitrary values and only determined by the expectant eigenvalues. Finally, the validity of the method is explained by the digital simulation.

2. Problem Formulation

Definition.2.1[9]. Define integral and order *m* differentiable function *f(t)* as $\alpha(\alpha<0)$ order *Caputo fractional order* derivative:

$$D^{\alpha} f(t)=\frac{1}{\Gamma(m-\alpha)}\int_{t_0}^{t}\frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha+1-m}}d\tau \tag{2.1}$$

for *m* is integer and $m-1<\alpha<m$, $\Gamma(*)$ is an Euler-Gammar function，$f^{(m)}(*)$ express *m* order derivative of function *f(*)*.

Definition 2.2. Define *Mittag-Leffler function* as: $E_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)}$ for $\alpha>0,\ \beta>0.$

Consider the commensurate fractional order LTI system Σ *be* governed by:

$$\Sigma:\begin{cases} D^{\alpha}x(t)=Ax(t)+Bu(t) \\ y(t)=Cx(t)+Du(t),\ x(0)=x_0\in R^n \end{cases} \tag{2.2}$$

where for $0<\alpha<1,\ t\in T,\ x(t)\in R$ is the state vector, $u(t)\in R$ is the control input, $y(t)\in R$ is the output, constant matrix A, B, C, D are the system matrix. The paper aims at designing a class of state feedback control laws for linear time invariant systems which can be used to ensure that the output $y(t)\in R$ of the system without overshoot for any step response $r\in R$ and adjoined zero stability. As such, make the following assumption.

Assumption 2.1. System Σ is stable and invertible and has no invariant zero at the origin. B isfull column rank and C is full column rank.

Lemma 2.1[10][11]. Let A be a real matrix.The necessary and sufficient conditions to the asymptotic stability of $\frac{d^{\alpha}x(t)}{dt^{\alpha}}=A\,x(t)$ is

$$|arg(spec(A))|>\frac{\alpha\pi}{2} \tag{2.3}$$

where, *spec(A)* is the domain of all eigenvalues of matrix A.

According o **Lemma 2.1,** nonovershooting tracking controller about step response r designed by eigenvalue and eigenvector placementmethods. Let $|arg(spec(A))|>\frac{\alpha\pi}{2}$, obtaina feedback gain matrix F and existence vector x_S ,$u_S$satisfied:

$$0=Ax_s+Bu_s \tag{2.4}$$

$$r=Cx_s+Du_s \tag{2.5}$$

Construct a state feedback control low:

$$u(t)=F(x(t)-x_s)+u_s \tag{2.6}$$

Homogeneous closed-loop system obtained from (2.2) and $\zeta(t):=x(t)-x_S$:

$$\Sigma_{hom}:\begin{cases} D^{\alpha}\xi(t)=(A+BF)\xi(t) \\ y(t)=(C+D)\xi(t)+r,\ \xi(0)=\xi_0=x_0-x_s\in R^n \end{cases} \tag{2.7}$$

Cause $|(A+BF)|>\frac{\alpha\pi}{2}$ state variable $x(t)$ tends to x_S and output $y(t)$ tends to r when t tending to infinity. The output y obtained a tracking error $\varepsilon(t)=r-y$ from x_0 which satisfied:

(i) $t\to\infty,\ \varepsilon(t)=r-y\to\infty$。

(ii) For anyinitial conditions x_0 and step response r, $\varepsilon(t)$ is stable.

From the above, system Σ has a non-overshooting step response from initial condition x_0 to r. It clears that Σ has a globally non-overshooting response for if r the output y is non-overshoot for all initial conditions x_0.

Lemma 2.2. Let $\lambda=\{\lambda_1,\lambda_2,\ldots,\lambda_n\}$ be a self-conjugate set of n distinct complex numbers. Let $s=\{s_1,s_2,\ldots,s_n\}$ be a set of n vectors in R^P.

Assumption 2.2. For $i\in\{1,2,\ldots,n\}$, the matrix equation:

$$\begin{bmatrix} A-\lambda_i I_n & B \\ C & D \end{bmatrix}\begin{bmatrix} v_i \\ W_i \end{bmatrix}=\begin{bmatrix} 0 \\ s_i \end{bmatrix} \tag{2.8}$$

Has solution set $V=\{v_1,v_2,\ldots,v_n\}\in C$ and $\in W=\{w_1,w_2,\ldots,w_n\}\in$ C. Then, provided v is linearly independent, an unique real feedback matrix $F=WV^{-1}$ exits, such that $i\in\{1,2,\ldots,n\}$,

$$(A+BF)v_i=\lambda_i v_i \tag{2.9}$$

$$(C+DF)v_i=s_i \tag{2.10}$$

3. Design of the Feedback Controller for Square Systems

3.1 System Σ has at the least n-p distinct invariant zeros.

Assumption 3.1. System Σ here we considered is the square $(p=q)$.

