

# Threshold resummation for polarized high- $p_T$ hadron production at COMPASS

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We study the cross section for the photoproduction process  $\gamma N \rightarrow hX$  where the incident photon and nucleon are longitudinally polarized and a hadron  $h$  is observed at high transverse momentum. Specifically, we address the “direct” part of the cross section, for which the photon interacts in a pointlike way. For this contribution we perform an all-order resummation of logarithmic threshold corrections generated by soft or collinear gluon emission to next-to-leading logarithmic accuracy. We present phenomenological results relevant for the COMPASS experiment and compare to recent COMPASS data.

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## 1. Introduction

To obtain information about the nucleon's gluon helicity distribution  $\Delta g$  and to explore its contribution to the proton's spin is the main focus of CERN's COMPASS experiment,  $\mu N \rightarrow \mu' hX$ , where the muon  $\mu$  and the nucleon  $N$  are both longitudinally polarized and where  $h$  denotes a charged hadron produced at high transverse momentum  $p_T$ . Thanks to the large  $p_T$ , we can use perturbative methods. Demanding the scattered muon  $\mu'$  to have a low scattering angle with respect to the incoming one, the main contributions come from almost on-shell photons exchanged between the muon and the nucleon, so that the scattering may then be treated as a *photoproduction* process  $\gamma N \rightarrow hX$ . COMPASS has presented data for the spin-averaged cross section [1], as well as for its double-longitudinal spin asymmetry  $A_{LL}$  [2], which is directly sensitive to  $\Delta g$ . As is well known [3], hard photoproduction cross sections receive contributions from two sources, the “direct” ones, for which the photon interacts in the usual pointlike way in the hard scattering, and the “resolved” ones, for which the photon reveals its own partonic structure. As discussed in [4], in the kinematic regime accessible at COMPASS perturbative corrections beyond NLO are important, as the partonic hard-scattering cross sections are largely probed in the “threshold”-regime. There, the initial photon and parton have just enough energy to produce a pair of recoiling high- $p_T$  partons, one of which subsequently fragments into the observed hadron. The phase space for radiation of additional gluons then becomes small, allowing radiation of only soft and/or collinear gluons. As a result, the cancelation of infrared singularities between real and virtual diagrams leaves behind large logarithmic corrections to the partonic cross sections. These logarithms appear for the first time at NLO and then recur with increasing power at every order of perturbation theory. Threshold resummation [5, 6] allows to sum the logarithms to all orders to a certain logarithmic accuracy. As it was applied to the spin-averaged cross section at COMPASS at next-to-leading logarithm (NLL) level in Ref. [4], we examine threshold resummation also for polarized scattering and with that for the spin asymmetry  $A_{LL}$  measured at COMPASS [2]. As a first step, we will consider the direct contributions to the cross section, which formally dominate over the resolved ones near partonic threshold, afterwards we plan to complete our resummation study for  $A_{LL}$  by performing threshold resummation also for the resolved contribution.

## 2. Photoproduction cross section and the resummed cross section

The underlying theoretical framework for the process  $\ell N \rightarrow \ell' hX$  can be found in detail in our publication, see Ref. [7]. We introduce the spin-averaged and spin-dependent cross sections for the lepton-nucleon process as  $d\sigma_{\ell N} \equiv \frac{1}{2} [d\sigma_{\ell N}^{++} + d\sigma_{\ell N}^{+-}]$  and  $d\Delta\sigma_{\ell N} \equiv \frac{1}{2} [d\sigma_{\ell N}^{++} - d\sigma_{\ell N}^{+-}]$ , where the superscripts  $(++)$ ,  $(+-)$  denote the helicities of the incoming particles. Using factorization, the differential spin-dependent cross section may be written as [8, 4]:

$$\frac{p_T^3 d\Delta\sigma}{dp_T d\eta} = \int_{x_\ell^{\min}}^1 dx_\ell \int_{x_n^{\min}}^1 dx_n \int_x^1 dz \frac{\hat{x}_T^4 z^2}{8v} \frac{\hat{s} d\Delta\hat{\sigma}_{ab \rightarrow cX}(v, w, \hat{s}, \mu_{r,fi,ff})}{dv dw} \Delta f_{a/\ell}(x_\ell, \mu_{fi}) \Delta f_{b/N}(x_n, \mu_{fi}) D_{h/c}(z, \mu_{ff}), \quad (2.1)$$

