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The Breaking $\mu \leftrightarrow \tau$ Symmetry through the $\mathbf{Q}_6$ Flavour Group

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In a supersymmetric scenario, we study masses and mixings for leptons through the $\mathbf{Q}_6$ flavour symmetry. In the simplest case, the $\mathbf{M}_\nu$ effective neutrino mass matrix, that comes from the type I see-saw mechanism, breaks the $\mu \leftrightarrow \tau$ interchange symmetry. As consequence, the reactor and atmospheric angles deviate from 0° and 45°, respectively. At first glance, the model might accommodate very well the reactor and atmospheric angles in good agreement with the experimental data.

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†A footnote may follow.

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Introduction

A new age in particle physics was opened with the discovery of neutrino oscillations and the precision measurements of corresponding parameters in this phenomena. Two of the main parameters that characterize the ordinary neutrino oscillations are the difference of the squared neutrino masses, as well as the flavour mixing angles. For the latter ones, the numerical values obtained as result of a global fit of current experimental data on neutrino oscillations [1], at Best Fit Point (BFP) $\pm 1\sigma$, are the following

$$\sin^2\theta_{12}/10^{-1} = 3.23 \pm 0.16, \quad \sin^2\theta_{23}/10^{-1} = \begin{cases} 5.67^{+0.32}_{-1.24} \\ 5.73^{+0.25}_{-0.39} \end{cases}, \quad \sin^2\theta_{13}/10^{-2} = \begin{cases} 2.26 \pm 0.12 \\ 2.29 \pm 0.12 \end{cases}, \tag{1}$$

The values given in the upper (lower) row are for a normal (inverted) hierarchy of the neutrino mass spectrum. This experimental evidence was enough to show that neutrinos have a tiny mass, whereby it was very easy to conclude that there is physics beyond the Standard Model (SM).

In the model building context, the $\mu \leftrightarrow \tau$ flavour symmetry has been widely used to propose possible extensions of the SM. In these extensions the $\mu \leftrightarrow \tau$ flavour symmetry can be defined in two different ways: *i*) the $\mu \leftrightarrow \tau$ *permutation symmetry* [2] where the neutrino mass term is unchanged under the transformations $\nu_e \rightarrow \nu_e$, $\nu_\mu \rightarrow \nu_\tau$ and $\nu_\tau \rightarrow \nu_\mu$. *ii*) the $\mu \leftrightarrow \tau$ *reflection symmetry* [3] where the neutrino mass term is unchanged under the transformations $\nu_e \rightarrow \nu_e^c$, $\nu_\mu \rightarrow \nu_\tau^c$ and $\nu_\tau \rightarrow \nu_\mu^c$, where c denotes the charge conjugation. But here we will only consider the first definition, hence in the following when we mention the $\mu \leftrightarrow \tau$ symmetry actually we mean the $\mu \leftrightarrow \tau$ permutation symmetry.

Historically, theoretical physicists have proposed the $\mu \leftrightarrow \tau$ symmetry in order to reproduce the experimental data on lepton mixing angles. Namely, the $\mu \leftrightarrow \tau$ symmetry is obtained if the reactor and atmospheric angles have the following values $\theta_{13} = 0^\circ$ and $\theta_{23} = 45^\circ$, respectively. However, this symmetry is ruled out by the current experimental data, but these same data suggest some possibles breakings of the $\mu \leftrightarrow \tau$ symmetry which have been explored recently [4–6]. Besides, many discrete groups have been proposed in order to understand the underlying flavour symmetry behind the lepton mixing angles [7].

In this line of thought, we build a $\mathbf{Q}_6$ flavoured supersymmetric model to study masses and mixing for quarks and leptons where the $\mu \leftrightarrow \tau$ symmetry is broken in the latter sector. As in early works on $\mathbf{Q}_6$ [8], it was necessary to extend the flavour label to the Higgs sector, this means that three families of doublets H_i^d and H_i^u are needed for the mixing. Our model is completely different from those already existing in the literature since the matter content assignment is very particular.

