

An Approach for Multi-attribute Group Decision Making under the Linguistic Environment

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In this paper, a conversion relationship between values in interval $[0, g]$ and elements in the linguistic multiset is proposed and a linguistic aggregation model is introduced. Based on the conversion relationship and the model, a linguistic weighted aggregation LWA_φ operator under the linguistic environment is established and its properties are studied. While the proposed aggregation operator is employed to deal with the multi-attribute group decision making problems. Finally, an illustrative example is provided and comparisons are made between the proposed method and the other existing methods.

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1. Introduction

Multiple attribute group decision-making (MAGDM) is a typical decision problem, which has been widely applied to many areas such as industry, commerce and society, etc. In the real life, because of complexity and uncertainty of the objective thing and the fuzziness of human beings, people tend to use linguistic terms to express their preferences [1]. For example, when evaluating the “price” of a TV, such linguistic terms as “high”, “fair” and “low” are usually used.

In the linguistic MAGDM process, aggregating each decision maker's opinion into the group's opinion is the key [2]. Up still now, different linguistic models have been proposed to aggregate the linguistic information.

(1)The approximate model based on the extension principle [3]. Based on the membership functions, this model transforms the linguistic assessment information into the fuzzy numbers [4]; however, it is too hard to determine the accurate membership functions and it will cause loss of information during the transformation process[5].

(2)The symbolic model based on the ordinal scales [6]. This model makes computations on the subscripts of linguistic terms and easy to operator [7]; however, the aggregation results is obtained according to an approximation process that makes it in shortage of accuracy [8].

(3)The 2-tuple linguistic fuzzy representation model [9]. This model first converts the linguistic assessment information into the 2-tuple linguistic variables, and then aggregate information based on some operators. The main advantage of this model is that it can avoid the information loss in linguistic information processing; however, Martinez and Herrera have said that the 2-tuple linguistic fuzzy representation model is only suitable for linguistic variables with equidistant labels [9]. In order to overcome this limitation, many researchers have made deep research. Herrera proposed a fuzzy linguistic methodology to deal with unbalanced linguistic term sets [10]. Wang and Hao proposed the 2-tuple proportional model [11]. Abchir developed the 2-tuple semantic model and the generalized symbolic modifiers [12]; however, Xu pointed out that the model was a bit of inconvenient in calculation [13].

(4) The model that computes with words directly, uses some operators to compute with words directly [14]. For example, Xu proposed the linguistic weighted arithmetic average (LWAA) operator [15], linguistic generalized (LGA) operator, linguistic hybrid aggregation (LHGA) operator [16] and Linguistic weights averaging (LWA_2) operator [17], etc. Zhou proposed linguistic power aggregation (LPA) operator [18]. Merigó developed induced linguistic generalized ordered weighted averaging (ILGOWA) operator [19]; moreover, many Linguistic MAGDM problems have been solved based on these operators. This model is straightforward and very convenient to calculate; however, it may have the aggregation results of linguistic information out of the original linguistic term set.

To overcome the above limitations, we proposed an approach for the linguistic MADGM problems. The paper is organized as follows. In Section 2, some basic concepts are briefly described and a new conversion relationship is provided. In Section 3, a linguistic weighted aggregation LWA_ϕ operator is proposed and its properties are studied. In Section 4, an approach for the linguistic MAGDM problems is proposed. In Section 5, an illustrative example is developed and comparisons are made between the proposed method and the other existing

methods, which can show the feasibility and applicability of the proposed approach. In Section 6, the conclusion is made.

2. Conversion Relationship

In this section, some basic concepts are briefly described and a new conversion relationship is provided.

Suppose that $L = \{l_i = 0, 1, \dots, g\}$ ($g = 2k, k \in N$) be a finite and totally ordered discrete term set, where l_i represents a possible value for a linguistic variable and N is a set of non-negative integers [20-21]. Usually, any element of the set L must satisfy four characteristics as follows [21]:

- (1) The set is ordered: $l_i \leq l_j$ if $i \leq j$;
- (2) There is a negation operator: $\text{neg}(l_i) = l_j$ such that $i = g - j$;
- (3) Maximization operator: $\max\{l_i, l_j\} = l_i$ if $i \geq j$;
- (4) Minimization operator: $\min\{l_i, l_j\} = l_i$ if $i \leq j$.

