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Naturalness, supersymmetry and dark matter

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We present the Bayesian naturalness prior to quantify electroweak fine tuning in the Next-to-Minimal Supersymmetric Standard Model (NMSSM). The naturalness prior arises automatically as an Occam razor in Bayesian model comparison quantifying the plausibility of electroweak symmetry breaking within the model. The prior incorporates the most widely used fine tuning measures as special cases. In particular, it captures features the Barbieri-Ellis-Giudice, and the Electroweak Fine Tuning measures.

We present the amount of Bayesian fine tuning over parameter space slices of the Constrained MSSM, the Constrained NMSSM, and an 11 parameter NMSSM scenario. According to the naturalness prior the constrained models are less tuned than other fine-tuning measures indicate.

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1. Introduction

The discovery of the Higgs boson fueled considerable interest in naturalness due to the apparent fine tuning in the Higgs sector of the Standard Model (SM) [1]. The masses of the standard matter and force carrier particles are protected against quantum fluctuations by chiral and gauge invariance in the SM. These symmetries thus separate the electroweak scale from the high scale of new physics, such as gravity. This separation of scales is considered to be natural, since such division of phenomena structures physics itself from cosmology, through astrophysics, condensed matter, atomic, and nuclear to elementary particle physics.

But the Higgs mass is unprotected against quantum fluctuations within the SM. The latter must be an effective description of nature, since it cannot account for various observations such as dark matter, the matter-antimatter asymmetry, gravity and more. When formulated as an effective field theory, with a cut-off scale Λ, due to the lack of a protective mechanism, the Higgs mass receives quantum corrections that sensitively depend on Λ. The SM violates the separation of scales: the electroweak size Higgs mass is directly connected to, in principle, arbitrarily high scales. This situation is considered to be unnatural: phenomena at disparate energy scales are fundamentally connected.

A simple way to express the unnaturalness of the SM Higgs sector is quantifying the fine tuning required to obtain a 125 GeV Higgs mass. The physical Higgs mass squared is the sum of a bare mass term and a correction

$$m_H^2 = m_0^2 + \delta m_H^2, \tag{1.1}$$

with $\delta m_H^2 \sim \Lambda^2$. The Large Hadron Collider is pushing the scale of new physics Λ beyond TeV, which requires a finely tuned cancellation between the bare mass and the quantum corrections. Simple algebra shows that the bare mass must be within a percent of TeV size quantum corrections to yield 125 GeV physical Higgs mass.

The above is an oversimplified measure of tuning. After all, the bare mass is non-physical, and it is impossible to argue about its value in a model independent way. A more sophisticated fine tuning measure was introduced by Barbieri, Ellis, and Giudice (BEG) [2, 3]. The prerequisite of this measure is the existence of an electroweak scale observable which is predicted by the theory. In the MSSM this quantity is chosen to be the mass of the Z boson, due to the fact that the electroweak symmetry breaking condition directly links it to the Lagrangian parameters of the theory:

$$\frac{m_Z^2}{2} = \frac{(m_{H_d}^2 + \delta m_{H_d}^2) - (m_{H_u}^2 + \delta m_{H_u}^2)\tan^2\beta}{\tan^2\beta - 1} - \mu^2. \tag{1.2}$$

The BEG measure quantifies the sensitivity of an electroweak scale observable to the change of a theory parameter. In the MSSM this measure is typically written as

$$\Delta_{BEG}(m_Z^2(\mu^2)) = \left|\frac{\partial m_Z^2}{\partial \mu^2}\right|, \tag{1.3}$$

with the $m_Z^2(\mu^2)$ function defined by the electroweak symmetry breaking condition Eq.(1.2) at tree level. This fine tuning measure accounts for correlations between m_Z and μ. In qualitative terms: if the m_Z prediction is sensitive to small changes in μ the theory is considered to be fine tuned. While the BEG fine tuning measure can be used for MSSM variants, such as constrained versions of the MSSM, when one goes beyond the MSSM it is unclear how to generalize it. This peoblem raises the question: Is there a fine tuning measure that can be applied to any extensions of the SM? Surprisingly, the answer may be simpler than expected.