The Model

The matter content and their respective assignment under the $\mathbf{Q_6}$ symmetry is displayed on Table 1. The present model is very peculiar in the sense that for the quarks and Higgs superfields, Q_I and $H_I^{u,d}$ stand for a doublet under the flavour symmetry $\mathbf{Q}_6$ where $I = 1,2$, this means explicitly for the former, $(Q_1, Q_2)^T$. The rest of fields should be understood in the same way if they have the label I, otherwise, the fields are singlets under the flavour symmetry. On the other hand, for leptons,

$\mathbf{Q_6}$	$\mathbf{1}_{+,0}$	$\mathbf{1}_{+,2}$	$\mathbf{1}_{-,1}$	$\mathbf{1}_{-,3}$	$\mathbf{2}_2$	$\mathbf{2}_1$
Matter	H^d_3	H^u_3 , Y_B	L_1 , N^c_1 , Q_3 , u^c_3	ℓ^c_1, d^c_3	L_J, ℓ^c_J , N^c_J , Q_I , d^c_I , u^c_I , H^d_I	H^u_I

Table 1: Matter Content.

L_J stands for a flavour doublet where $J = 2,3$; and the first family belongs to any of the singlets. Then, the superpotential, which is allowed by the gauge and flavour symmetry, is given by

$$\begin{aligned}\mathbf{W} &= y^u_1 (Q_1 u^c_2 - Q_2 u^c_1) H^u_3 + y^u_2 (Q_1 H^u_2 + Q_2 H^u_1) u^c_3 + y^u_3 Q_3 (u^c_1 H^u_2 + u^c_2 H^u_1) + y^u_4 Q_3 u^c_3 H^u_3 \\ &+ y^d_1 \left[Q_1 \left(-d^c_1 H^d_1 + d^c_2 H^d_2\right) + Q_2 \left(d^c_1 H^d_2 + d^c_2 H^d_1\right)\right] + y^d_2 (Q_1 d^c_1 + Q_2 d^c_2) H^d_3 + y^d_3 Q_3 d^c_3 H^d_3 \\ &+ y^\ell_1 L_1 e^c_1 H^d_3 + y^\ell_2 \left[L_2 \left(-e^c_2 H^d_1 + e^c_3 H^d_2\right) + L_3 \left(e^c_2 H^d_2 + e^c_3 H^d_1\right)\right] + y^\ell_3 (L_2 e^c_2 + L_3 e^c_3) H^d_3 + y^D_1 L_1 N^c_1 H^u_3 \\ &+ y^D_2 L_1 (N^c_2 H^u_2 + N^c_3 H^u_1) + y^D_3 (L_2 H^u_2 + L_3 H^u_1) N^c_1 + y^D_4 (L_2 N^c_3 - L_3 N^c_2) H^u_3 + y^m Y_B N^c_1 N^c_1 \\ &+ M_{R_2} (N^c_2 N^c_2 + N^c_3 N^c_3) \end{aligned} \tag{2}$$

Comments are in order: one flavon Y_{BK} (Babu-Kubo) has been included in order to build a flavour invariant Majorana mass matrix for the right-handed neutrinos (RHN's) [8]; a subtle problem appears with our peculiar assignment for the matter content; there is no μ term for the Higgs sector since these are not flavour invariant, but these kind of terms should be present since they are crucial to get the electroweak symmetry breaking. In order to fix this, extra gauge singlets will be included to construct a gauge and flavour invariant μ term. Due to our interest in studying masses and mixings in this model, for the moment some alignments in the vacuum expectation values (vev's) of the scalars will be assumed. Then, we will leave aside the full scalar superpotential that eventually has to be analysed to have a complete model.

Mases and mixings

Going back to the expression in Eq.(2) we obtain the Dirac fermion mass matrices which have the following form

$$\mathbf{M}_u = \begin{pmatrix} 0 & y^u_1 \langle \mathbf{H}^u_3 \rangle & y^u_2 \langle \mathbf{H}^u_2 \rangle \\ -y^u_1 \langle \mathbf{H}^u_3 \rangle & 0 & y^u_2 \langle \mathbf{H}^u_1 \rangle \\ y^u_3 \langle \mathbf{H}^u_2 \rangle & y^u_3 \langle \mathbf{H}^u_1 \rangle & y^u_4 \langle \mathbf{H}^u_3 \rangle \end{pmatrix}, \qquad \mathbf{M}_d = \begin{pmatrix} y^d_2 \langle \mathbf{H}^d_3 \rangle - y^d_1 \langle \mathbf{H}^d_1 \rangle & y^d_1 \langle \mathbf{H}^d_2 \rangle & 0 \\ y^d_1 \langle \mathbf{H}^d_2 \rangle & y^d_2 \langle \mathbf{H}^d_3 \rangle + y^d_1 \langle \mathbf{H}^d_1 \rangle & 0 \\ 0 & 0 & y^d_3 \langle \mathbf{H}^d_3 \rangle \end{pmatrix}, \tag{3}$$

