PROCEEDINGS OF SCIENCE

Non-singlet coefficient functions for charged-current deep-inelastic scattering to the third order in QCD

J. Davies*
Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK
E-mail: Joshua.Davies@liv.ac.uk

S. Moch
II. Institute for Theoretical Physics, University of Hamburg, D-22761 Hamburg, Germany
E-mail: Sven-Olaf.Moch@desy.de

J.A.M. Vermaseren
Nikhef Theory Group, Science Park 105, 1098 XG Amsterdam, The Netherlands
E-mail: t68@nikhef.nl

A. Vogt
Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK
E-mail: Andreas.Vogt@liv.ac.uk

We have calculated the coefficient functions for the structure functions F_2, F_L and F_3 in $\nu-\bar{\nu}$ charged-current deep-inelastic scattering (DIS) at the third order in the strong coupling α_s, thus completing the description of unpolarized inclusive $W^{\pm}$-exchange DIS to this order of massless perturbative QCD. In this brief note, our new results are presented in terms of compact approximate expressions that are sufficiently accurate for phenomenological analyses. For the benefit of such analyses we also collect, in a unified notation, the corresponding lower-order contributions and the flavour non-singlet coefficient functions for $\nu+\bar{\nu}$ charged-current DIS. The behaviour of all six third-order coefficient functions at small Bjorken-x is briefly discussed.

XXIV International Workshop on Deep-Inelastic Scattering and Related Subjects
11-15 April 2016, DESY Hamburg, Germany

*Speaker.

PoS(DIS2016)059

1. Introduction

With six structure functions, F_2, F_L and F_3 for W^+ and W^- exchange [1, 2], inclusive charged-current DIS is an important source of information on the parton structure of nucleons and nuclei and on Standard Model parameters such as the strong coupling α_s and the weak mixing $\sin^2\theta_W$ [3]. Future facilities, e.g., the LHeC [4], will be required to fully realize its phenomenological potential. Charged-current structure functions are, at very small values of the Bjorken variable x, also of interest for the scattering of high-energy cosmic neutrinos, see Ref. [5] and references therein.

Here we address the corresponding coefficient functions (mass-factorized partonic cross sections) in massless perturbative QCD. These functions are relevant also beyond DIS, e.g., for Higgs production via vector boson fusion [6, 7]. For recent progress on heavy-quark effects see Ref. [8].

2. First-order and $\nu+\bar{\nu}$ non-singlet coefficient functions

The first-order coefficient functions for unpolarized inclusive DIS were derived in the early days of QCD, see Refs. [1,9]. The second- and third-order contributions for the $\nu+\bar{\nu}$ charged-current case have been calculated in Refs. [10–17]. Here we collect the corresponding flavour non-singlet results in the $\overline{\mathrm{MS}}$ scheme for the standard choice $\mu_F = \mu_R = Q$ of the renormalization and factorization scales. The additional contributions at other values of μ_F and/or μ_R are determined by these results and the corresponding splitting functions [18, 19]; see, e.g., Eqs. (2.16) – (2.18) in Ref. [20].

We denote the non-singlet quark coefficient functions for the charged-current structure functions $F_{2,3,L}^{\nu p \pm \bar{\nu} p}(x,Q^2)$ in neutrino-proton DIS by $C_{a,\pm}$, and write their perturbative expansion as

$$C_{a,\pm}(x,Q^2) \,=\, \sum_{n=0} a_s^n\, c_{a,\pm}^{(n)}(x) \quad \text{with} \quad a_s \,\equiv\, \alpha_s(Q^2)/(4\pi)\,. \tag{2.1}$$

In this notation the zeroth- and first-order coefficient functions are given by

$$c_{2,\pm}^{(0)}(x) \,=\, c_{3,\pm}^{(0)}(x) \,=\, \delta(x_1)\,, \quad c_{L,\pm}^{(0)}(x) \,=\, 0\,, \quad c_{L,\pm}^{(1)}(x) \,=\, 4C_F\, x\,, \tag{2.2}$$

$$c_{2,\pm}^{(1)}(x) \,=\, C_F\{4\,\mathscr{D}_1 - 3\,\mathscr{D}_0 - (9+4\,\zeta_2)\,\delta(x_1) - 2\,(1+x)(L_1-L_0) - 4x_1^{-1}L_0 + 6 + 4x\}\,, \tag{2.3}$$

