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# $f(R)$ cosmology and dark matter

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In this paper is discussed the thermal abundance and evolution of relic particles (WIMPs) assuming that the background is described by $f(R)$ cosmology. As a reliable model, the marginally deformed Starobinsky model $f(R) = R + \alpha R^n$, with $1 < n \leq 2$ is considered.

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## 1. Introduction

Despite all fundamental results of General Relativity (GR), the observational data of the present Universe indicate that strong deviations from the standard Hilbert-Einstein picture [1] are nedeed, and that new unknown form of (dark) matter and energy are necessary. Several modified theories of gravity have been proposed in last years, which try to address, at the same time, the shortcomings of the Cosmological Standard Model (for example, higher order curvature invariants allow to get inflationary-like solutions of early Universe, as well as to explain the flatness and horizon problems) [2, 3, 4, 5]. In the framework of models that extend GR, $f(R)$ gravity is one of the favorite candidate. It provides an unified description of dark energy and dark matter, without invoking exotic sources as dark matter, and allows for the unification of the early-time (inflation) and the later-time acceleration of the Universe [6, 7]. The gravitational action for $f(R)$ gravity is [8, 9]

$$S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,f(R)+S_m[g_{\mu\nu},\psi_m]\,, \tag{1.1}$$

where $S_m$ is the action of the standard matter and $\kappa^2=8\pi G=8\pi/M_{Pl}^2$, with $M_{Pl}\simeq 10^{19}$GeV. One of the consequences of dealing with alternative cosmologies is that the thermal history of (relic) particles is modified. In these models the expansion rates $H$ of the Universecan be written as $H(T)=A(T)H_{GR}(T)$, where $H_{GR}$ is the expansion rate in GR, while the factor $A(T)$ encodes the information about the particular model of gravity extending or modifying GR. Usually, the factor $A(T)$ is defined in order that the successful predictions of the Big Bang Nucleosynthesis (BBN) are preserved, that is $A(T)\neq 1$ at early time, and $A(T)\to 1$ before BBN begins (one refers to the pre-BBN epoch since it is not directly constrained by cosmological observations). On the other hand, an enhanced (pre-BBN) expansion of the Universe can reconcile observed dark matter cosmic relic abundance with contraints provided by indirect dark matter detection experiments (such as PAMELA [10] and the more recent AMS-02) [11]. These aspects will be studied in this paper assuming thta the evolution of the Universe is governed by

$$f(R)=R+\alpha R^n\,, \tag{1.2}$$

where the case $n=2$ corresponds to the Starobinsky model. Models of the form (1.2) may generate sizable primordial tensor modes provided $1<n<2$ [12]. Remarkably, these models might emerge from Supergravity [13, 14, 15] or dilaton dynamics in brane cosmology scenarios based on string theory [17], and are in agreement with BICEP2 [14, 16] and Planck data [18] ($R^2$-inflation is indeed fully consistent with observations [2, 19, 20]).

The paper is organized as follows. In Section 2 we derive the $f(R)$ gravity field equations and solve them in the radiation dominated era. Section 3 is devoted to the study of thermal relics abundance and their evolution. Conclusions are drawn in Section 4.

## 2. Field equations in $f(R)$ gravity

The field equations for $f(R)$ gravity follow by varying the action (1.1) with respect to the tensor metric $g_{\mu\nu}$

$$G^c_{\mu\nu}=\kappa^2T^m_{\mu\nu}\,,\quad G^c_{\mu\nu}\equiv f'R_{\mu\nu}-\frac{f}{2}g_{\mu\nu}-\nabla_\mu\nabla_\nu f'+g_{\mu\nu}\Box f'\,, \tag{2.1}$$

where $f' \equiv \dfrac{\partial f}{\partial R}$, and $T^m_{\mu\nu}$ is the energy-momentum tensor for matter. The trace equation is

$$3\Box f' + f'R - 2f = \kappa^2 T^m\,, \qquad T^m = \rho - 3p\,. \tag{2.2}$$

The Bianchi's identities are satisfied: $\nabla^\mu G^c_{\mu\nu} = 0 = \nabla^\mu T^m_{\mu\nu} = 0$. We work in the regime $\alpha R^n > R$, with $1 < n < 2$ according to Ref. [12]. We look for solutions of the form $a(t) = a_0 t^\beta$. In a (spatially flat) Friedman-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2]\,, \tag{2.3}$$