In order to keep all the given information, L is extended to a continuous linguistic term set $\tilde{L} = \{l_x | x \in [0, g]\}$ [15]. If $l_x \in L$, l_x is called an original linguistic term; if $l_x (l_x \notin L)$, l_x is called a virtual linguistic term. Usually, the linguistic term $l_x (l_x \in L)$ is given by the decision maker, while the virtual linguistic term $l_x (l_x \notin L)$ only appears in computation [14].

Definition 1. For any $l_{x_1}, l_{x_2} \in \tilde{L}$, a binary relation \preceq on \tilde{L} is defined by

$$l_{x_1} \preceq l_{x_2} \Leftrightarrow x_1 \leq x_2 \quad \text{and} \quad l_{x_1} = l_{x_2} \Leftrightarrow x_1 = x_2. \quad (2.1)$$

Property 1. For any $l_{x_1}, l_{x_2}, l_{x_3} \in \tilde{L}$, the binary relation \preceq satisfies the following three properties: (1) (reflexive): $l_{x_1} \preceq l_{x_1}$;

(2) (antisymmetry): $l_{x_1} \preceq l_{x_2}$ and $l_{x_2} \preceq l_{x_1}$ implies $l_{x_1} = l_{x_2}$;

(3) (transitivity): $l_{x_1} \preceq l_{x_2}$ and $l_{x_2} \preceq l_{x_3}$ implies $l_{x_1} \preceq l_{x_3}$.

Definition 2.[6] Let \tilde{L} be the extended continuous linguistic term set, $\alpha_x = R^+ (R^+ = r | r \geq 0, r \in R)$ be a numeric value, $\tilde{u}: \tilde{L} \rightarrow R^+$ is a mapping. For any $l_x \in \tilde{L}$, \tilde{u} is defined by

$$\tilde{u}(l_x) = \alpha_x \quad (2.2)$$

where $0 \leq \alpha_0 < \alpha_1 < \dots < \alpha_g$. \tilde{u} is called the linguistic scale function.

Clearly, \tilde{u} is a strictly monotonically increasing continuous function. The inverse function of \tilde{u} exists, which is denoted by \tilde{u}^{-1} .

To date, several linguistic scale functions have been proposed. Such as Saaty scale, Exponential scale, power scale [17].

Definition 3. Let $L = \{l_i = 0, 1, \dots, g\}$ be a linguistic term set and $I = [0, g]$, the linguistic multiset L_r is defined by

$$L_r = L \times I = \{(l_{\text{round}(x)}, x) | x \in [0, g]\} \quad (2.3)$$

where $\text{round}(\cdot)$ is the usual round operation.

Definition 4. Suppose that linguistic multiset $L_r = \{(l_{round(x)}, x) | x \in [0, g]\}$ and $\varphi: \tilde{L} \rightarrow L_r$ be a mapping. For any $l_x \in \tilde{L}$, φ is defined by

$$\varphi(l_x) = (l_{round(x)}, x) \quad (2.4)$$

where $round(\cdot)$ is the usual round operation.

Definition 5. For any $(l_{round(x_1)}, x_1), (l_{round(x_2)}, x_2) \in L_r$, a binary relation \leq_φ on L_r is defined by

$$(l_{round(x_1)}, x_1) \leq_\varphi (l_{round(x_2)}, x_2) \Leftrightarrow x_1 \leq x_2, \quad (l_{round(x_1)}, x_1) =_\varphi (l_{round(x_2)}, x_2) \Leftrightarrow x_1 = x_2 \quad (2.5)$$

where $x_1, x_2 \in [0, g]$.

For any $l_{x_1}, l_{x_2} \in \tilde{L}$, $l_{x_1} \neq l_{x_2}$, we have $x_1 \neq x_2$. So $\varphi(l_{x_1}) \neq \varphi(l_{x_2})$. For any, $(l_{round(x)}, x) \in L_r$, there is only $l_x \in \tilde{L}$ so that $\varphi(l_x) = (l_{round(x)}, x)$. Thus the inverse mapping of φ exists, which is denoted by φ^{-1} .