$$c_{3,\pm}^{(1)}(x) \,=\, c_{2,\pm}^{(1)}(x) - 2C_F\,(1+x) \tag{2.4}$$

with $C_F = (n_c^2-1)/(2n_c) = 4/3$ in QCD. Here and below we use the abbreviations

$$x_1 \,=\, 1-x\,, \quad L_0 \,=\, \ln x\,, \quad L_1 \,=\, \ln x_1\,, \quad \mathscr{D}_k \,=\, [x_1^{-1}L_1^k]_+\,, \tag{2.5}$$

where $[a(x)]_+$ denotes +-distributions defined via $\int_0^1 dx\,[a(x)]_+\, f(x) \,\equiv\, \int_0^1 dx\, a(x)\{\, f(x) - f(1)\}$.

The coefficient functions $c_{a,+}^{(n)}$ and $c_{a,-}^{(n)}$ differ at $n>1$. The 2[nd]- and 3[rd]-order contributions to the former quantities read, in an approximate but sufficiently accurate form given in Refs. [15–17],

$$\begin{aligned} c_{2,+}^{(2)}(x) \,\cong\, & 128/9\,\mathscr{D}_3 - 184/3\,\mathscr{D}_2 - 31.1052\,\mathscr{D}_1 + 188.641\,\mathscr{D}_0 - 338.513\,\delta(x_1) - 17.74L_1^3 \\ & + 72.24L_1^2 - 628.8L_1 - 181.0 - 806.7x + L_0L_1(37.75L_0 - 147.1L_1) \\ & + 0.719xL_0^4 - 28.384L_0 - 20.70L_0^2 - 80/27L_0^3 \\ + n_f\, \{ & 16/9\,\mathscr{D}_2 - 232/27\,\mathscr{D}_1 + 6.34888\,\mathscr{D}_0 + 46.8531\,\delta(x_1) - 1.500L_1^2 + 24.87L_1 \\ & - 7.8109 - 17.82x - 12.97x^2 + 8.113L_0L_1 - 0.185xL_0^3 + 16/3L_0 + 20/9L_0^2\}\,, \end{aligned} \tag{2.6}$$

$$\begin{aligned} c_{L,+}^{(2)}(x) \,\cong\, & -37.338 + 89.53x + 33.82x^2 + 128/9xL_1^2 - 46.50xL_1 + xL_0(32.90 + 18.41L_0) \\ & - 84.094L_0L_1 - 128/9L_0 - 0.012\,\delta(x_1) \,+\, 16/27n_f\,\{6xL_1 - 12xL_0 - 25x + 6\}\,, \end{aligned} \tag{2.7}$$

$$
\begin{aligned}
c^{(2)}_{3,+}(x) \;\cong\;& 128/9\,\mathcal{D}_3 - 184/3\,\mathcal{D}_2 - 31.1052\,\mathcal{D}_1 + 188.641\,\mathcal{D}_0 - 338.572\,\delta(x_1) - 16.40\,L_1^3 \\
& + 78.46\,L_1^2 - 470.6\,L_1 - 149.75 - 693.2\,x + 0.218\,xL_0^4 + L_0L_1(33.62\,L_0 - 117.8\,L_1) \\
& - 49.30\,L_0 - 94/3\,L_0^2 - 104/27\,L_0^3 \qquad (2.8)\\
& + n_f \big\{ 16/9\,\mathcal{D}_2 - 232/27\,\mathcal{D}_1 + 6.34888\,\mathcal{D}_0 + 46.8464\,\delta(x_1) + 0.066\,L_1^3 - 0.663\,L_1^2 \\
& + 24.86\,L_1 - 5.738 - 5.845\,x - 10.235\,x^2 - 0.190\,xL_0^3 + 4.265\,L_0L_1 + 20/9\,L_0(4+L_0) \big\}
\end{aligned}
$$