we find that the $0-0$ field equation and the trace equation read (in the very early Universe $t \to 0$)

$$\alpha\Omega_{\beta,n} R^n = \kappa^2 \rho\,, \qquad \alpha\Gamma_{\beta,n} R^n = \kappa^2 T^m\,, \tag{2.4}$$

where $\rho$ is the energy density and

$$\Omega_{\beta,n} \equiv \frac{1}{2}\left[\frac{n(\beta+2n-3)}{2\beta-1} - 1\right], \qquad \Gamma_{\beta,n} \equiv n - 2 - \frac{n(n-1)(2n-1)}{\beta(2\beta-1)} + \frac{3n(n-1)}{2\beta-1}\,. \tag{2.5}$$

During the radiation dominated era $\rho = \dfrac{\pi^2 g_*}{30} T^4$ ($g_* \sim 10^2$) and $T^m = 0$, hence $\Gamma_{\beta,n} = 0$. One therefore gets

$$\beta = \frac{n}{2} \qquad \Omega_{\beta=\frac{n}{2},n} = \frac{5n^2 - 8n + 2}{4(n-1)}\,. \tag{2.6}$$

The function $\Omega_{\beta=\frac{n}{2},n}$ is positive for $n \geqslant 1.289$. It is worth to note that one can also consider scenarios in which $T^m \neq 0$, which are related, for example, to bulk viscosity effects [21]. From (2.4) one gets the relation between the cosmic time $t$ and the temperature $T$

$$t(T) = \Sigma\left(\frac{T}{M_{Pl}}\right)^{-\frac{2}{n}} M_{Pl}^{-1}\,, \qquad \Sigma \equiv [6|\beta(1-2\beta)|]^{1/2}\left(\frac{15\tilde{\alpha}\Omega_{\beta,n}}{4\pi^3 g_*}\right)^{\frac{1}{2n}}\,, \quad \tilde{\alpha} = \frac{\alpha}{M_{Pl}^{2(1-n)}}\,. \tag{2.7}$$

We assume that at the instant $t_*$ (the transition time) the Universe passes from the cosmic evolution described by $f(R)$ cosmology to the cosmic evolution described by the standard cosmological model. The transition time $t_*$ or the transition temperature $T_*$ are determined via the relation $\alpha\Omega_{\beta,n} R^n(t_*) = H^2_{GR}(t_*)$, yielding

$$t_* = \left[4\tilde{\alpha}\Omega_{\beta,n}[6|\beta(2\beta-1)|]^n\right]^{\frac{1}{2(n-1)}} M_{Pl}^{-1}\,, \tag{2.8}$$

$$T_* \equiv M_{Pl}[6(|\beta(1-2\beta)|)]^{-\frac{n}{4(n-1)}}\left[\frac{15}{16\pi^3 g_*}\right]^{\frac{1}{4}}\left[4\tilde{\alpha}\Omega_{\beta,n}\right]^{-\frac{1}{4(n-1)}}\,. \tag{2.9}$$

The relation (2.7) can be then cast in the form

$$t = t_*\left(\frac{T}{T_*}\right)^{-\frac{2}{n}}\,, \quad \text{with} \quad \frac{t_* T_*^2}{M_{Pl}} = \sqrt{\frac{15}{16\pi^3 g_*}}\,. \tag{2.10}$$

The expansion rate of the Universe in $f(R)$ cosmology is

$$H(T) = A(T) H_{GR}(T)\,, \qquad A(T) \equiv 2\sqrt{3}\beta\left(\frac{T}{T_*}\right)^p\,, \quad p \equiv \frac{2}{n} - 2 \tag{2.11}$$

where the factor $A(T)$ is the so called enhancement factor.