Assumption 3.2. System Σ has at least n-p distinct invariant zeroes lying in the region: $|arg(A+BF)|>\frac{\alpha\pi}{2}$.

Let $\lambda=\{\lambda_1,\lambda_2,\ldots,\lambda_n\}$ be a self-conjugate set of independent steady-state closed-loop eigenvalues of matrix $A+BF$ and $\{z_1,z_2,\ldots,z_{n-p}\}$ be n-p independent stable zeroes of system Σ. Let $\lambda_i=Z_i$, $i\in\{1,2,\ldots,n-p\}$, for $i\in\{n-p+1,\ldots,n\}$, independent eigenvalue λ_i can be optional selected from C_s and differ in the invariant zeroes of system Σ.

Remark 3.1. As the plural invariant zero exists in pairs, in order to ensure λ that shall belong to the self-conjuga te set, for $i\in\{n-p+1,\ldots,n\}$:

(i) Eigenvalues λ_i located at arbitrary position in the open left half part of the complex plane.

(ii) Select eigenvalues λ_i arbitrarily existing in pairs while it`s plural.

Let $\{e_1,e_2,\ldots,e_P\}$ be the canonical base of R^P, $s=\{s_1,s_2,\ldots,s_n\}$ as:

$$s=\begin{cases} 0, i\in\{1,2,\ldots,p\} \\ e_{1,} i=n-p+1 \\ \cdot \\ \cdot \\ \cdot \\ e_p, i=n \end{cases} \tag{3.1}$$

$V=\{v_1,v_2,\ldots,v_n\}$ *and* $W=\{w_1,w_2,\ldots,w_n\}$ obtained by λ_i selected from (2.8), for $i\in\{1,2,\ldots,n\}$.

According to **Lemma2.1,** if V is linear independent, state feedback gain matrix $F=WV^{-1}$ can *ensure* the stability of the closed-loop system. V satisfies:

$$(A+BF)v_i=\lambda_i v_i,\quad i\in\{1,2,\ldots,n\} \tag{3.2}$$

$$(C+DF)w_i=\begin{cases} 0 & i\in\{1,2,\ldots,n\} \\ e_{i-(n-p)} & i\in\{n-p+1,\ldots,n\} \end{cases} \tag{3.3}$$

Theory 3.1. System Σ satisfies **Assumption 2.1, 3.1 and 3.2.** Let L be expected closed-loop pole set, and associated eigenvector set V obtained from (2.8) which is linear and independent. Let r be step response and x_0 be initial condition. Matrix F is obtained by Moore arithmetic. The output $y(t)$ obtained by $u(t)$ from (2.6) to system Σ is non-overshooting.

Proof: Homogeneous system Σ_{hom} in (2.7) obtained from which the state feedback control low (2.6) applying to system Σ. For any initial condition $\zeta_0=x_0-x_S$ and step response r , the tracking error $\varepsilon(t)=r-y$ is obtained by:

$$\varepsilon(t)=(C+DF)\xi_0 E_\alpha[(A+BF)t_\alpha] \tag{3.4}$$

According toeigenvalue of matrix $(A+BF)$ is independent that:

$$\Lambda=V_{-1}(A+BF)V=diag(\lambda_1,\lambda_2,\dots,\lambda_n) \tag{3.5}$$

so

$$E_{\alpha,1}[(A+BF)t^\alpha]=\sum_{k=0}^{\infty}\frac{[V\Lambda V_{-1t_\alpha}]^k}{\Gamma(k\alpha+1)}=V[\sum_{k=0}^{\infty}\frac{[\Lambda t^\alpha]_k}{\Gamma(k\alpha+1)}]V_{-1}=VE_{\alpha,1}(\Lambda t^\alpha)V_{-1} \tag{3.6}$$

for

$$E_{\alpha,1}=diag[E_{\alpha,1}(\lambda_1 t^\alpha),E_{\alpha,1}(\lambda_2 t^\alpha),\dots,E_{\alpha,1}(\lambda_n t^\alpha)] \tag{3.7}$$

According to (13-16) , the tracking error obtained by $\delta:=[\delta_1,\delta_2,\dots,\delta_n]^T=V^{1}\zeta_0$ is given by

$$\begin{aligned}\varepsilon(t)&=(C+DF)VE_{\alpha,1}[\Lambda t^\alpha]V^{-1}\xi_0\\ &=\sum_{i=n-p+1}^{n}e_{i-(n-p)}\delta E_{\alpha,1}[\lambda_t t^\alpha]=\begin{bmatrix}\delta_{n-p+1}E_{\alpha,1}(\lambda_{n-p+1}t^\alpha)\\ \dots\\ \delta_n E_{\alpha,1}(\lambda_n t^\alpha)\end{bmatrix}\end{aligned} \tag{3.8}$$

From that, every part of p of $\varepsilon(t)$ only contains one modality:$\varepsilon(t)_i=\delta_{n-p+1}E_{\alpha,1}[\lambda_{n-p+1}t^\alpha]$. System Σ_{hom} tends to be stable gradually when all eigenvalues are included to C_S and $t\to\infty$, $\varepsilon(t)\to 0$. For $i\in\{n-p+1,\dots,n\}$, λ_i is determined and $E_{\alpha,1}[\lambda_i t^\alpha]$ do not change the sign. $\varepsilon(t)$ doesn't change sign in any part and output y is non-overshooting to step response r.