Let us assume the existence of a SM extension that adds only a single parameter μ to those of the SM, and that this model predicts the mass of the Z boson in terms of μ^2: $m_Z = m_Z(\mu^2)$. The Bayesian evidence for this theory is

$$\mathscr{E} = \mathscr{V}_{\mu^2}^{-1} \int_{\mu^2_{\rm min}}^{\mu^2_{\rm max}} \mathscr{L}(m_Z^2(\mu^2))\, d\mu^2, \tag{1.4}$$

treating, for simplicity, all the SM parameters as nuisances. The evidence $\mathscr{E}$ reflects the plausibility of this single parameter theory in light of the measured Z mass. The likelihood function $\mathscr{L}$ measures how well the model can predict m_Z over the parameter space of the model. We assumed a constant prior for the μ parameter, which yielded the constant normalization factor

$$\mathscr{V}_{\mu^2} = \int_{\mu^2_{\rm min}}^{\mu^2_{\rm max}} d\mu^2. \tag{1.5}$$

It is reasonable to assume that the $m_Z^2(\mu^2)$ function is differentiable and invertible. Then, via a variable change, one can express the evidence integral as

$$\mathscr{E} = \mathscr{V}_{\mu}^{-1} \int_{m_Z^2(\mu^2_{\rm min})}^{m_Z^2(\mu^2_{\rm max})} \mathscr{L}(m_Z^2)\, \Delta_{BEG}^{-1}(m_Z^2)\, dm_Z^2. \tag{1.6}$$

The variable change reveals the connection of the evidence integral to naturalness since it induced the derivative $\Delta_{BEG}(m_Z^2(\mu^2)) = dm_Z^2/d\mu^2$ which is the single parameter version of the above defined BEG measure. This measure here plays the role of a Bayesian prior of the theoretically predicted m_Z values.

In the Bayesian formalism the meaning of the prior $\Delta_{BEG}^{-1}(m_Z^2)$ is the probability distribution of the predicted m_Z values within the theory. If the average value of the $\Delta_{BEG}(m_Z^2(\mu^2))$ function is low over the parameter space then the evidence integral is enhanced. This situation corresponds to a case when the theory has low fine tuning. Thus the value of the Bayesian evidence is clearly correlated with the naturalness of the theory. Casting the evidence into an integral over the observable reveals its meaning as the plausibility of the theory in terms of observation and naturalness. Conversely, naturalness in the Bayesian framework is understood as the plausibility that the theory predicts the correct value of a given observable.

The Bayesian evidence not only calculable for any parametric model but also reveals some implicit properties of the BEG fine tuning measure. Perhaps most importantly, Bayesian inference justifies the derivative form of Δ_{BEG}. By definition the evidence is an integral over the parameters of the model. When it is recast as an integral over the predicted observables Δ_{BEG}^{-1} automatically emerges as the Jacobian of the variable transformation.

Bayesian hypothesis testing sheds light on the normalization, or scale, of Δ_{BEG}. In model comparison the ratio of evidences is known as the Bayes factor, which quantifies the plausibility of a model over another. This ratio is measured on Jeffreys' scale. In this context it is clear that naturalness is the ability of a given model to predict electroweak scale observables, and it has to be compared to the naturalness of another model. The traditional BEG measure, at best, could only be interpreted as probability density, which has to be integrated to become an objective measure of plausibility.