## 3. Relic abundance and WIMP particles

Current observations indicate that not only our Universe is dominated by dark matter, responsible of galactic and extragalactic dynamics, but also by dark energy, responsible of the accelerated expansion of the Universe [24]

$$0.092 \leq \Omega_{CDM}h^2 \leq 0.124\,, \quad 0.30 \leq \Omega_{DE}h^2 \leq 0.46\,, \tag{3.1}$$

where $h = 100\text{Km s}^{-1}\text{ Mpc}^{-1}$ is the Hubble constant. Favorite candidates for non-baryonic cold dark matter are the WIMPs (weakly interacting massive particles). The interest about these particles as dark matter follows from the fact that WIMPs in chemical equilibrium in the early Universe have the abundance which agrees with the expected one in the context of cold dark matter. This kind of studies are motivated by recent astrophysical results which involve cosmic ray electron and positrons, antiprotons, and $\gamma$-rays. Particular attention is devoted to the rising behavior of the positron fraction observed in PAMELA experiment [10] (the astrophysical interpretation of this phenomenon is discussed in [22], while the possibility of dark matter annihilation into lepton is discussed in [23]).

The general analysis that accounts for the enhancement of the expansion rates in alternative cosmological models has been performed in [11] (see also [25]). Typically the expansion rate is written in the for $H = A(T)H_{GR}$, where the function $A(T) = \eta(T/T_f)^\nu$ encodes parameters characterizing a particular model (here $\eta$ and $\nu$ are free parameters, while $T_f \sim 10$GeV is the decoupling temperature). In our $f(R)$ model, we have $\nu = 2/n-2$, so that $-1 \leqslant \nu \leqslant 0$ for $1 \leqslant n \leqslant 2$. To compute the DM abundance, one has to numerically solve the Boltzmann equation for the number density of thermal relic $Y$. Fixing $\eta = \{-1,0,1,2\}$, $\langle\sigma_{ann}v\rangle \sim 2.1\times 10^{-26}\text{ cm}^3\text{ sec}^{-1}$, and $m_\chi = [10, 10^4]$TeV, one determines the values of parameter $\eta$ vs the DM masses [11] required to infer the correct relic abundance of DM particles $\Omega_\chi h^2 = \Omega h^2\big|_{CDM}^{WMAP} = 0.1131 \pm 0.0034$ [26]. The relic abundance is given by

$$\Omega_\chi h^2 = \frac{m_\chi s_0 Y_0}{\rho_c}\,,$$

where $\rho_c = 3H_0^2 M_{Pl}^2/8\pi$ is the critical density of the Universe, $s_0$ is the present value of the entropy density $s = \frac{2\pi^2}{45} g_\chi(T)T^3$, and $Y_0$ is the present value of the WIMP abundance for comoving volume

$$\frac{1}{Y_0} = \frac{1}{Y_f} + \sqrt{\frac{\pi M_{Pl}^2}{45}}\, m_\chi \int_{x_f}^{\infty} \frac{g_\chi(x)\langle\sigma_{ann}v\rangle}{\sqrt{g_*(x)}A(x)x^2}dx\,,$$

in which $Y_f$ is the value of the WIMP abundance for comoving volume at the freeze-out, $x = m_\chi/T$, $\{g_\chi(T), g_*(T)\}$ counts the effective number of degrees of freedom at temperature $T$, and, $x_f = \ln\left[0.0038 g_\chi \frac{M_{Pl} m_\chi \langle\sigma_{ann}v\rangle_f}{A(x_f)\sqrt{x_f g_*(x_f)}}\right]$ (computations refer to non relativistic DM particles). Finally one gets [11]

$$\eta \geqslant 0.1 \quad \text{for} \quad m_\chi \gtrsim 10^2\text{GeV}\,. \tag{3.2}$$

More precisely, the analysis shows that DM masses are in the range $[10^2 - 10^4]$GeV, and the allowed region for the parameter $\eta$ is $0.1 \leqslant \eta \lesssim 10^3 - 10^6$, where the upper bounds on $\eta$ vary for the different cosmological models labelled by $\nu$ [11].