and

$$\mathbf{M}_D = \begin{pmatrix} y^D_1 \langle \mathbf{H}^u_3 \rangle & y^D_2 \langle \mathbf{H}^u_2 \rangle & y^D_2 \langle \mathbf{H}^u_1 \rangle \\ y^D_3 \langle \mathbf{H}^u_2 \rangle & 0 & y^D_4 \langle \mathbf{H}^u_3 \rangle \\ y^f_3 \langle \mathbf{H}^u_1 \rangle & -y^D_4 \langle \mathbf{H}^u_3 \rangle & 0 \end{pmatrix}, \quad \mathbf{M}_\ell = \begin{pmatrix} y^\ell_1 \langle \mathbf{H}^d_3 \rangle & 0 & 0 \\ 0 & y^\ell_3 \langle \mathbf{H}^d_3 \rangle - y^\ell_2 \langle \mathbf{H}^d_1 \rangle & y^\ell_2 \langle \mathbf{H}^d_2 \rangle \\ 0 & y^\ell_2 \langle \mathbf{H}^d_2 \rangle & y^\ell_3 \langle \mathbf{H}^d_3 \rangle + y^\ell_2 \langle \mathbf{H}^d_1 \rangle \end{pmatrix}. \tag{4}$$

In addition, the RHN mass matrix is diagonal in the flavour space, $\mathbf{M}_R = \mathrm{diag.}(M_{R_1}, M_{R_2}, M_{R_2})$; here, $M_{R_1} = y^n \langle Y_{BK} \rangle$.

Now, assuming the degeneracy between $\langle \mathbf{H}^u_1 \rangle = \langle \mathbf{H}^u_2 \rangle$, and $\langle \mathbf{H}^d_2 \rangle = 0$, the resultant matrix, $\mathbf{M}_u$, has implicitly the NNI textures and the $\mathbf{M}_d$ mass matrix is diagonal. At the first glance the CKM mixing matrix will not be accommodated with great accuracy. Of course, we have to make sure that this is the case so that an χ^2 analysis has to be done. On the orden hand, the charged lepton mass matrix is also diagonal, then the PMNS mixing matrix comes only from the neutrino sector. Under these assumptions, we will focus on the lepton sector for the moment. Therefore,

$$\mathbf{M}_D = \begin{pmatrix} a_D & b_D & b_D \\ c_D & 0 & d_D \\ c_D & -d_D & 0 \end{pmatrix}, \qquad \mathbf{M}_\ell = \begin{pmatrix} a_\ell & 0 & 0 \\ 0 & c_\ell - b_\ell & 0 \\ 0 & 0 & c_\ell + b_\ell \end{pmatrix} \equiv \begin{pmatrix} a_\ell & 0 & 0 \\ 0 & B_\ell & 0 \\ 0 & 0 & D_\ell \end{pmatrix} \tag{5}$$

and the effective neutrino mass matrix, that comes from the type I see-saw mechanism, is given by

$$\mathbf{M}_\nu = \mathbf{M}_D \mathbf{M}_R^{-1} \mathbf{M}_D^T = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{22} \end{pmatrix}, \qquad \mathbf{M}_R^{-1} \equiv \mathrm{diag}(x, y, y). \tag{6}$$

Let us exhibit an appealing feature about $\mathbf{M}_\nu$: the block matrix $2-3$ provides a $\pi/4$ angle (to the mixing matrix) that may be identified with the atmospheric one. Moreover, if $|B_\nu| = |C_\nu|$ were true, $\mathbf{M}_\nu$ would have the $\mu - \tau$ symmetry, so that the PMNS mixing matrix would have $\theta_{13} = 0^\circ$ and $\theta_{23} = 45^\circ$ for the reactor and atmospheric angle, respectively. Strictly speaking, in the present model, $\mathbf{M}_\nu$ does not possess the $\mu \leftrightarrow \tau$ symmetry since $m_{12} \neq m_{13}$. This fact, actually, is crucial to get $\theta_{13} \neq 0^\circ$ and $\theta_{23} \neq 45^\circ$ in the PMNS matrix as we will see next.