The Bayesian framework also shows us that there is some amount of subjectivity involved when one selects which fundamental parameter of the theory and which (electroweak) observable is used to define Δ_{BEG}. It seems that a different kind of fine tuning is measured by the different possible choices. It is enlightening to see, for example, that the exact form of Δ_{BEG} depends not only on the choice of parameter (such as μ or μ^2 or $B\mu$), but also on the initial prior for the given parameter. If, for example, the parameter value spans several orders of magnitude in the theory (before considering any observational constrains), then it is customary to choose a logarithmic prior for it. In this case, from the Bayesian point of view, the theoretical parameter is $\log\mu$ and the induced Jacobian should be $d(\log\mu)/d(\log m_Z)$. According to this, whether the following forms of the fine tuning measure are 'correct'

$$\Delta_{BEG}(m_Z) = \frac{dm_Z}{d\mu} \quad or \quad \frac{dm_Z^2}{d\mu^2} \quad or \quad \frac{d\log m_Z}{d\log\mu} \quad or \quad \frac{d\log m_Z^2}{d\log\mu^2}, \tag{1.7}$$

depends on our definition of the theoretical parameter, its prior, and the experimental observable that we want to use to quantify naturalness. Finally, when $n > 1$ theoretical parameters $\{p_1,...,p_n\}$ are 'fixed' in terms of n observables $\{o_1,...,o_n\}$ the naturalness prior takes the form of a $n\times n$ determinant

$$\Delta_J(o_1,...,o_n) = \begin{vmatrix} \frac{\partial o_1}{\partial p_1} & \cdots & \frac{\partial o_1}{\partial p_n} \\ & \ddots & \\ \frac{\partial o_n}{\partial p_1} & \cdots & \frac{\partial o_n}{\partial p_n} \end{vmatrix}. \tag{1.8}$$

We can measure the fine tuning within a model with respect of several observables and parameters simultaneously. But when we do that the fine tuning is measured by the above determinant. Most interestingly, within this determinant not all terms are positive! In other words, it is not the trace of the matrix rather the full determinant that quantifies fine tuning.

2. Naturalness prior for the NMSSM

In this section we derive the naturalness prior for the constrained and an 11 dimensional version of the NMSSM (CNMSSM and NMSSM-11). As indicated above, Δ_J depends on the choice of parameters, which in turn is the function of the definition of the model. In this work we define the CNMSSM at the GUT scale to have a universal gaugino mass ($M_{1/2}$), a universal soft tri-linear coupling (A_0), with all MSSM-like soft scalar masses being equal (M_0). The new soft singlet mass ($m_{S_0} = m_S(M_{GUT})$), however, is left unconstrained at the GUT scale. Thus the model is parametrized by

$$\{p_1,...,p_6\}_{CNMSSM} = \{M_0, M_{1/2}, A_0, \lambda_0, \kappa_0, m_S\}, \tag{2.1}$$

in contrast with the CMSSM

$$\{p_1,...,p_5\}_{CMSSM} = \{M_0, M_{1/2}, A_0, \mu_0, B_0\}. \tag{2.2}$$

Spectrum generators, such as `NMSPEC` and `Next-to-Minimal SOFTSUSY` [4], trade the GUT scale parameters λ_0, κ_0 and m_S^2 for weak scale λ, m_Z and $\tan\beta$ giving the user the mixed scale input parameters of $\{M_0, M_{1/2}, A_0, \tan\beta, \lambda, m_Z\}$. This transformation gives rise to a Jacobian

$$d\lambda_0 d\kappa_0 dm_{S_0}^2 = J_{\mathscr{T}_0} d\lambda\, dm_Z^2 d\tan\beta, \tag{2.3}$$

which may be written as

$$J_{\mathscr{T}_0} = J_{\mathscr{T}^{\lambda}_{\kappa m_S}} J_{RG} \quad = \left| \begin{matrix} \frac{\partial\kappa}{\partial m_Z^2} & \frac{\partial m_S^2}{\partial m_Z^2} \\ \frac{\partial\kappa}{\partial\tan\beta} & \frac{\partial m_S^2}{\partial\tan\beta} \end{matrix} \right|_{\lambda} \left| \begin{matrix} \frac{\partial\lambda_0}{\partial\lambda} & \frac{\partial\kappa_0}{\partial\lambda} \\ \frac{\partial\lambda_0}{\partial\kappa} & \frac{\partial\kappa_0}{\partial\kappa} \end{matrix} \right| \left| \frac{\partial m_{S_0}^2}{\partial m_S^2} \right|. \tag{2.4}$$

The Jacobian $J_{\mathscr{T}^{\lambda}_{\kappa m_S}}$ can be rewritten in terms of simpler coefficients embedded in the determinant of a 3×3 matrix. The coefficients appearing in this determinant are given in the appendix of Ref. [5]. The second Jacobian J_{RG} transforms the input parameters from the GUT scale to the electroweak scale, and factorizes as shown due to the supersymmetric non-renormalization theorem. The subscript λ indicates that this parameter is kept constant in the derivatives.