where we sum over all possible partonic channels  $ab \rightarrow cX$ . The  $\Delta f_{b/N}(x_n, \mu_{fi})$  are the polarized parton distribution functions of the nucleon, and the  $D_{h/c}(z, \mu_{ff})$  are the parton-to-hadron fragmentation functions, which both depend on a momentum fraction  $x_n$  or  $z$ , and on an initial- or final-state factorization scale  $\mu_{fi}$  or  $\mu_{ff}$ . Finally, the perturbative  $d\Delta\hat{\sigma}_{ab \rightarrow cX}$  are the spin-dependent partonic cross sections, expanded in terms of the strong coupling constant  $\alpha_s$ ,  $d\Delta\hat{\sigma}_{ab \rightarrow cX} = d\Delta\hat{\sigma}_{ab \rightarrow cX}^{(0)} +$

$\frac{\alpha_s}{\pi} d\Delta\hat{\sigma}_{ab\rightarrow cX}^{(1)} + \dots$ . They have been written differential in  $v \equiv 1 + \hat{t}/\hat{s}$  and  $w \equiv -\hat{u}/(\hat{s} + \hat{t})$ , with the Mandelstam variables  $\hat{s} = x_\ell x_n \mathcal{S}$ ,  $\hat{t} = -\hat{s}\hat{x}_T/2e^{-\hat{\eta}}$  and  $\hat{u} = -\hat{s}\hat{x}_T/2e^{\hat{\eta}}$ . Furthermore, we have  $\hat{x}_T \equiv x_T/(z\sqrt{x_\ell x_n})$ , where  $x_T \equiv 2p_T/\sqrt{S}$ , and  $\hat{\eta} = \eta + 1/2\ln(x_n/x_\ell)$ . Finally, the lower integration bounds in Eq. (2.1) can be found in [8, 4]. As mentioned before, we have to distinguish between direct (at LO:  $\gamma q \rightarrow g(q)$ ,  $\gamma q \rightarrow q(g)$ ,  $\gamma g \rightarrow q(\bar{q})$ ) and resolved contributions. The physical cross section is the sum of both parts,  $d\Delta\sigma = d\Delta\sigma_{\text{dir}} + d\Delta\sigma_{\text{res}}$ . Analytical expressions for the NLO partonic cross section have been obtained in Refs. [9, 10]. For each process the result may be cast into the form

$$\frac{\hat{s}d\Delta\hat{\sigma}_{ab\rightarrow cX}^{(1)}(v,w)}{dv dw} = A(v)\delta(1-w) + B(v)\left(\frac{\ln(1-w)}{1-w}\right)_+ + C(v)\left(\frac{1}{1-w}\right)_+ + F(v,w), \quad (2.2)$$

where the coefficients  $A(v), B(v), C(v), F(v,w)$  depend on the process under consideration, and where the plus-distributions are defined as usual by  $\int_0^1 dw f(w) [g(w)]_+ \equiv \int_0^1 dw [f(w) - f(1)]g(w)$ . The terms with plus-distributions give rise to the large double-logarithmic corrections that are addressed by threshold resummation. Their origin lies in soft-gluon radiation, and they recur with higher power at every higher order of perturbation theory. For the  $k$ -th order QCD correction, the leading terms are proportional to  $\alpha_s^k [\ln^{2k-1}(1-w)/(1-w)]_+$ . In the following we discuss the all-order resummation of the threshold logarithms in the direct part of the cross section, which we separate from the resolved part adopting the  $\overline{\text{MS}}$  scheme. We perform the resummation to next-to-leading logarithm (NLL), which means that the three ‘‘towers’’  $\alpha_s^k [\ln^{2k-1}(1-w)/(1-w)]_+$ ,  $\alpha_s^k [\ln^{2k-2}(1-w)/(1-w)]_+$ ,  $\alpha_s^k [\ln^{2k-3}(1-w)/(1-w)]_+$  are taken into account to all orders in the strong coupling. They turn then into the NLL towers  $\alpha_s^k \ln^{2k}(N)$ ,  $\alpha_s^k \ln^{2k-1}(N)$ , and  $\alpha_s^k \ln^{2k-2}(N)$  in Mellin moment space, where resummation takes place. Hence we write for the direct contribution:

$$\frac{p_T^3 d\sigma}{dp_T d\eta} = \int_0^1 dx_\ell \int_0^1 dx_n \Delta f_{\gamma/\ell}(x_\ell, \mu_{fi}) \Delta f_{b/N}(x_n, \mu_{fi}) \int_{\mathcal{C}} \frac{dN}{2\pi i} (x^2)^{-N} D_{h/c}^{2N+3}(\mu_{ff}) \Delta \tilde{w}_{\gamma b \rightarrow cX}^{2N}(\hat{\eta}), \quad (2.3)$$