**Table 1:** Estimations of $\alpha$ for fixed values of the transition temperature $T_* = (1-10^2)$MeV.

| $n$ | $T$(GeV) | $\alpha$ |
|---|---|---|
| 1.3 | $10^{-3}$ | $10^{14}\text{GeV}^{-0.6}$ |
| | $10^{-1}$ | $10^{12}\text{GeV}^{-0.6}$ |
| 2 | $10^{-3}$ | $10^{44}\text{GeV}^{-2}$ |
| | $10^{-1}$ | $10^{36}\text{GeV}^{-2}$ |

**Applications to $f(R)$ cosmology**

According to the above results, we rewrite the factor $A(T)$ (see Eqs. (2.9) and (2.11)) in the form

$$A(T) = \eta\left(\frac{T}{T_f}\right)^{\nu}, \qquad \eta \equiv 2\sqrt{3}\beta\left(\frac{T_f}{T_*}\right)^{\nu}, \qquad \nu = \frac{2}{n} - 2. \tag{3.3}$$

The transition temperature $T_*$ is fixed for values greater than the free-out temperature $T_f$. Therefore we set $T_* = (1 \div 10^2)T_{BBN}$. From (2.9) we get

$$\alpha = \left(\frac{15}{16\pi^3 g_*}\right)^{n-1} \frac{[6|\beta(1-2\beta)|]^{-n}}{4\Omega_{\beta=\frac{n}{2},n}} \left(\frac{M_{Pl}}{T_*}\right)^{4(n-1)} M_{Pl}^{2(1-n)}.$$

The order of magnitudes of $\alpha$ are reported in Table I for $\Omega_{\beta=\frac{n}{2},n}$ given in (2.6). The function $\eta$ vs $n$ is plotted in Fig. 1. As we can see, the parameter $\eta$ assumes values of the order $\mathcal{O}(0.1-1)$, so that the mass of WIMPs particles is of the order $10^2$GeV.

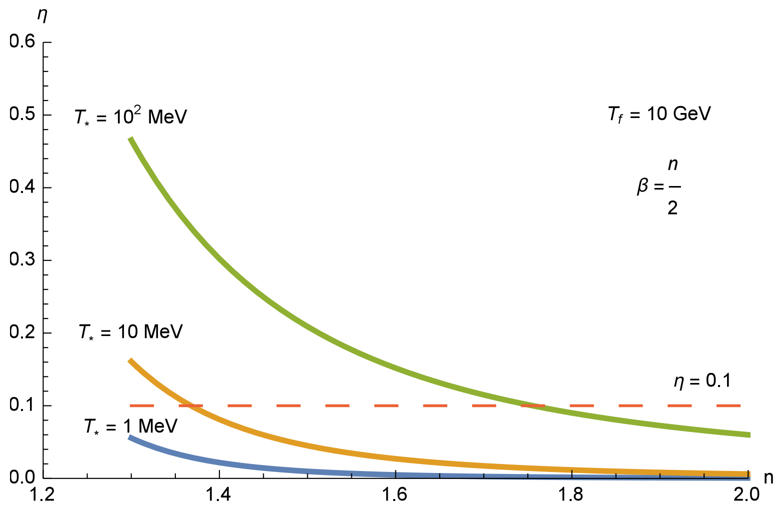


**Figure 1:** $\eta$ vs $n$ for $\beta = n/2$ and transition temperatures $T = \{1, 10, 10^2\}$MeV. $T_f = 10$GeV is the freeze-out temperature, while $\eta = 0.1$ is the lower bound on $\eta$ (see (3.2)).

## 4. Conclusions

In this paper we have studied the evolution of thermal relic particles in $f(R)$ cosmology, assuming $f(R) = R + \alpha R^n$ ($n = 2$ corresponds to Starobinsky's model, while the so called marginally deformed model ($n \neq 1$) produces sizable primordial tensor modes provided the exponent $n$ falls down in the range $n \in [1,2]$). As we have shown, if the cosmic evolution of the early Universe is described by modified field equations, as provided indeed by $f(R)$ gravity, then the expansion rate gets modified by a factor $A(T)$ ($H(T) = A(T)H_{GR}(T)$, see Eq. (2.11)). This quantity essentially weights how much the expansion rate of the Universe in $f(R)$ cosmology deviates from the expansion rate derived in the standard cosmology, and affects, in turn, the production of relic particles (thermal relics decouple with larger relic abundances). As a consequence, the latter is obtained for larger annihilation cross section, and therefore also the indirect detection rates get enhanced. This effect may have its imprint on supersymmetric candidates for DM.