As explained, we can choose to work with the logarithms of parameters (as is natural if we choose logarithmic priors) so that we obtain a new factor in the denominator, which is the inverse of the Jacobian with logarithms inserted inside the derivatives. This gives us

$$\Delta_J^{\text{CNMSSM}} = \left| \frac{\partial \ln(m_Z^2, \tan\beta, \lambda)}{\partial \ln(\kappa_0, m_{S_0}^2, \lambda_0)} \right| \quad = \frac{\kappa_0 m_{S_0}^2 \lambda_0}{m_Z^2 \tan\beta\,\lambda} J_{\mathscr{T}_0}^{-1} \tag{2.5}$$

It is well known that the top quark Yukawa coupling can play a significant role in fine tuning so we also considered this by extending the transformation to include the top quark mass and (unified) Yukawa coupling: $\{\kappa_0, m_{S_0}^2, \lambda_0, y_0\} \to \{m_Z^2, \tan\beta, \lambda, m_t\}$. Nonetheless as was already observed in the MSSM case [6, 7], we found that all the derivatives, other than $\frac{\partial m_t}{\partial y_t}$, that involve m_t and y_t cancel, so this only changes the Jacobian by a single multiplicative factor of $\frac{\partial m_t}{\partial y_t}$. Finally when logarithmic priors are chosen this factor will disappear entirely because $\frac{\partial \ln m_t}{\partial \ln y_t} = 1$, and the Yukawa renormalization group evolution (RGE) factor $\frac{\partial \ln y_t}{\partial \ln y_0}$ is the same order one constant (at 1-loop) as in the CMSSM case so we neglect it.

Therefore we write our NMSSM Jacobian based tuning measure as

$$\Delta_J^{\text{CNMSSM}} = \left| \frac{\partial \ln(m_Z^2, \tan\beta, \lambda, m_t^2)}{\partial \ln(\kappa_0, m_{S_0}^2, \lambda_0, y_0^2)} \right|, \tag{2.6}$$

with the additional transformation between m_t and y_0 included to emphasise that we have also considered these, since the cancellation will prove to be rather important (in both the MSSM and NMSSM) when we compare against the BEG tuning measure in the focus point (FP) region. There we will show that due to this cancellation we do not see a large tuning penalty in the much discussed FP region, which appears in the BEG measure when one includes y_t as a parameter [8, 9, 10, 11].

The expression given here is formally the Jacobian which should be used in the Bayesian analysis of any NMSSM model when $(\lambda_0, \kappa_0, m_{S_0}^2, y_0^2)$ are traded for $(m_Z^2, \tan\beta, \lambda, m_t^2)$. At the same time Δ_J^{CNMSSM} can be interpreted as a measure of the naturalness of the NMSSM, which may be applied to the CNMSSM, the general NMSSM and λ-SUSY scenarios.