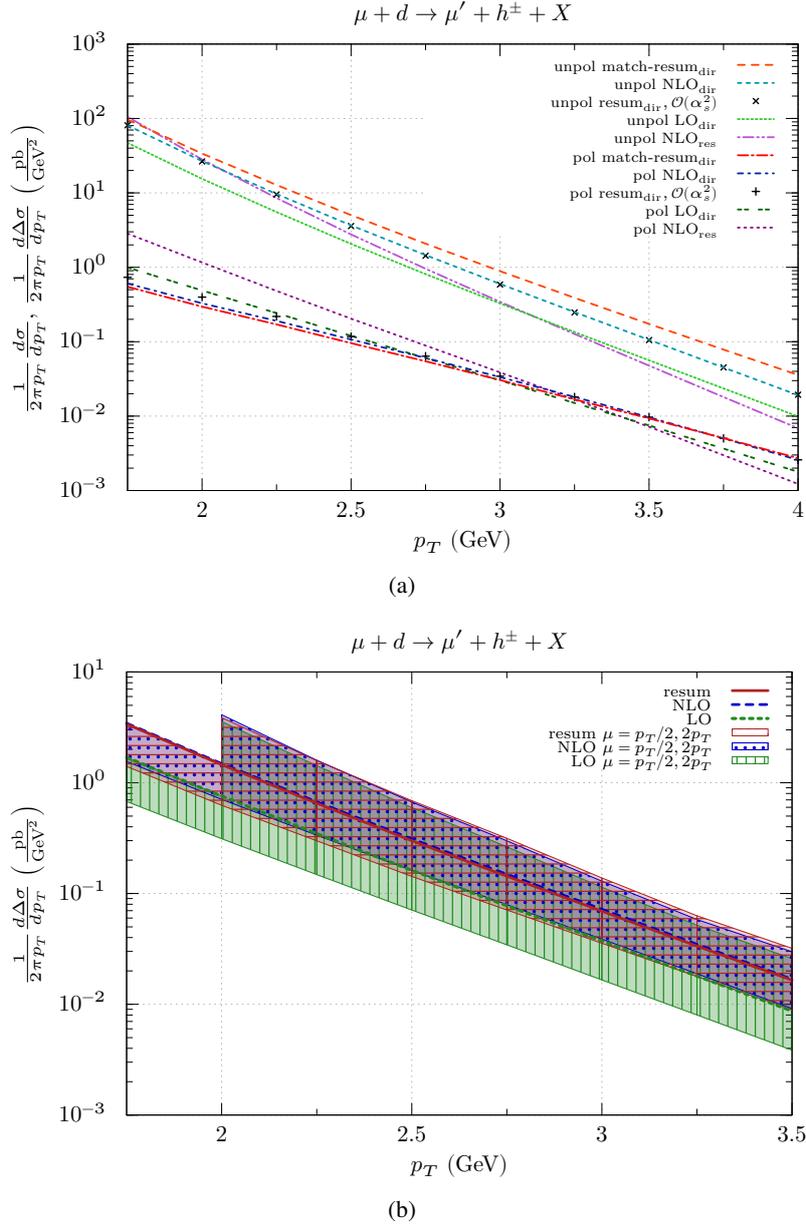
where  $D_{h/c}^N(\mu) \equiv \int_0^1 dz z^{N-1} D_{h/c}(z, \mu)$  and where we have for the hard scattering function:

$$\Delta \tilde{w}_{\gamma b \rightarrow cX}^N(\hat{\eta}) \equiv 2 \int_0^1 d\frac{\hat{s}_4}{\hat{s}} \left(1 - \frac{\hat{s}_4}{\hat{s}}\right)^{N-1} \frac{\hat{x}_T^4 z^2}{8v} \frac{\hat{s}d\Delta\hat{\sigma}_{\gamma b \rightarrow cX}}{dv dw}, \quad (2.4)$$

with  $\hat{s}_4 = \hat{s} + \hat{t} + \hat{u} = \hat{s}v(1-w) = \hat{s}(1 - \hat{x}_T \cosh \hat{\eta})$ . At NLL one has [11, 12, 4]:

$$\begin{aligned} \Delta \tilde{w}_{\gamma b \rightarrow cd}^{N, \text{resum}}(\hat{\eta}) &= \left(1 + \frac{\alpha_s}{\pi} \Delta C_{\gamma b \rightarrow cd}^{(1)}\right) \Delta \hat{\sigma}_{\gamma b \rightarrow cd}^{(0)}(N, \hat{\eta}) \Delta_b^{(-\hat{t}/\hat{s})N}(\hat{s}, \mu_{fi}, \mu_r) \\ &\times \Delta_c^N(\hat{s}, \mu_{ff}, \mu_r) J_d^N(\hat{s}) \exp \left[ \int_{\mu_r}^{\sqrt{\hat{s}}/N} \frac{d\mu'}{\mu'} 2\text{Re}\Gamma_{\gamma b \rightarrow cd}(\hat{\eta}, \alpha_s(\mu')) \right]. \end{aligned} \quad (2.5)$$

Only  $\Delta C_{\gamma b \rightarrow cd}^{(1)}$ , which matches the resummed cross section to the NLO one, and  $\Delta \hat{\sigma}_{\gamma b \rightarrow cd}^{(0)}$  depend on the polarizations of the incoming partons; all other factors are spin-independent. The functions  $\Delta_b^{(-\hat{t}/\hat{s})N}$  and  $\Delta_c^N$  are exponentials and account for soft radiation collinear to the parton  $b$  or  $c$ , respectively, and the function  $J_d^N$  describes collinear emission, soft and hard, off the unobserved recoiling parton  $d$ . Finally, emission of soft gluons at large angles is accounted for by the last factor in (2.5), containing the soft anomalous dimension  $\Gamma_{\gamma b \rightarrow cd}$ . As usual we match our resummed cross section to the NLO one by subtracting all NLO contributions that are present in the resummed



**Figure 1:** (a) Direct parts of the LO, NLO and matched resummed cross sections for  $\mu d \rightarrow \mu' h^\pm X$ . We also show the NLO expansions of the resummed results (symbols), as well as the NLO resolved contributions. (b) Scale dependence of the spin-dependent cross sections. For the resummed case we include the resolved contributions at NLO. We vary the scale, so that  $p_T/2 \leq \mu \leq 2p_T$ , and show results only for  $\mu \geq 1$  GeV.

result and adding instead the full NLO cross section, see [7]. This procedure makes sure that NLO is fully included in the theoretical predictions, as well as all soft-gluon contributions beyond NLO to NLL accuracy.