Suppose that f be a monotone increasing and continuous function $f(x) = \alpha_x$ ($\alpha_x \in R^+$). The inverse function of f exists, which is denoted by f^{-1} . From the discussion above, we obtain the conversion relationship as follows:

$$\boxed{x \in [0, g]} \xrightleftharpoons[f^{-1}]{f} \boxed{\alpha_x \in R^+} \xrightleftharpoons[\tilde{u}]{\tilde{u}^{-1}} \boxed{l_x \in \tilde{L}} \xrightleftharpoons[\varphi^{-1}]{\varphi} \boxed{(l_{round(x)}, x) \in L_r}$$

Figure 1 : the conversion relationship between $x \in [0, g]$ and $(l_{round(x)}, x) \in L_r$.

In fact, as to any $x_1, x_2 \in [0, g]$,

$$x_1 \leq x_2 \Leftrightarrow \alpha_{x_1} \leq \alpha_{x_2} \Leftrightarrow l_{x_1} \preceq l_{x_2} \Leftrightarrow \varphi(l_{x_1}) \preceq \varphi(l_{x_2}) \quad (2.6)$$

$l_{round(x)}$ has the closest index label to l_x and x is the value that supports the ordering relation.

3. A New Linguistic Aggregation Operator

In this section, a linguistic weighted aggregation LWA_φ operator is developed and its properties are studied.

Definition 6. Let $\tilde{L} = \{l_x | x \in [0, g]\}$ be a extended linguistic term set and $d: \tilde{L} \times \tilde{L} \rightarrow R^+$ be a binary function. For any $l_{x_1}, l_{x_2} \in \tilde{L}$, d which is defined by

$$d(l_{x_1}, l_{x_2}) = |\tilde{u}(l_{x_1}) - \tilde{u}(l_{x_2})| \quad (3.1)$$

where $\tilde{u}(l_x) = \alpha_x$ ($\alpha_x \in R^+$).

In the group decision making, $(l_{x_1}, l_{x_2}, \dots, l_{x_n}) \in L^n$ is the decision information for the alternative A given by the decision makers $\{e_1, e_2, \dots, e_n\}$, and $w = (w_1, w_2, \dots, w_n)^T$ is the weighting vector of $(l_{x_1}, l_{x_2}, \dots, l_{x_n})$ satisfying $w_i > 0, \sum_{i=1}^n w_i = 1$. In order to aggregate the group's opinion, we consider the following mathematical programming model:

$$(MI) \min_{l_x \in \tilde{L}} F(l_x) = \sum_{i=1}^n w_i (d(l_{x_i}, l_x))^2 = \sum_{i=1}^n w_i (\tilde{u}(l_{x_i}) - \tilde{u}(l_x))^2$$

where $x \in [0, g]$.

$$\begin{aligned} \text{Let } h(\tilde{u}(l_x)) &= \sum_{i=1}^n w_i (\tilde{u}(l_{x_i}) - \tilde{u}(l_x))^2 \\ &= \tilde{u}^2(l_x) - 2 \sum_{i=1}^n w_i \tilde{u}(l_{x_i}) \tilde{u}(l_x) + \sum_{i=1}^n w_i \tilde{u}^2(l_{x_i}) \end{aligned} \quad (3.2)$$

Obviously, h is a quadratic function of $\tilde{u}(l_x)$, the axis of symmetry about $h(\tilde{u}(l_x))$ is $\tilde{u}(l_{\bar{x}}) = \sum_{i=1}^n w_i \tilde{u}(l_{x_i})$ and $\min_{l_x \in \tilde{L}} h(\tilde{u}(l_x)) = h(\tilde{u}(l_{\bar{x}}))$; moreover, $l_{\bar{x}} = \tilde{u}^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{x_i}))$.

According to the conversion relationship above, we have $\bar{x} = f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{x_i}))$.

Inference 1. The optimal solution of (M1) is $l_{\bar{x}} = l_{f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{x_i}))}$.