To diagonalize $\mathbf{M}_\nu$, a perturbative analysis will be done as follows: $\mathbf{M}_\nu$ is written as $\mathbf{M}_\nu = \mathbf{M}_\nu^0 + \mathbf{M}_\nu^\delta$ where

$$\mathbf{M}_\nu^0 = \begin{pmatrix} m_{11} & m_{12} & m_{12} \\ m_{12} & m_{22} & m_{23} \\ m_{12} & m_{23} & m_{22} \end{pmatrix} \qquad \text{and} \qquad \mathbf{M}_\nu^\delta = \begin{pmatrix} 0 & 0 & m_{13} - m_{12} \\ 0 & 0 & 0 \\ m_{13} - m_{12} & 0 & 0 \end{pmatrix} \tag{7}$$

The former mass matrix possesses the $\mu \leftrightarrow \tau$ symmetry. Here, we assume that $\mathbf{M}_\nu^\delta$ contains a perturbation parameter which will be defined later. In general, $\mathbf{M}_\nu$ is diagonalized by $\mathbf{U}_\nu \approx \mathbf{U}_\nu^0 \mathbf{U}_\nu^\delta$ with $\hat{\mathbf{M}}_\nu = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \approx \mathbf{U}_\nu^\dagger \mathbf{M}_\nu \mathbf{U}_\nu^*$, where $\mathbf{U}_\nu^0$ diagonalizes to $\mathbf{M}_\nu^0$ and $\mathbf{U}_\nu^\delta$ makes the same for the resultant matrix that depends on the difference $m_{12} \neq m_{13}$.

Explicitly, when the perturbation is switched off ($m_{12} = m_{13}$), we have

$$\mathbf{U}_\nu^0 = \begin{pmatrix} \cos\theta_\nu e^{i\eta_\nu} & \sin\theta_\nu e^{i\eta_\nu} & 0 \\ -\frac{\sin\theta_\nu}{\sqrt{2}} & \frac{\cos\theta_\nu}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sin\theta_\nu}{\sqrt{2}} & \frac{\cos\theta_\nu}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{8}$$

At the same time, the matrix elements of $\mathbf{M}_\nu^0$ may be written in terms of neutrino mass eigenvalues and the θ_ν angle as

$$m_{11} = \left(m_1^0 \cos^2\theta_\nu + m_2^0 \sin^2\theta_\nu\right) e^{2i\eta_\nu}, \qquad m_{12} = \frac{\sin\theta_\nu \cos\theta_\nu (m_2^0 - m_1^0)}{\sqrt{2}} e^{i\eta_\nu},$$
$$m_{22} + m_{23} = m_1^0 \sin^2\theta_\nu + m_2^0 \cos^2\theta_\nu, \qquad m_{22} - m_{23} = m_3^0. \tag{9}$$

In general, the m_i^0 active neutrino masses are complex due to the presence of Majorana phases, and the θ_ν angle is a free parameter. Going back to Eq. (7), in principle, $m_{12} \neq m_{13}$ so that it breaks the $\mu \leftrightarrow \tau$ symmetry but we will assume that $m_{13} - m_{12}$ is very small in order to apply a perturbative analysis.

Having pointed that, we rewrite $\mathbf{M}_\nu^\delta$ as

$$\mathbf{M}_\nu^\delta = \begin{pmatrix} 0 & 0 & \sqrt{2}\bar{m}_{12}\delta \\ 0 & 0 & 0 \\ \sqrt{2}\bar{m}_{12}\delta & 0 & 0 \end{pmatrix}, \quad \text{and} \quad \delta \equiv \frac{(m_{13} - m_{12})/2}{\bar{m}_{12}}, \tag{10}$$

where $\bar{m}_{12} = (m_{13} + m_{12})/2$ and $|\delta| \ll 1$, then, this will be our perturbation parameter. Thus, applying $\mathbf{U}^0_\nu$ to $\mathbf{M}_\nu$ one obtains $\mathbf{U}^{0\dagger}_\nu \mathbf{M}^0_\nu \mathbf{U}^{0*}_\nu + \mathbf{U}^{0\dagger}_\nu \mathbf{M}^\delta_\nu \mathbf{U}^{0*}_\nu$ where we can write explicitly as

$$\mathrm{diag}(m^0_{\nu_1}, m^0_{\nu_2}, m^0_{\nu_3}) + \begin{pmatrix} 0 & 0 & \sqrt{2}\bar{m}_{12}\cos\theta_\nu \delta e^{-i\eta_\nu} \\ 0 & 0 & \sqrt{2}\bar{m}_{12}\sin\theta_\nu \delta e^{-i\eta_\nu} \\ \sqrt{2}\bar{m}_{12}\cos\theta_\nu \delta e^{-i\eta_\nu} & \sqrt{2}\bar{m}_{12}\sin\theta_\nu \delta e^{-i\eta_\nu} & 0 \end{pmatrix} \tag{11}$$