and

$$
\begin{aligned}
c^{(3)}_{2,+}(x) \;\cong\;& 512/27\,\mathcal{D}_5 - 5440/27\,\mathcal{D}_4 + 501.099\,\mathcal{D}_3 + 1171.54\,\mathcal{D}_2 - 7328.45\,\mathcal{D}_1 + 4442.76\,\mathcal{D}_0 \\
& - 9170.38\,\delta(x_1) - 512/27\,L_1^5 + 704/3\,L_1^4 - 3368\,L_1^3 - 2978\,L_1^2 + 18832\,L_1 - 4926 \\
& + 7725\,x + 57256\,x^2 + 12898\,x^3 - 56000\,x_1L_1^2 - L_0L_1(6158 + 1836\,L_0) + 4.719\,xL_0^5 \\
& - 775.8\,L_0 - 899.6\,L_0^2 - 309.1\,L_0^3 - 2932/81\,L_0^4 - 32/27\,L_0^5 \\
& + n_f \big\{ 640/81\,\mathcal{D}_4 - 6592/81\,\mathcal{D}_3 + 220.573\,\mathcal{D}_2 + 294.906\,\mathcal{D}_1 - 729.359\,\mathcal{D}_0 \\
& + 2574.687\,\delta(x_1) - 640/81\,L_1^4 + 153.5\,L_1^3 - 828.7\,L_1^2 - 501.1\,L_1 + 831.6 - 6752\,x \\
& - 2778\,x^2 + 171.0\,x_1L_1^4 + L_0L_1\,(4365 + 716.2\,L_0 - 5983\,L_1) + 4.102\,xL_0^4 + 275.6\,L_0 \\
& + 187.3\,L_0^2 + 12224/243\,L_0^3 + 728/243\,L_0^4 \big\} \qquad (2.9)\\
& + n_f^2 \big\{ 64/81\,\mathcal{D}_3 - 464/81\,\mathcal{D}_2 + 7.67505\,\mathcal{D}_1 + 1.00830\,\mathcal{D}_0 - 103.2366\,\delta(x_1) - 64/81\,L_1^3 \\
& + 18.21\,L_1^2 - 19.09\,L_1 + 129.2\,x + 102.5\,x^2 + L_0L_1\,(-96.07 - 12.46\,L_0 + 85.88\,L_1) \\
& - 8.042\,L_0 - 1984/243\,L_0^2 - 368/243\,L_0^3 \big\}\,,
\end{aligned}
$$

$$
\begin{aligned}
c^{(3)}_{L,+}(x) \;\cong\;& 512/27\,L_1^4 - 177.40\,L_1^3 + 650.6\,L_1^2 - 2729\,L_1 - 2220.5 - 7884\,x + 4168\,x^2 \\
& - (844.7\,L_0 + 517.3\,L_1)L_0L_1 + (195.6\,L_1 - 125.3)\,x_1L_1^3 + 208.3\,xL_0^3 - 1355.7\,L_0 \\
& - 7456/27\,L_0^2 - 1280/81\,L_0^3 + 0.113\,\delta(x_1) \\
& + n_f \big\{ 1024/81\,L_1^3 - 112.35\,L_1^2 + 344.1\,L_1 + 408.4 - 9.345\,x - 919.3\,x^2 \\
& + (239.7 + 20.63\,L_1)\,x_1L_1^2 + (887.3 + 294.5\,L_0 - 59.14\,L_1)L_0L_1 - 1792/81\,xL_0^3 \\
& + 200.73\,L_0 + 64/3\,L_0^2 + 0.006\,\delta(x_1) \big\} \qquad (2.10)\\
& + n_f^2 \big\{ 3xL_1^2 + (6-25x)L_1 - 19 + (317/6 - 12\,\zeta_2)\,x - 6xL_0L_1 + 6x\,\mathrm{Li}_2(x) + 9xL_0^2 \\
& - (6-50x)L_0 \big\}\, 64/81\,,
\end{aligned}
$$