Moreover, $l_{f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{x_i}))}$ can be converted to

$$\varphi(l_{f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{x_i}))}) = (l_{\text{round}(f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{x_i})))}, f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{x_i}))). \quad (3.3)$$

Form the above discussion, linguistic information can be aggregated as follows:

Definition 7. An linguistic weighted aggregation LWA_{φ} operator of dimension n is a mapping $LWA_{\varphi}: L^n \rightarrow L_r$, furthermore,

$$LWA_{\varphi}(l_{x_1}, l_{x_2}, \dots, l_{x_n}) = (l_{\text{round}(f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{x_i})))}, f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{x_i}))) \quad (3.4)$$

where $w = (w_1, w_2, \dots, w_n)^T$ be the weight vector of $(l_{x_1}, l_{x_2}, \dots, l_{x_n})$ satisfying $w_i > 0, \sum_{i=1}^n w_i = 1, \tilde{u}(l_x) = \alpha_x (\alpha_x \in R^+), f^{-1}(\alpha_x) = x (x \in [0, g])$.

In fact $LWA_{\varphi}(l_{x_1}, l_{x_2}, \dots, l_{x_n}) = \varphi(l_{f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{x_i}))})$.

Based on the binary relation \leq_{φ} on L_r , we define the following ordering relation \leq_{ψ} on L^n to rank the alternatives:

Definition 8. Let $L = (l_0, l_1, \dots, l_g)$ be a linguistic term set. For any $(l_{x_1}, l_{x_2}, \dots, l_{x_n}), (l_{y_1}, l_{y_2}, \dots, l_{y_n}) \in L^n$ a binary relation \leq_{ψ} is defined by

$$(l_{x_1}, l_{x_2}, \dots, l_{x_n}) \leq_{\psi} (l_{y_1}, l_{y_2}, \dots, l_{y_n}) \Leftrightarrow \varphi(l_{f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{x_i}))}) \leq_{\varphi} \varphi(l_{f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{y_i}))}) \quad (3.5)$$

$$(l_{x_1}, l_{x_2}, \dots, l_{x_n}) =_{\psi} (l_{y_1}, l_{y_2}, \dots, l_{y_n}) \Leftrightarrow \varphi(l_{f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{x_i}))}) =_{\varphi} \varphi(l_{f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{y_i}))}) \quad (3.6)$$

$l_{\text{round}(f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{x_i}))})} \in L$ is the aggregation result of $(l_{x_1}, l_{x_2}, \dots, l_{x_n})$ and

$f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{x_i}))$ is the value supporting of the ordering relation.

Property 2. Let $(l_{x_1}, l_{x_2}, \dots, l_{x_n}), (l_{y_1}, l_{y_2}, \dots, l_{y_n})$ be two linguistic argument collections, then the operator satisfies:

(1)(Monotonic) If $l_{x_i} \preceq l_{y_i}$ for all i , then

$$LWA_{\varphi}(l_{x_1}, l_{x_2}, \dots, l_{x_n}) \preceq LWA_{\varphi}(l_{y_1}, l_{y_2}, \dots, l_{y_n}).$$

(2)(Bounded) For any weighting vector $w = (w_1, w_2, \dots, w_n)^T$, the following inequality holds: $\min(l_{x_i}, \preceq_i) LWA_{\varphi}(l_{x_1}, l_{x_2}, \dots, \preceq_{x_n}) \max(l_{x_i}, x_i)$.

(3)(Idempotent) If $l_{x_i} = l$ for all i , then $LWA_{\varphi}(l_{x_1}, l_{x_2}, \dots, l_{x_n}) = l$.

(4)(Commutative) If $w=(1/n, \dots, 1/n)^T$, $LWA_\varphi(l_{x_1}, \dots, l_{x_n})=LWA_\varphi(l_{y_1}, \dots, l_{y_n})$.

(5)(Focus effect) If w_t is sufficiently close to 1 and $l_{x_t} \prec l_{y_t}$, then

$$LWA_\varphi(l_{x_1}, l_{x_2}, \dots, l_{x_n}) \prec LWA_\varphi(l_{y_1}, l_{y_2}, \dots, l_{y_n}).$$

Proof. the proofs are obvious and thus omitted.