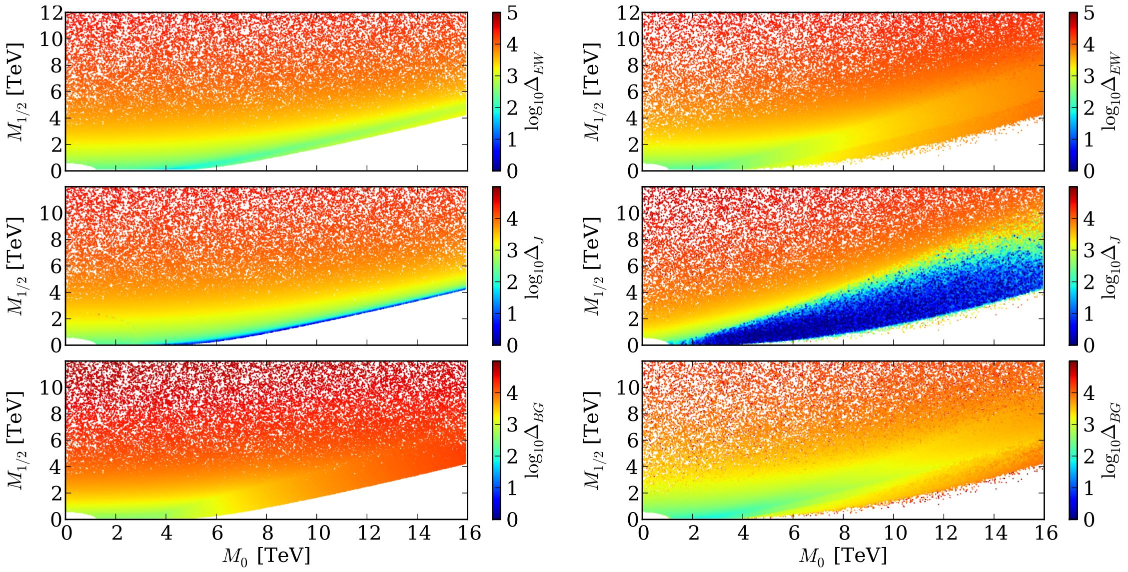

Figure 1: The left frame shows maps of fine tuning measures Δ_{BEG} (top), Δ_J (middle), Δ_{EW} (bottom) in the M_0 vs. $M_{1/2}$ plane for $A_0 = -2.5$ TeV, $\tan\beta = 10$ and sgn(μ)=1 in the CMSSM. The color code quantifies the value of Δ_{EW} and Δ_J. Since Δ_{BEG} is dominated by the μ derivative it is low in the small M_0 and $M_{1/2}$ region. Although Δ_{BEG}, by definition, is formally part of Δ_J the numerical behavior of the latter is similar to that of Δ_{EW}. No experimental constraints applied except that the lightest supersymmetric particle is electrically neutral and the EWSB condition is satisfied. Right frame: Same as the left frame except for the constrained NMSSM. $A_{0,\kappa,\lambda} = -2.5$ TeV and $\tan\beta = 10$ are assumed. λ is sampled from the range [0,0.8].

3. Numerical results

For our numerical calculations we use `SOFTSUSY 3.3.5` [13], `NMSPEC` [14] in `NMSSM-Tools 4.1.2`, `Next-to-Minimal SOFTSUSY` [4], and `MultiNest 3.3` [15, 16]. The spectrum generators provide Δ_{BEG} with renormalization group flow improvement. For Δ_{BEG} in the CMSSM we include individual sensitivities, $\Delta_{BEG}(p_i)$, for the parameters $M_0, M_{1/2}, A_0, \mu, B, y_t$. For the CNMSSM we use the set $M_0, M_{1/2}, A_0, \lambda, \kappa, y_t$.

First we examine how the tuning measures vary with M_0 and $M_{1/2}$, without requiring a 125 GeV Higgs. We fix $\tan\beta = 10$, where the extra NMSSM F-term contribution is small, but there is interesting focus point (FP) behavior [8, 9, 10, 11]. Previous studies [17] show that large and negative A_0 is favoured, so to simplify the analysis we choose[1] $A_0 = -2.5$ TeV.