For a power law scale factor, solutions of the modified field equations, and parameterizing the enhancement factor as $A(T) = \eta \left(\frac{T}{T_f}\right)^{\nu}$, we find that the $f(R)$ model is consistent with PAMELA constraints, and the abundance of relic DM $\Omega_\chi h^2 \simeq 0.11$, provided that $\eta \sim \mathscr{O}(0.1-1)$ (and $-1 \leqslant \nu \lesssim -0.46$). According to (3.2), the corresponding WIMPs masses are $m_\chi \gtrsim 10^2\,$GeV.

## References


[1] S. Capozziello and M. De Laurentis Phys. Rept. **509**, 167 (2011).

[2] A. A. Starobinsky, Phys. Lett. B **91**, 99 (1980).

[3] N.D. Birrell and P.C.W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge, UK (1982).

[4] I.L. Buchbinder, S.D. Odintsov, I.L. Shapiro, *Effective Action in Quantum Gravity*, IOP, Bristol, UK (1992).

[5] N.H. Barth and S. Christensen, Phys. Rev. D **28**, 8 (1983).

[6] A.G. Reiss *et al.*, Astron. J. **116**, 1009 (1998). S. Perlmutter *et al.*, Nature (London) **391**, 51 (1998).

[7] S. Nojiri and S.D. Odintsov, Phys. Rep. **505**, 59 (2011). A. Silvestri and M. Trodden, Rep. Prog. Phys. **72**, 096901 (2009). J.A. Frieman, M.S. Turner, and D. Huterer, Annu. Rev. Astron. Astrophys. **46**, 385 (2008). S. Capozziello and G. Lambiase, e-Print: arXiv:1304.5640 [astro-ph.CO]. M. Sami, Lect.Notes Phys. **720**, 219 (2007). E.J. Copeland, M. Sami, and Sh. Tsujikawa, Int. J. Mod. Phys. **D** 15, 1753 (2006).

[8] L. Amendola and S. Tsujikawa, *Dark Energy: Theory and Observations*, Cambridge University Press, 2010.

[9] S. Nojiri, S.D. Odintsov, Phys. Rev. D **77**, 026007 (2008). L. Pogosian and A. Silvestri, Phys. Rev. D **77**, 023503 (2008). I. Sawicki and W. Hu, Phys. Rev. D **75**, 127502 (2007). B. Li and J.D. Barrow, Phys. Rev. D **75**, 084010 (2007). S. Capozziello and G. Lambiase, Gen. Relat. Grav. **32**, 295 (2000); Gen. Relat. Grav. **31**, 1005 (1999); Phys. Lett. B **750**, 344 (2015). S. Capozziello, G. Lambiase, and H.J. Schmidt, Annalen Phys. **9**, 39 (2000). SI. Brevik, E. Elizalde, S. Nojiri, and S.D. Odintsov, Phys. Rev. D **84**, 103508 (2011). S. Nojiri and S.D. Odintsov, Prog. Theor. Phys. Suppl. **190**, 155 (2011). T.P. Sotiriou and V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010). A. De Felice and S. Tsujikawa, Living Rev. Rel. **13**, 3 (2010). G. Lambiase, Phys. Rev. D **90**, 064050 (2014). G. Lambiase, S. Mohanty, and A.R. Prasanna, Int. J. Mod. Phys. D **22**, 1330030 (2013). G. Lambiase and G. Scarpetta, Phys. Rev. D **74**, 087504 (2006). B. Jain, V. Vikram, and J. Sakstein, Astrophys. J. **779**, 39 (2013).. L. Lombriser, A. Slosar, U. Seljak, and W. Hu, Phys. Rev. D **85**, 124038 (2012). S. Derraro, F. Schmidt, and W. Hu,