$$
\begin{aligned}
c^{(3)}_{3,+}(x) \;\cong\;& 512/27\,\mathcal{D}_5 - 5440/27\,\mathcal{D}_4 + 501.099\,\mathcal{D}_3 + 1171.54\,\mathcal{D}_2 - 7328.45\,\mathcal{D}_1 + 4442.76\,\mathcal{D}_0 \\
& - 9172.68\,\delta(x_1) - 512/27\,L_1^5 + 8896/27\,L_1^4 - 1396\,L_1^3 + 3990\,L_1^2 + 14363\,L_1 \\
& - 1853 - 5709\,x + x\,x_1(5600 - 1432\,x) - L_0L_1(4007 + 1312\,L_0) - 0.463\,xL_0^6 \\
& - 293.3\,L_0 - 1488\,L_0^2 - 496.95\,L_0^3 - 4036/81\,L_0^4 - 536/405\,L_0^5 \\
& + n_f \big\{ 640/81\,\mathcal{D}_4 - 6592/81\,\mathcal{D}_3 + 220.573\,\mathcal{D}_2 + 294.906\,\mathcal{D}_1 - 729.359\,\mathcal{D}_0 \\
& + 2575.46\,\delta(x_1) - 640/81\,L_1^4 + 32576/243\,L_1^3 - 660.7\,L_1^2 + 959.1\,L_1 + 31.95\,x_1L_1^4 \\
& + 516.1 - 465.2\,x + x\,x_1(635.3 + 310.4\,x) + L_0L_1\,(1496 + 270.1\,L_0 - 1191\,L_1) \\
& - 1.200\,xL_0^4 + 366.9\,L_0 + 305.32\,L_0^2 + 48512/729\,L_0^3 + 304/81\,L_0^4 \big\} \qquad (2.11)\\
& + n_f^2 \big\{ 64/81\,\mathcal{D}_3 - 464/81\,\mathcal{D}_2 + 7.67505\,\mathcal{D}_1 + 1.00830\,\mathcal{D}_0 - 103.2602\,\delta(x_1) - 64/81\,L_1^3 \\
& + 992/81\,L_1^2 - 49.65\,L_1 + 11.32 - x\,x_1(44.52 + 11.05\,x) + 51.94\,x + 0.0647\,xL_0^4 \\
& - L_0L_1\,(39.99 + 5.103\,L_0 - 16.30\,L_1) - 16.00\,L_0 - 2848/243\,L_0^2 - 368/243\,L_0^3 \big\} \\
& + fl_{02}\,n_f \big\{ 2.147\,L_1^2 - 24.57\,L_1 + 48.79 - x_1(242.4 - 150.7\,x) - L_0L_1\,(81.70 + 9.412\,L_1) \\
& + xL_0\,(218.1 + 82.27\,L_0^2) - 477.0\,L_0 - 113.4\,L_0^2 + 17.26\,L_0^3 - 16/27\,L_0^5 \big\}\, x_1\,.
\end{aligned}
$$

3. $\nu - \bar{\nu}$ non-singlet coefficient functions

The differences between corresponding $\nu + \bar{\nu}$ and $\nu - \bar{\nu}$ coefficient functions are, as conjectured in Ref. [23], suppressed at large x by two powers of $1-x$. Hence it is convenient to present the coefficient functions for $\nu - \bar{\nu}$ charged-current DIS in terms of differences which we define as

$$\delta C_{2,L} \;\equiv\; C_{2,L}^{\nu p+\bar{\nu}p} - C_{2,L}^{\nu p-\bar{\nu}p} \;, \qquad \delta C_{3} \;\equiv\; C_{3}^{\nu p-\bar{\nu}p} - C_{3}^{\nu p+\bar{\nu}p} \;. \tag{3.1}$$

The flavour class fl_{02}, see Fig. 1 of Ref. [16], does not contribute to the flavour asymmetries probed in the $\nu - \bar{\nu}$ combinations, hence it is understood that the corresponding part of Eq. (2.11) is removed before the difference for F_3 is formed. The δC_a can be perturbatively expanded as

$$\delta C_a \;=\; \sum_{n=2} a_{\rm s}^n \, \delta c_a^{(n)} \;, \tag{3.2}$$

where $a_{\rm s}$ is defined in Eq. (2.1) above. The second-order results were already given in Ref. [21] in exact and parametrized form. The later results, written in terms of the abbreviations (2.5), read

$$\delta c_2^{(2)}(x) \;\cong\; \{-9.1587 - 57.70\,x + 72.29\,x^2 - 5.689\,x^3 - xL_0\,(\,68.804 + 24.40\,L_0 + 2.958\,L_0^2\,) + 0.249\,L_0 + 8/9\,L_0^2\,(2+L_0)\,\}\,x_1 \;, \tag{3.3}$$

$$\delta c_L^{(2)}(x) \;\cong\; \{\,10.663 - 5.248\,x - 7.500\,x^2 + 0.823\,x^3 + xL_0\,(\,11.10 + 2.225\,L_0 - 0.128\,L_0^2\,) + 64/9\,L_0\,\}\,x_1^2 \;, \tag{3.4}$$

$$\delta c_3^{(2)}(x) \;\cong\; \{-29.65 + 116.05\,x - 71.74\,x^2 - 16.18\,x^3 + xL_0\,(\,14.60 + 69.90\,x - 0.378\,L_0^2\,) - 8.560\,L_0 + 8/9\,L_0^2\,(4+L_0)\,\}\,x_1 \;. \tag{3.5}$$