The results for the CMSSM are shown in FIG. 1. The value of Δ_{EW} [18] is governed by the $m^2_{H_u}$ and μ^2 contributions since $m_Z^2/2 \approx -\overline{m}^2_{H_u} - \mu^2$, where $\overline{m}^2_{H_u}$ includes the radiative corrections. In general Δ_{EW} is dominated by μ^2, while the crossover to the $m^2_{H_u}$ dominance occurs in the vicinity of the EWSB boundary. For this measure there is low fine tuning even at large M_0. This may seem counterintuitive, but for $\tan\beta = 10$ at large M_0 we are close to the FP. In this region the dependence on M_0 which appears from RG evolution of m_{H_u} vanishes. For example in the CMSSM

[1] We checked that with alternative A_0 choices the behaviour is similar. The main difference is with the Higgs masses where a large and negative A_0 was chosen to increase the lightest Higgs mass.

semi-analytical solution to the renormalisation group equations (RGEs),

$$m_{H_u}^2 = c_1 M_0^2 + c_2 M_{1/2}^2 + c_3 A_0^2 + c_4 M_{1/2} A_0, \tag{3.1}$$

the coefficients c_i are functions of Yukawa and gauge couplings, and $\tan\beta$ and c_1 can be close to zero. Such regions then appear to have low fine tuning even with large M_0 since the small size of c_1 means there is no need to cancel the large M_0 in Eq. (1.2) to obtain the correct m_Z^2.

In Δ_{BEG} the sensitivity to the top quark Yukawa coupling is also included. Since the RG coefficients depend on this Yukawa coupling, the large stop corrections from the RGEs feeding into $m_{H_u}^2$ lead to a large $\Delta_{BEG}(y_t)$ even in the focus point region. Δ_{EW} is not sensitive to this effect since it does not take into account such RG effects. Interestingly $\Delta_J^{\rm CMSSM}$ exhibits similar behavior to Δ_{EW} despite containing derivatives from Δ_{BEG}. This is because $\Delta_J^{\rm CMSSM}$ does not contain the derivative of m_Z with respect y_t [5]. As a result Δ_J in the MSSM can remain small in the FP.

Fine tuning measures for the CNMSSM are shown in the right frame of FIG. 1. Here $\Delta_J^{\rm CNMSSM}$ is defined by Eq. (2.6) and Δ_{BEG} is defined in Ref. ([5]), while Δ_{EW} is defined the same as for the MSSM. The parameter μ dominates electroweak tuning, Δ_{EW}, throughout the M_0 vs. $M_{1/2}$ plane. Since μ values and related derivatives are similar in the CMSSM and CNMSSM the fine tuning measures are qualitatively similar for the two models. As in the CMSSM the Jacobian derived tuning Δ_J increases with $M_{1/2}$, as anticipated since for large $M_{1/2}$ large cancellation is required to keep m_Z light. At large M_0 values Δ_J can still be low seeming to favour the FP region, which is a result of the same cancellation that happened in the MSSM Jacobian.

Interestingly the region where the tuning can be very low extends further in the NMSSM. Note this is not a result of raising the Higgs mass with λ since we impose no Higgs constraint yet and have large $\tan\beta$. However λ is varied across the plane and affects the EWSB condition and the renormalization group evolution. However since the number of parameters are different in the CNMSSM and CMSSM, to determine whether the CNMSSM is preferred over the CMSSM, we have to compare Bayesian evidences.

4. Conclusions

In this work we presented Bayesian naturalness priors to quantify fine tuning in the (N)MSSM. These priors emerge automatically during model comparison within the Bayesian evidence. We compared the Bayesian measure of fine tuning (Δ_J) to the Barbieri-Giudice (Δ_{BEG}) and ratio (Δ_{EW}) measures. Even though the Bayesian prior is closely related to the Barbieri-Giudice measure, the numerical value of the Bayesian measure reproduces important features of Δ_{EW}. Both Δ_{EW} and Δ_J are low in focus point scenarios.

Our numerical analysis is limited to fixed $(A_0, \tan\beta)$ slices of the constrained parameter space. For these slices we show that, according to the naturalness prior, the constrained version of the NMSSM is less tuned than the CMSSM. This statement, however, has to be confirmed by comparing Bayesian evidences of the models. The complete parameter space scan and the full Bayesian analysis for the NMSSM is deferred to a later work.

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