Inference2. Let $(l_{x_1}, l_{x_2}, \dots, l_{x_n}), (l_{y_1}, l_{y_2}, \dots, l_{y_n}) \in L^n$,

If $f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{x_i})) \leq f^{-1}(\sum_{i=1}^n w_i \tilde{u}(l_{y_i}))$. Then

$$LWA_\varphi(l_{x_1}, l_{x_2}, \dots, l_{x_n}) \preceq LWA_\varphi(l_{y_1}, l_{y_2}, \dots, l_{y_n}). \quad (3.7)$$

Proof. By Eq. (2.6) the proof is obvious and thus omitted.

4. An Approach for Multi-attribute Group Decision Making under Linguistic Environment

Considering a MAGDA problem with linguistic information, let $A=\{A_1, A_2, \dots, A_m\}$ be the discrete set of alternatives, $C=\{C_1, C_2, \dots, C_n\}$ be the set of attributes and

$E=(e_1, e_2, \dots, e_t)$ be the set of decision makers. Let $w=(w_1, w_2, \dots, w_n)^T$ be the weight vector of attributes satisfying $w_j \in [0, 1]$, $\sum_{j=1}^n w_j = 1$. And let $v=(v_1, v_2, \dots, v_t)^T$ be the weight vector of decision makers satisfying $v_k \in [0, 1]$, $\sum_{k=1}^t v_k = 1$. Assume each decision makers $e_k \in E (k=1, 2, \dots, t)$ provides his linguistic decision matrix $M_k=(l_{ij}^k)_{m \times n}$, where $l_{ij}^k \in L (i=1, 2, \dots, m; j=1, 2, \dots, n)$ are given by $e_k \in E$ for the alternative $A_i \in A$ with respect to the attribute $C_j \in C$.

Then, using the LWA_φ operator to aggregate all decision maker's linguistic decision information into a group's decision matrix G :

$$G = \begin{bmatrix} (l_{r11}, g_{11}) & (l_{r12}, g_{12}) & \cdots & (l_{r1n}, g_{1n}) \\ (l_{r21}, g_{21}) & (l_{r22}, g_{22}) & \cdots & (l_{r2n}, g_{2n}) \\ \vdots & \vdots & \cdots & \vdots \\ (l_{rm1}, g_{m1}) & (l_{rm2}, g_{m2}) & \cdots & (l_{rmn}, g_{mn}) \end{bmatrix} \quad (4.1)$$

where

$$(l_{rij}, g_{ij}) = LWA_\varphi(l_{ij}^1, l_{ij}^2, \dots, l_{ij}^t) = (l_{\text{round}(f^{-1}(\sum_{i=1}^m v_k \tilde{u}(l_{ij}^k)))}, f^{-1}(\sum_{i=1}^m v_k \tilde{u}(l_{ij}^k))) \quad (4.2)$$

and v_k is the weight of the decision maker e_k .

Therefore, G corresponds to the group's comprehensive linguistic decision matrix \tilde{G} and the value supporting matrix \tilde{G}_S as follows:

$$\tilde{G} = \begin{bmatrix} l_{r11} & l_{r12} & \cdots & l_{r1n} \\ l_{r21} & l_{r22} & \cdots & l_{r2n} \\ \vdots & \vdots & \cdots & \vdots \\ l_{rm1} & l_{rm2} & \cdots & l_{rmn} \end{bmatrix}, \quad \tilde{G}_S = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ g_{m1} & g_{m2} & \cdots & g_{mn} \end{bmatrix} \quad (4.3)$$

where

$$rij = \text{round} \left(f^{-1} \left(\sum_{i=1}^m v_k \tilde{u}(l_{ij}^k) \right) \right), g_{ij} = f^{-1} \left(\sum_{i=1}^m v_k \tilde{u}(l_{ij}^k) \right), j=1,2,\dots,n, k=1,2,\dots,t.$$

Furthermore, based on the value supporting matrix \tilde{G}_S we can calculate the value supporting $N_{A_i} (i=1,2,\dots,m)$ of each alternative by the weighted average method:

$$N_{A_i} = \sum_{j=1}^n w_j g_{ij}, i=1,2,\dots,m. \quad (4.4)$$

Moreover, we can calculate the overall assessment $L_{A_i} (i=1,2,\dots,m)$ of each alternative by **Fig.1**.

$$L_{A_i} = (l_{\text{round}(N_{A_i})}, N_{A_i}) \quad (4.5)$$

Then the alternatives $A_i (i=1,2,\dots,m)$ can be ranked based on the N_{A_i} .