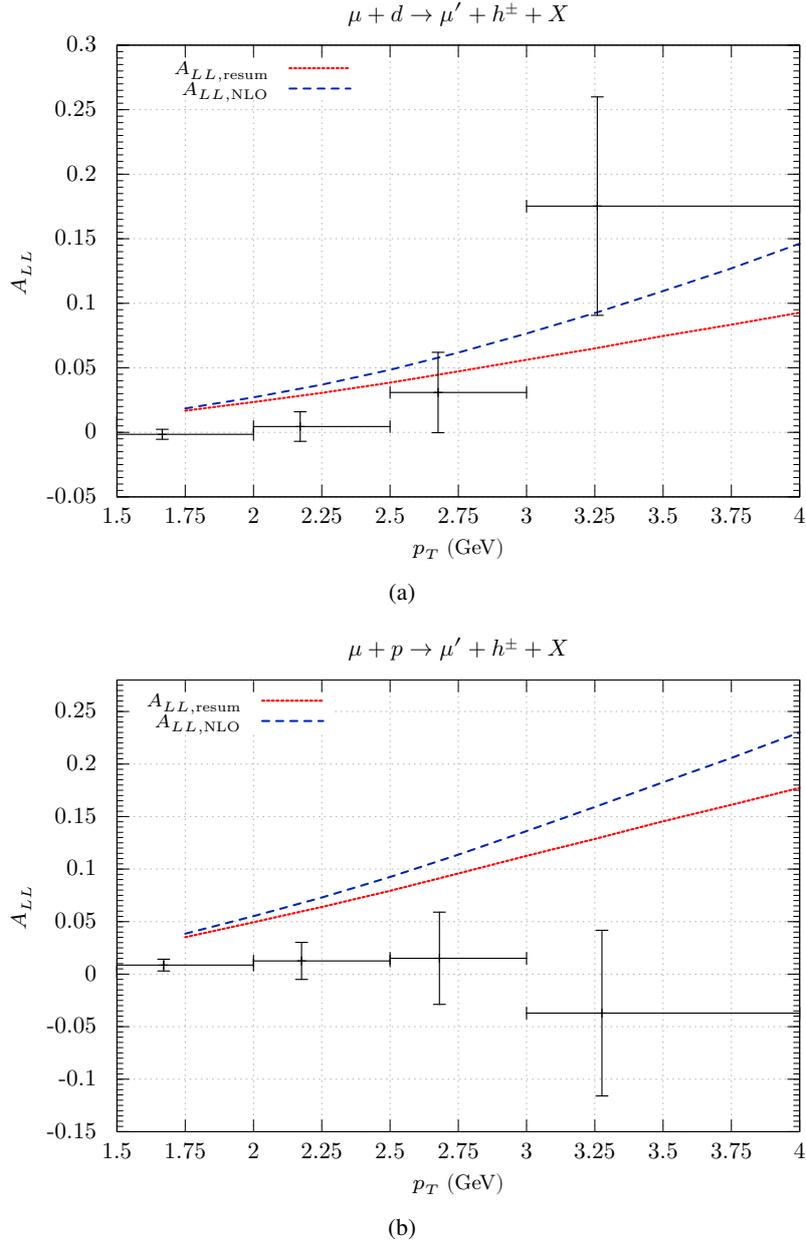
### 3. Phenomenological Results

COMPASS uses a muon mean beam energy of  $E_\mu = 160$  GeV, resulting in  $\sqrt{S} = 17.4$  GeV. Both deuteron and proton targets are available. COMPASS imposes the cuts  $Q_{\text{max}}^2 = 1$  GeV<sup>2</sup> on the virtuality of the exchanged photon,  $0.2 \leq y \leq 0.9$  on the fraction of the lepton's momentum carried by the photon, and  $0.2 \leq z \leq 0.8$  for the photon's energy fraction carried by the hadron.

Charged hadrons are detected in COMPASS if their scattering angle is between  $10 \leq \theta \leq 120$  mrad, corresponding to  $-0.1 \leq \eta \leq 2.38$  in the hadron's pseudorapidity. We integrate over this range and sum over the charges. Our choice for the helicity parton distributions is the set of [13] (DSSV2014), further, we adopt the fragmentation functions of Ref. [14] (DSS) throughout this work. In the calculations of the NLL resummed unpolarized cross sections we follow Ref. [4] and use the numerical code of that work. We employ the unpolarized parton distribution functions of Ref. [15] (MSTW). For comparisons we will also present results for the NLO resolved contributions, for which we will adopt the unpolarized and polarized photonic parton distributions of Refs. [16] and [17], respectively. We furthermore choose all factorization/renormalization scales to be equal,  $\mu_r = \mu_{fi} = \mu_{ff} \equiv \mu$ . We usually choose  $\mu = p_T$ , except when investigating the scale dependence of the theoretical predictions. Figure 1 shows the direct parts (defined in  $\overline{\text{MS}}$ ) of the spin-averaged and spin-dependent cross sections for  $\mu d \rightarrow \mu' h^\pm X$  at LO, NLO, and resummed with matching implemented as described in [7]. The symbols in the figure show the NLO-expansions of the non-matched resummed cross sections, and for comparison the figure also presents the NLO resolved contributions. As can be seen, in the unpolarized case the difference between the LO and NLO results is very large, and resummation adds another equally sizable correction that increases relative to the NLO result as one goes to larger  $p_T$ , that is, closer to threshold. The NLO expansion of the resummed cross section shows excellent agreement with the full NLO result, demonstrating that the threshold terms correctly reproduce the dominant part of the cross section. These findings are as reported in [4]. In the polarized case, the higher-order corrections are overall much more modest. The resummation effects are smaller here, leading to only a modest further enhancement over NLO as one gets closer to threshold. This implies that the higher-order resummation effects will not cancel in the spin asymmetry for the process. Again the NLO expansion of the resummed cross section reproduces the full NLO result faithfully, although not quite as well as in the unpolarized case. These features that we observe for the direct part of the polarized cross section may be understood from the fact that the two competing LO subprocesses  $\gamma q \rightarrow qg$  and  $\gamma g \rightarrow q\bar{q}$  enter with opposite sign and thus cancel to some extent. This was already observed in Ref. [8] in the context of the NLO calculation. In Fig. 1 (b) we examine the scale dependence of the spin-dependent cross section, where we vary the scales in the range  $p_T/2 \leq \mu \leq 2p_T$ . For the resummed cross section we include the resolved contributions at NLO, so that  $d\Delta\sigma_{\text{resum}} = d\Delta\sigma_{\text{dir,resum}} + d\Delta\sigma_{\text{res,NLO}}$ ; the LO and NLO cross sections, however, contain as usual their full direct and resolved contributions. One can observe that the scale uncertainty is large. There is a clear improvement when going from LO to NLO, but no further improvement when we include resummation, a feature that will require further attention in the future. We now investigate the double-longitudinal spin asymmetry, defined as the ratio  $A_{LL} = d\Delta\sigma/d\sigma$ . We include the NLO resolved contributions, so that at the present stage the “resummed” spin asymmetry is given by