4. Example

Let system Σ_1 as fractional order linear invariant system and $\alpha=0.9$.

$$\Sigma_1:\begin{cases}D^{0.9}x(t)=\begin{bmatrix}-9&-9&5&0&-3\\-8&0&0&-7&0\\-10&0&8&-5&0\\-10&0&8&-5&0\\1&0&0&0&-7\end{bmatrix}x(t)+\begin{bmatrix}0&0\\6&0\\9&0\\2&-10\\0&0\end{bmatrix}u(t)\ ,x(0)=x_0\in R^n\\ y(t)=\begin{bmatrix}10&0&0&0&-1\\0&0&0&-1&-4\end{bmatrix}x(t)+\begin{bmatrix}1&2\\0&-1\end{bmatrix}u(t)\end{cases} \tag{4.1}$$

Assume that tracking response as $[5\ -5]^T$. It's clear that system Σ_1 has five invariant zeroes at 14.4207±3.0814*i*, -17.1802+11.9843*i*, -6.4809. Exists n-p=3 stable invariant zeroes in the domain of C_S. Choose -17.1802+11.9843*i*, -6.4809 as closed-loop system eigenvalues. The last eigenvalues can be selected from any real number in the open and left half part complex plane.

According to **Lemma2.1,** state feedback gain matrix *F* given by choosing -8 and -10:

$$F = \begin{bmatrix} 1.3942 & -8.8883 & 3.1104 & 2.7451 & -3.2564 \\ -0.4510 & -2.1672 & 1.2501 & 1.2996 & -0.06328 \end{bmatrix} \tag{4.2}$$

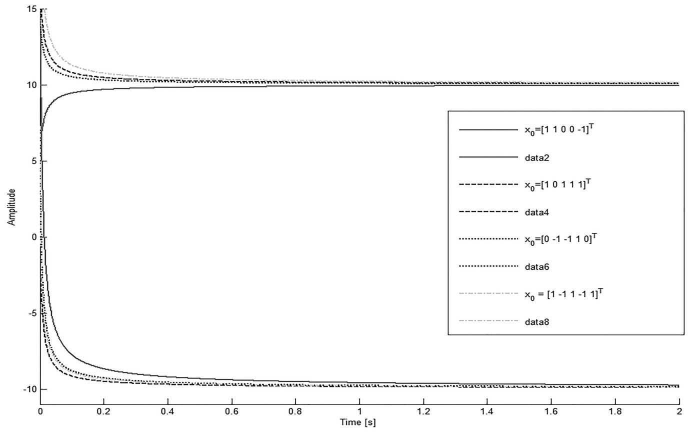

Figure 1 : Simulation

Let system Σ_1 as fractional order linear invariant system and $\alpha=0.7$.

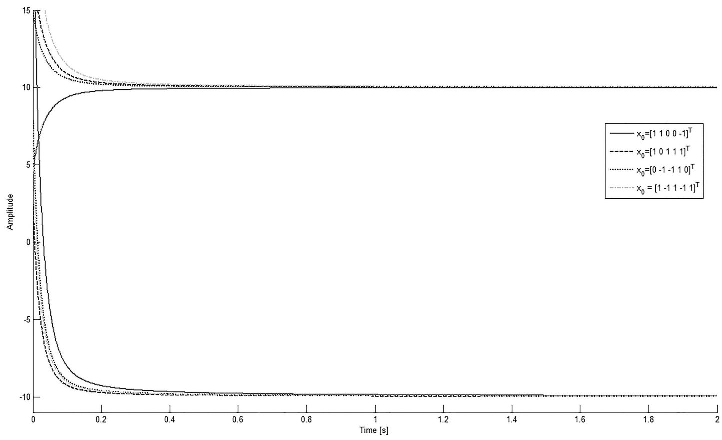

Figure 2 : Simulation

By comparing the simulation obtained that the difference of decadent rates to the non-overshooting controller by different values of the alpha.

5. Conclution

The state feedback gain matrix *F* obtained by eigenvalue and eigenvector placement methods can make the step response which is non-overshooting to the linear system and verify its correctness by Matlab simulation. It can be used as a new method to study the system without overshoot in the control of the state feedback control. The design of the controller can be applied to a square linear time invariant system with continuous time. The appropriate feedback gain matrix *F* satisfying the state feedback controller can be obtained by using Moore algorithm. The controller make for the linear system can be used under any initial condition to obtain a non-overshooting input. If the design can be applied to the manufacturing industry, itwill greatly reduce the product damage.

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