To get the best alternative(s), the follows steps are involved:

Step1. Choose the function \tilde{u} and f^{-1} .

Step2. Calculate the group's decision matrix G by Eq.(4.1). Obtain the group's comprehensive linguistic decision matrix \tilde{G} and the value supporting matrix \tilde{G}_S .

Step3. Calculate the value supporting N_{A_i} and the overall assessment L_{A_i} of each alternative by Eq. (4.4-4.5).

Step4. Rank all the alternatives based on the value supporting N_{A_i} by Eq. (2.6) and obtain the best alternative(s).

5. Illustrative Example

Let us suppose an investment company, which wants to invest a sum of money in the best option (adapted from [21]). There is a panel with five possible alternatives to invest the money:

A_1 is a car industry, A_2 is a food company, A_3 is a computer company, A_4 is an arms company, A_5 is a TV company. The investment company considers the following four attributes (with the weighting vector as $w=(0.35,0.15,0.20,0.30)^T$): C_1 is the risk analysis, C_2 is the growth analysis, C_3 is the social-political impact analysis, C_4 is the environmental impact analysis. The five possible alternatives $A_i (i=1,2,\dots,5)$ are to be evaluated by using the linguistic term set

$$L = \{l_0 = \text{extremely poor}, l_1 = \text{very poor}, l_2 = \text{poor}, l_3 = \text{slightly poor}, l_4 = \text{fair}, l_5 = \text{slightly good}, l_6 = \text{good}, l_7 = \text{very good}, l_8 = \text{extremely good}\} \quad (5.1)$$

By the three decision makers $e_k (k=1,2,3)$ (whose weighting vector $v=(0.35,0.25,0.4)^T$) under the above four attributes $C_j (j=1,2,3,4)$ and decision matrix $M_k = (l_{ij}^k)_{5 \times 4}, (k=1,2,3)$ are listed as follows:

$$M_1 = \begin{bmatrix} l_4 & l_5 & l_3 & l_2 \\ l_8 & l_7 & l_7 & l_4 \\ l_4 & l_5 & l_5 & l_6 \\ l_8 & l_6 & l_3 & l_2 \\ l_6 & l_4 & l_7 & l_5 \end{bmatrix}, M_2 = \begin{bmatrix} l_4 & l_5 & l_2 & l_2 \\ l_6 & l_6 & l_6 & l_3 \\ l_5 & l_4 & l_4 & l_5 \\ l_7 & l_6 & l_2 & l_4 \\ l_5 & l_3 & l_6 & l_4 \end{bmatrix}, M_3 = \begin{bmatrix} l_4 & l_6 & l_2 & l_2 \\ l_6 & l_4 & l_5 & l_5 \\ l_5 & l_5 & l_7 & l_6 \\ l_7 & l_5 & l_6 & l_3 \\ l_5 & l_7 & l_6 & l_6 \end{bmatrix} \quad (5.2)$$

Step1. Take the linguistic scale function $\tilde{u}(l_x)=0.8^x$ and obtain the function $f^{-1}=\log_{0.8}x$.

Step 2. Calculate the group's decision matrix G by Eq.(4.1). Obtain the group's comprehensive linguistic decision matrix \tilde{G} and the value supporting matrix \tilde{G}_S .

$$\tilde{G}=\begin{bmatrix} l_4 & l_5 & l_2 & l_2 \\ l_7 & l_5 & l_6 & l_4 \\ l_5 & l_5 & l_5 & l_6 \\ l_7 & l_6 & l_4 & l_3 \\ l_5 & l_5 & l_6 & l_5 \end{bmatrix}, \tilde{G}_S=\begin{bmatrix} 4.0000 & 5.3737 & 2.3252 & 2.0000 \\ 6.6035 & 5.3542 & 5.8675 & 4.3810 \\ 4.6241 & 4.7283 & 5.3810 & 5.7283 \\ 7.3252 & 5.5729 & 3.6380 & 2.8350 \\ 5.3252 & 4.6380 & 6.3252 & 5.0791 \end{bmatrix} \quad (5.3)$$