$$A_{LL,\text{resum}} = \frac{d\Delta\sigma_{\text{dir,resum}} + d\Delta\sigma_{\text{res,NLO}}}{d\sigma_{\text{dir,resum}} + d\sigma_{\text{res,NLO}}}, \quad \text{while} \quad A_{LL,\text{NLO}} = \frac{d\Delta\sigma_{\text{dir,NLO}} + d\Delta\sigma_{\text{res,NLO}}}{d\sigma_{\text{dir,NLO}} + d\sigma_{\text{res,NLO}}}. \quad (3.1)$$

Our results are shown in Figs. 2 (a) and (b). The different size of the resummation effects for the polarized and unpolarized cross sections, see Fig. 1, clearly implies that the resummed threshold logarithm contributions do not cancel in the double-spin asymmetry. Indeed, as Fig. 2 shows, the deuteron asymmetry is reduced by almost a factor of two at high  $p_T$ , when going from NLO to NLL. For a proton target, there also is a substantial, albeit somewhat less dramatic, decrease. In Fig. 2 we compare our theoretical results with the COMPASS data presented in [2]. As one can



**Figure 2:** Double-longitudinal spin asymmetries  $A_{LL}$  for (a) a deuteron and (b) a proton target for COMPASS kinematics with the full rapidity range  $-0.1 \leq \eta \leq 2.38$ . The asymmetries include the resolved contributions at NLO. The theoretical results are compared to the recent COMPASS data [2].

see, while the asymmetries for deuterons are in marginal agreement, the very small asymmetry seen by COMPASS for protons is incompatible with any of our predictions. As shown in Ref. [2], this problem appears to be especially pronounced in the rapidity range  $-0.1 \leq \eta \leq 0.45$  and for positively charged hadrons. While the higher order resummed corrections that we have included ameliorate the situation, they are clearly not sufficient. Given the rather large decrease of the spin asymmetry generated by resummation of the direct contributions, it is arguably not possible to draw any reliable conclusions from this observation before also the resummation for the resolved part of the cross sections has been carried out. It appears unlikely, however, that the resummation of the resolved contribution will bring the data and the theoretical results into good agreement

since they affect the asymmetries for both targets in similar ways. If, for instance, the polarized resolved contribution were so large and negative that the proton data could be accommodated, the description of the deuteron asymmetry would vastly deteriorate [18].

#### 4. Conclusion and Outlook

We have studied the impact of threshold resummation at NLL on the spin-dependent cross section for  $\gamma N \rightarrow hX$  at high transverse momentum  $p_T$  of the hadron  $h$ , and on the resulting double-longitudinal spin asymmetry  $A_{LL}$ . For the kinematics relevant for the COMPASS experiment we find that the spin-dependent cross section receives much smaller enhancements by resummation than the spin-averaged one treated in Ref. [4]. As a result, threshold effects do not cancel in the asymmetry, and the prediction for  $A_{LL}$  decreases when resummation is taken into account. Definite conclusions about the impact of resummation on the spin asymmetry will become possible only when also the resolved component has been resummed, which we plan to do in a future work. We also note that the scale dependence of the perturbative cross section remains uncomfortably large. In order to improve this it may, eventually, be necessary to extend resummation to next-to-next-to-leading logarithmic level, following the techniques developed in Ref. [19]. Comparison to the recent COMPASS data [2] shows that the theoretically predicted spin asymmetries fail to reproduce the data well. Especially for the proton target the data show a nearly vanishing asymmetry, while the theoretical result appears to be always clearly positive. In fact, it is worth stressing that each of the theoretical results shown in Fig. 2 predicts a *larger* spin asymmetry for the proton than for the deuteron, in contrast to the trend seen in the data. This feature of the theoretical predictions is likely no accident, as a simple study of the LO direct contributions shows [18]. Clearly, future work is needed in order to clarify in how far the leading-twist perturbative-QCD framework can accommodate a larger spin asymmetry for  $\mu d \rightarrow \mu' hX$  than for  $\mu p \rightarrow \mu' hX$ .

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