The corresponding third-order corrections are the new results of the present contribution. They supersede the previous approximate expressions in Eqs. (3.7) – (3.9) of Ref. [21], which were based on the lowest five even- integer and odd-integer Mellin moments of $C_{3,-}$ and $C_{a,-}$, $a = 2, L$, respectively, computed in Ref. [22]. Our new exact results can parametrized as

$$\begin{aligned}\delta c_2^{(3)} \;\cong\; & \big\{\,273.59 - 44.95\,x - 73.56\,x^2 + 40.68\,x^3 + 0.1356\,L_0^5 + 8.483\,L_0^4 + 55.90\,L_0^3 \\ & +120.67\,L_0^2 + 388.0\,L_0 - 329.8\,L_0L_1 - xL_0(316.2 + 71.63\,L_0) + 46.30\,L_1 \\ & +5.447\,L_1^2\big\}\,x_1 - 0.0008\,\delta(x_1) \\ & + n_f\,\big\{\big(-19.093 + 12.97\,x + 36.44\,x^2 - 29.256\,x^3 - 0.76\,L_0^4 - 5.317\,L_0^3 - 19.82\,L_0^2 \\ & -38.958\,L_0 - 13.395\,L_0L_1 + xL_0(14.44 + 17.74\,L_0) + 1.395\,L_1\big)\,x_1 + 0.0001\,\delta(x_1)\big\} \;,\end{aligned} \tag{3.6}$$

$$\begin{aligned}\delta c_L^{(3)} \;\cong\; & \big\{-620.53 - 394.5\,x + 1609\,x^2 - 596.2\,x^3 + 0.217\,L_0^3 + 62.18\,L_0^2 + 208.47\,L_0 \\ & -482.5\,L_0L_1 - xL_0(1751 - 197.5\,L_0) + 105.5\,L_1 + 0.442\,L_1^2\big\}\,x_1^2 \\ & + n_f\,\big\{-6.500 - 12.435\,x + 23.66\,x^2 + 0.914\,x^3 + 0.015\,L_0^3 - 6.627\,L_0^2 - 31.91\,L_0 \\ & - xL_0(5.711 + 28.635\,L_0)\big\}\,x_1^2 \;,\end{aligned} \tag{3.7}$$

$$\begin{aligned}\delta c_3^{(3)} \;\cong\; & \big\{-553.5 + 1412.5\,x - 990.3\,x^2 + 361.1\,x^3 + 0.1458\,L_0^5 + 9.688\,L_0^4 + 90.62\,L_0^3 \\ & +83.684\,L_0^2 - 602.32\,L_0 - 382.5\,L_0L_1 - xL_0(2.805 + 325.92\,L_0) + 133.5\,L_1 \\ & +10.135\,L_1^2\big\}\,x_1 - 0.0029\,\delta(x_1) \\ & + n_f\,\big\{\big(-16.777 + 77.78\,x - 24.81\,x^2 - 28.89\,x^3 - 0.7714\,L_0^4 - 7.701\,L_0^3 - 21.522\,L_0^2 \\ & -7.897\,L_0 - 16.17\,L_0L_1 + xL_0(43.21 + 67.04\,L_0) + 1.519\,L_1\big)\,x_1 + 0.00006\,\delta(x_1)\big\} \;.\end{aligned} \tag{3.8}$$

4. Discussion

With the exception of the n_f part of Eq. (2.7) and the n_f^2 part of Eq. (2.10) which are exact, the second- and third-order expressions in sections 2 and 3 have been obtained by fitting the coefficients not written as fractions in the non-distribution parts to the exact coefficient functions at $x \geq 10^{-6}$. Where useful, the coefficients of $\delta(1-x)$ have been adjusted (even from zero) to fine-tune the accuracy of Mellin moments and convolutions. The resulting accuracy of Eqs. (2.6) – (2.11) and (3.3) – (3.8) and their convolutions with typical quark distributions of hadrons is 0.1% or better except where the functions are very small. Towards smaller x the accuracy deteriorates, but the results are still accurate to about 1% and 3% at $x = 10^{-8}$ and $x = 10^{-10}$, respectively.