In this way, the active neutrino masses get corrections up to the second order in the δ parameter, and these are given by

$$m_{\nu_1} = m^0_{\nu_1} + \frac{2|\bar{m}_{12}|^2\cos^2\theta|\delta|^2}{m^0_{\nu_1} - m^0_{\nu_3}}, \quad m_{\nu_2} = m^0_{\nu_2} + \frac{2|\bar{m}_{12}|^2\sin^2\theta|\delta|^2}{m^0_{\nu_2} - m^0_{\nu_3}},$$
$$m_{\nu_3} = m^0_{\nu_3} + 2|\bar{m}_{12}|^2|\delta|^2\left[\frac{\cos^2\theta}{m^0_{\nu_3} - m^0_{\nu_1}} + \frac{\sin^2\theta}{m^0_{\nu_3} - m^0_{\nu_2}}\right]. \tag{12}$$

At the same time, in Eq. (11) the second mass matrix will modify the $\mathbf{U}^0_\nu$ mixing matrix so that after a lengthy task the correction is written as

$$\mathbf{U}^\delta_\nu = \begin{pmatrix} N_1 & N_2 k_1 k_2 r_2 \delta^2 & N_3 k_1 r_1 \delta \\ N_1 k_1 k_2 r_1 \delta^2 & N_2 & N_3 k_2 r_2 \delta \\ -N_1 k_1 r_1 \delta & -N_2 k_2 r_2 \delta & N_3 \end{pmatrix}, \tag{13}$$

where $r_{1,2} \equiv (m^0_2 - m^0_1)/(m^0_3 - m^0_{1,2})$, $k_1 \equiv \sin\theta_\nu\cos^2\theta_\nu$ and $k_2 \equiv \sin^2\theta_\nu\cos\theta_\nu$. Besides, N_i with $i = 1,2,3$, stands for the normalization factor for each eigenvector of $\mathbf{U}^\delta_\nu$. At the end of the day, the full mixing matrix, which has to be compared with the standard parametrization, is $\mathbf{V} = \mathbf{U}^0_\nu \mathbf{U}^\delta_\nu$. Comparing the magnitude of entries of $\mathbf{V}$ with the mixing matrix in the standard parametrization of the PMNS, give us the following expressions for the lepton mixing angles

$$|\sin\theta_{23}|^2 = \left|\frac{N_3}{\sqrt{2}}\frac{[1 - \cos\theta_\nu k_2 r_1 r_2 \delta]}{\sqrt{1 - |\sin\theta_{13}|^2}}\right|^2, \qquad \sin\theta_{12} = N_2\frac{\sin\theta_\nu}{\sqrt{1 - |\sin\theta_{13}|^2}},$$
$$\sin\theta_{13} = N_3 k_1 r_1 \cos\theta_\nu \delta\left[1 + \tan\theta_\nu \frac{k_2 r_2}{k_1 r_1}\right]. \tag{14}$$

For simplicity, we neglected terms that are proportional to $|\delta|^2$ since we have assumed that $|\delta| \ll 1$. So far, these mixing angles depend directly of several free parameters namely: the active neutrino masses (m^0_i), the $|\delta|$ and θ_ν parameters; and the Majorana and η_ν phases. Actually, the neutrino masses may be used as inputs in order to reduce the free parameters. At the end of the day, an χ^2 analysis has to be done in order to explore the allowed regions for the free parameters.

Conclusions

We have constructed a supersymmetric model, with Q_6 flavour symmetry and extended flavoured Higgs sector, where the breaking of the $\mu - \tau$ symmetry leads to a deviation of 0° and 45° of the reactor and atmospheric angles respectively. The mixing angles depend on the active neutrino masses, as well as on the difference of two of the neutrino mass matrix elements $|\delta|$, the angle θ_ν, and the Majorana and η_ν phases.

The crucial difference with other discrete symmetry models that use Q_6 or S_3 as symmetry group is that for the quarks and Higgs fields we have assigned the first two families to a doublet representation and the third one to a singlet, but we have reversed the assignment for the leptons, i.e. the second and third family are in a doublet and the first in a singlet irrep. This breaks the $\mu - \tau$ symmetry, thus giving the possibility of realistic values for the reactor and atmospheric angles. A χ^2 analysis is in order to determine the experimentally allowed regions in parameter space. A complete model has to accommodate both lepton and quark sectors simultaneously, this work is still in progress.

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