Phys. Rev. D **83**, 063503 (2011). L. Lombriser, F. Schmidt, T. Baldauf, R. Mandelbaum, U. Seljak, and R.E. Smith, Phys. Rev. D **85**, 102001 (2012). F. Schmidt, A. Vikhlinin, and W. Hu, Phys. Rev. D **80**, 083505 (2009). H. Motohashi, A.A. Starobinsky, and J. Yakoyama, Phys. Rev. Lett. **110**, 121302 (2013). G. Lambiase et al., JCAP 1507 (2015) no.07, 003. S. Capozziello et al., Phys. Rev. D **91**, 044012 (2015). G. Lambiase, M. Sakellariadou, An. Stabile, JCAP 1312 (2013) 020. K. Bamba et al., JCAP **01**, 008 (2014). M. Novello and S.E.P. Bergliaffa, Phys. Rep. **463**, 127 (2008).

[10] O. Adriani *et al.*, Nature (London) **458**, 607 (2009).

[11] R. Catena et al. Phys. Rev. D **81**, 123522 (2010).

[12] A. Codello, J. Joergensen, F. Sannino and O. Svendsen, JHEP **02**, 050 (2015).

[13] J. Ellis, D. V. Nanopoulos and K. A. Olive, Phys. Rev. Lett. **111** (2013) 111301 [Erratum-ibid. **111** (2013) 12, 129902].

[14] S. Ferrara, A. Kehagias, and A. Riotto, Fortsch. Phys. **62**, 573 (2014); Fortsch. Phys. **63**, 2 (2015).

[15] G.K. Chakravarty et al., e-Print: arXiv:1511.03121 [hep-ph].

[16] S. Basilakos, J. A. Sales Lima, J. Sola Int. J. Mod. Phys. D **23**, 1442011 (2014).

[17] J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, Phys.Lett. B **732**, 380 (2014).

[18] P.A.R. Ade *et al.* (Planck Collaboration), A&A **57**, A22 (2014).

[19] V.F. Mukhanov and G. Chibisov, JEPT Lett. **33**, 532 (1981).

[20] A.A. Starobinsky, Sov. Astron. Lett. **9**, 302 (1983).

[21] W. Zimdahl, Phys. Rev. D **53**, 5483 (1996). R. Opher and A. Pelisan, Phys. Rev. D **74**, 023505 (2006). G. Lambiase and S. Mohanty, Phys. Rev. D **84**, 023509 (2011). J.R. Wilson, G.J. Mathews, and G.M. Fuller, Phys. Rev. D **75**, 043521 (2007). S. Weinberg, Astrophysical J. **168**, 175 (1971). R. Maartens, arXiv:astro-ph/9609119.

[22] D. Hooper, P. Blasi, P.D. Serpico, JCAP **0901**, 025 (2010). P. Blasi, Phys. Rev. Lett. **103**, 051104 (2009).

[23] N. Arkani-Hamed et al., Phys. Rev. D **79**, 015014 (2009). B.E. Robertson and A.R. Zentner, Phys. Rev. D **79**, 083525 (2009). M. Cirelli et al., Nucl. Phys. B **813**, 1 (2009). I. Cholis et al., Phys. Rev. D **80**, 123511 (2009). F. Donato et al., Phys. Rev. Lett. **102**, 071301 (2009). K. Ishiwata, S. Matsumoto, and T. Moroi, Phys. Lett. B **675**, 446 (2009).

[24] D.N. Spergel, *et al.*, (WMAP Collaboration), Astrophys. J. Suppl. **170**, 377 (2007).

[25] G. Gelmini and P. Gondolo, arXiv:1009.3690 [astro-ph.CO]. G. D'Amico, M. Kamionkowski, and K. Sigurdson, arXiv:0907.1912 [astro-ph.CO]. R. Catena et al., JHEP **10**, 003 (2008). D.I. Santiago et a., Phys. Rev. D **58**, 124005 (1998). P. Salati, Phys. Lett. B **571**, 121 (2003). G. Lambiase, Phys. Rev. D **83**, 107501 (2011); J. Cosmol. Astropart. Phys. 10 (2012) 028. S. Capozziello et al., Phys. Lett. B **715**, 1 (2012). A. Iorio, G. Lambiase, Eur. Phys. J. C **75**, 115 (2015). S. Capozziello et al. Phys. Rev. D 92, 084006 (2015).

[26] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **180**, 330 (2009).