FORTRAN subroutines of these functions can be obtained from the preprint server ARXIV.ORG by downloading the source of this note. They are also available from the authors upon request.

Analogous parametrizations for the pure-singlet quark and gluon coefficient functions for F_2 and F_L have been given in Ref. [15] and section 4 of Ref. [16]. The partly very lengthy exact expression corresponding to Eqs. (2.6) – (2.11) and (3.3) – (3.5) can be found in Ref. [16] – where the fl_{11} contribution has to be disregarded for the present charged-current case – and Refs. [17,21]; those for Eqs. (3.6) – (3.8) will be presented in Ref. [25]. Only the latter expressions allow for the analytical calculation of all integer Mellin moments of the coefficient functions.

The second moments of $\delta c_2^{(3)}$ and $\delta c_L^{(3)}$ are of particular relevance, since they enter the QCD corrections to the Paschos-Wolfenstein relation [24] for the determination of $\sin^2\theta_W$ from charged-current DIS [3]. The truncated numerical values of these moments for n_f light flavours are

$$\delta c_2^{(3)}(N=2) \;=\; -20.4001 + 0.72202\, n_f \,, \qquad \delta c_L^{(3)}(N=2) \;=\; -24.7755 + 0.80134\, n_f \,. \qquad (4.1)$$

Within their error estimates, the previous approximate results for these moments [21] agree with Eq. (4.1). The corresponding analytical expressions will also by presented in Ref. [25].

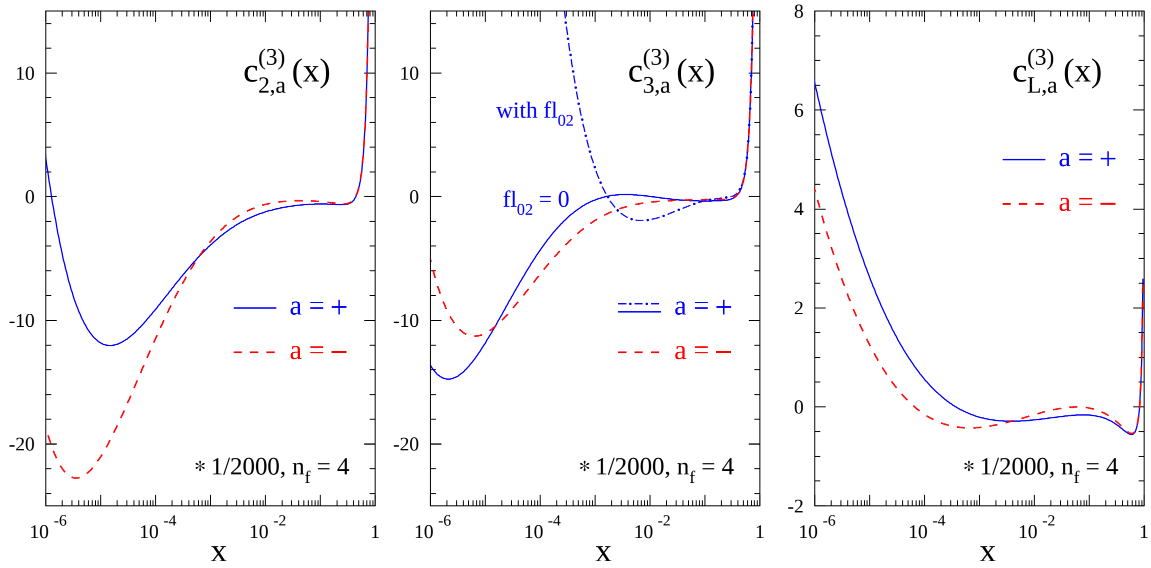

Figure 1: The third-order non-singlet coefficient functions for F_2, F_L and F_3 in $\nu p + \bar{\nu} p$ ($a = +$) and $\nu p - \bar{\nu} p$ ($a = -$) charged-current DIS for four light flavours. The function $c_{3,+}^{(3)}(x)$ in Eq. (2.11) is shown with and without the fl_{02} contribution. The factor 1/2000 approximately converts the curves to an expansion in α_s.