Step 3. Calculate the value supporting of N_{A_i} each alternative by Eq.(4.4):

$$N_{A_1}=3.8085, N_{A_2}=6.1376, N_{A_3}=5.5952, N_{A_4}=5.5351, N_{A_5}=5.8121. \quad (5.4)$$

Moreover, the overall assessment L_{A_i} of each alternative can be calculated by **Figure. 1**

$$L_{A_1}=(l_4, 3.8085), L_{A_2}=(l_6, 6.1376), L_{A_3}=(l_6, 5.5952), \\ L_{A_4}=(l_6, 5.5351), L_{A_5}=(l_6, 5.8121). \quad (5.5)$$

Step 4. Rank the value supporting $N_{A_i}(i=1,2,\dots,5)$ of each alternative:

$$N_{A_2} > N_{A_5} > N_{A_3} > N_{A_4} > N_{A_1}. \quad (5.6)$$

By Eq. (2.6) we can rank all the alternatives $A_i(i=1,2,\dots,5)$:

$$A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1. \quad (5.7)$$

The best alternative is A_2 .

5.1. Comparative and Discussion

To test the validity of the proposed linguistic MAGDM method, the comparisons are made between the proposed approach and the existing methods (Xu' method [17] and Perk' method [21]). The analysis is based on the above illustrative example.

(1)When taking the linguistic scale function $\tilde{u}(l_x)=x/8(x \in [0,8])$, the linguistic label L is equidistant. Using the proposed approach, the rank ordering among the alternatives is

$$A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1.$$

Using the Park' method [21], the overall assessments of all alternatives can be obtained:

$$A_1=(l_3, -0.193), A_2=(l_5, 0.255), A_3=(l_5, -0.196), A_4=(l_5, -0.407), A_5=(l_2, 0.008)$$

And the rank ordering among the alternatives is $A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1$.

Obviously, the proposed approach and the Park' method have the same ranking results; therefore, the proposed method is rational and valid.

From above example, when taking the linguistic scale function $\tilde{u}(l_x)=0.8^x$, the absolute deviation between the adjacent linguistic subscripts increases, so the linguistic label L is non-equidistant. In such a case, as Herrera and Martinez themselves said that the 2-tuple linguistic representation model was unable to deal with the non-equidistant labels [9]; however, the approach which we proposed is able to overcome this limitation.

(2)Using Xu' method [20], the overall assessments of all alternatives can be obtained:

$$A_1=l_{3.82}, A_2=l_{6.2875}, A_3=l_{5.62}, A_4=l_{5.6325}, A_5=l_{5.925}.$$

And the rank among the alternatives is $A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1$.

Although the selection order is the same, the results of Xu' method [17] may not be out of the initial term sets. For example, $l_{3.82} \notin L$ is the virtual linguistic term which only appears in comparison and in operation, then $l_{3.82}$ does not have any syntax or semantics assigned [1]. In the proposed approach, based on the conversion relationship, the linguistic multiset is used to express the overall assessment of each alternative (eg. $L_{A_1}=(l_4, 3.8085), l_4 \in L$), which contains the linguistic evaluation of the alternative and the value supporting of the alternative, that can make the aggregation results match with the initial linguistic terms; therefore, the approach which we proposed is more exactly in dealing with linguistic MAGDM problems than the Park' method and the Xu' method.

6. Conclusion

We provided a linguistic weighted aggregation LWA_ϕ operator and studied its properties. The main advantages of the proposed operator are that it can be used to aggregate linguistic variables with equidistant and non-equidistant labels, and it can make the aggregation results to match with the initial linguistic terms. Based on the LWA_ϕ operator and the conversion relationship, we obtained a new linguistic MAGDM approach. Finally, an illustrative example was given and some comparisons were made to show the applicability and effectiveness of the proposed approach.

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