The a_s^3 coefficients $c^{(3)}_{a,\pm}$ in Eq. (2.1) are illustrated in Fig. 1 over a wide range in x. All six functions exhibit a sharp small-x rise, but only at $x < 10^{-5}$ for F_2 and F_3 and at $x \lesssim 10^{-4}$ for F_L. With the exception of the flavour structure fl_{02} that occurs at three loops for the first time but dominates $c^{(3)}_{a,+}$ at small x, at least four of five $\ln^k x$ small-x terms are required for a good approximation for $c^{(3)}_{2,\pm}$ and $c^{(3)}_{3,\pm}$, and all three such terms for $c^{(3)}_{L,\pm}$, even below the x-range shown in the figure. Further discussions and illustrations of these coefficient functions will be presented in Ref. [25].

Acknowledgements

This work has been supported by the UK *Science & Technology Facilities Council* (STFC) grants ST/L000431/1 and ST/K502145/1, by the *Deutsche Forschungsgemeinschaft* (DFG) via contract MO 1801/1-1, and by the *European Research Council* (ERC) Advanced Grant 320651, HEPGAME.

References

[1] A.J. Buras, Rev. Mod. Phys. 52 (1980) 199, and references therein

[2] K.A. Olive et al., Particle Data Group, Chin. Phys. C38 (2014) 090001

[3] G. Zeller *et al.*, NuTeV, Phys. Rev. Lett. 88 (2002) 091802 E: *ibid.* 90 (2003) 239902, hep-ex/0110059

[4] J.L. Abelleira Fernandez *et al.*, LHeC Study Group, J. Phys. G39 (2012) 075001, arXiv:1206.2913

[5] A.Y. Illarionov, B.A. Kniehl and A.V. Kotikov, Phys. Rev. Lett. 106 (2011) 231802, arXiv:1105.2829

[6] P. Bolzoni, F. Maltoni, S. Moch and M. Zaro, Phys. Rev. D85 (2012) 035002, arXiv:1109.3717

[7] F.A. Dreyer and A. Karlberg, arXiv:1606.00840

[8] J. Blümlein, G. Falcioni and A. De Freitas, arXiv:1605.05541

[9] W. Furmanski and R. Petronzio, Z. Phys. C11 (1982) 293, and references therein

[10] J. Sanchez Guillen et al., Nucl. Phys. B353 (1991) 337

[11] W.L. van Neerven and E.B. Zijlstra, Phys. Lett. B272 (1991) 127

[12] E.B. Zijlstra and W.L. van Neerven, Phys. Lett. B273 (1991) 476

[13] E.B. Zijlstra and W.L. van Neerven, Phys. Lett. B297 (1992) 377

[14] S. Moch and J.A.M. Vermaseren, Nucl. Phys. B573 (2000) 853, hep-ph/9912355

[15] S. Moch, J.A.M. Vermaseren and A. Vogt, Phys. Lett. B606 (2005) 123, hep-ph/0411112

[16] J.A.M. Vermaseren, A. Vogt and S. Moch, Nucl. Phys. B724 (2005) 3, hep-ph/0504242

[17] S. Moch, J.A.M. Vermaseren and A. Vogt, Nucl. Phys. B813 (2009) 220, arXiv:0812.4168

[18] S. Moch, J.A.M. Vermaseren and A. Vogt, Nucl. Phys. B688 (2004) 101, hep-ph/0403192

[19] A. Vogt, S. Moch and J.A.M. Vermaseren, Nucl. Phys. B691 (2004) 129, hep-ph/0404111

[20] W.L. van Neerven and A. Vogt, Nucl. Phys. B588 (2000) 345, hep-ph/0006154

[21] S. Moch, M. Rogal and A. Vogt, Nucl. Phys. B790 (2008) 317, arXiv:0708.3731

[22] S. Moch and M. Rogal, Nucl. Phys. B782 (2007) 51, arXiv:0704.1740

[23] D.J. Broadhurst, A.L. Kataev and C.J. Maxwell, Phys. Lett. B590 (2004) 76, hep-ph/0403037

[24] E.A. Paschos and L. Wolfenstein, Phys. Rev. D7 (1973) 91

[25] J. Davies, S. Moch, J.A.M. Vermaseren and A. Vogt, to appear