

The Renaissance of Kaon Flavour Physics

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There is no doubt that in the coming years we will witness the renaissance of kaon flavour physics with crucial measurements of the branching ratios for the rare decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ that are very sensitive to new physics (NP) and are theoretically very clean. Simultaneously the role of ϵ'/ϵ , ϵ_K , ΔM_K , $K_L \rightarrow \mu^+ \mu^-$ and $K_L \rightarrow \pi^0 \ell^+ \ell^-$ in searching for NP will significantly increase through their improved estimates within the SM. In fact the hints for NP contributing to ϵ'/ϵ have been signalled last year through improved estimates of hadronic matrix elements of QCD and electroweak penguin operators Q_6 and Q_8 by lattice QCD and large N dual QCD approach. Also recent increased tensions between ϵ_K and $\Delta M_{s,d}$ mass differences in $B_{s,d}^0 - \bar{B}_{s,d}^0$ mixing within the SM and models with constrained MFV tell us that we should have tremendous fun in chasing NP through all these observables. The present talk discusses the hinted anomalies in ϵ'/ϵ and $\Delta F = 2$ transitions and summarizes possible implications of simultaneous anomalies in ϵ'/ϵ and ϵ_K for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and ΔM_K in models with tree-level Z and Z' exchanges. The anomalies in ϵ'/ϵ and ϵ_K , if confirmed, would signal new sources of CP violation. This is in contrast to the recent anomalies in B physics that signal lepton flavour non-universality and NP in CP-conserving observables. Highlights from recent flavour analyses of 331 models are briefly described. The correlations of kaon flavour observables with those in B^\pm , $B_{s,d}$ and D meson systems will be crucial for the identification of flavour dynamics at very short distance scales.

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1. Overture

Flavour physics continues to play an important role in identifying new physics (NP) at short distance scales [1–4]. The full picture will only be gained through the study of flavour violating processes in all meson systems and including also lepton flavour violation, breakdown of lepton flavour universality, electric dipole moments and $(g-2)_{e,\mu}$. But in this talk I will discuss dominantly K meson flavour physics which is gaining impetus through the recent developments mentioned in the abstract.

In my talk at the EPS conference on HEP in Vienna last year I have discussed the following topics [5]:

- The present status of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ within the SM [6] presenting parametric expressions for the branching ratios for these two decays in terms of the CKM input and the correlations between $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $B_s \rightarrow \mu^+ \mu^-$ and between $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and ε_K in the SM. This was followed by the results for both decays obtained in simplified models with flavour violating couplings of the SM Z and of a heavy Z' [7].
- The 2015 picture of quark flavour observables in the LHT model with T-parity after LHC Run 1 [8].
- New results on ε'/ε from lattice QCD [9, 10] and large N approach [11] which have shown the emerging anomaly in ε'/ε [12] with its value in the SM being significantly below the experimental world average from NA48 [13] and KTeV [14, 15] collaborations.
- The impact of these results on the correlation between ε'/ε and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ in the LHT model [8] and the first look at such a correlation within simplified models in [7].

The present talk can be considered as the continuation of the discussion in [5] and will include the following topics

- A significant tension between ε_K and $\Delta M_{s,d}$ within the SM and models with constrained MFV (CMFV) [16] implied by new lattice QCD results from Fermilab Lattice and MILC Collaborations [17] on $B_{s,d}^0 - \bar{B}_{s,d}^0$ hadronic matrix elements.
- A strategy for a systematic analysis of the implications of the ε'/ε anomaly, mentioned above, in conjunction with a possible anomaly in ε_K on $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and ΔM_K in models with flavour violating couplings of the SM Z and of a heavy Z' [18].
- New results on ε'/ε and their correlations with other flavour observables in 331 models based on the gauge group $SU(3)_C \times SU(3)_L \times U(1)_X$, recently presented in [19, 20].

In view of space limitations I will try to reduce the overlap with [5] to a minimum so that potential readers are asked to look up also the EPS15 writing [5], in particular many references included there. For a very nice summary of other anomalies in flavour physics see [21].

$\Delta M_s = 17.757(21) \text{ ps}^{-1}$	[22]	$\Delta M_d = 0.5055(20) \text{ ps}^{-1}$	[22]
$S_{\psi K_S} = 0.691(17)$	[22]	$S_{\psi\phi} = 0.015(35)$	[22]
$ V_{us} = 0.2253(8)$	[23]	$ \varepsilon_K = 2.228(11) \cdot 10^{-3}$	[23]
$F_{B_s} = 228.6(3.8) \text{ MeV}$	[24]	$F_{B_d} = 193.6(4.2) \text{ MeV}$	[24]
$F_{B_s} \sqrt{\hat{B}_{B_s}} = (274.6 \pm 8.8) \text{ MeV}$	[17]	$F_{B_d} \sqrt{\hat{B}_{B_d}} = (227.7 \pm 9.8) \text{ MeV}$	[17]
$\eta_{cc} = 1.87(76)$	[25]	$\tilde{\kappa}_\varepsilon = 0.94(2)$	[26]

Table 1: Values of the experimental and theoretical quantities used as input parameters.

2. Tensions between ε_K and $\Delta M_{s,d}$ in the SM and CMFV Models

The five observables of interest are in this case

$$\Delta M_s, \quad \Delta M_d, \quad S_{\psi K_S}, \quad S_{\psi\phi}, \quad \varepsilon_K \quad (2.1)$$

with $\Delta M_{s,d}$ being the mass differences in $B_{s,d}^0 - \bar{B}_{s,d}^0$ mixings and $S_{\psi K_S}$ and $S_{\psi\phi}$ the corresponding mixing induced CP-asymmetries. ε_K describes the size of the indirect CP violation in $K^0 - \bar{K}^0$ mixing. As seen in Table 1 $\Delta M_{s,d}$ and ε_K are already known with impressive precision. The asymmetries $S_{\psi K_S}$ and $S_{\psi\phi}$ are less precisely measured but have the advantage of being subject to only very small hadronic uncertainties. The $K_L - K_S$ mass difference ΔM_K is also very precisely determined but is subject to much larger theoretical uncertainties than the five observables in (2.1). Still as we will see in the next section it could play an important role in distinguishing between various BSM models in the future.

The hadronic uncertainties in $\Delta M_{s,d}$ and ε_K within the SM and CMFV models reside within a good approximation in the parameters

$$F_{B_s} \sqrt{\hat{B}_{B_s}}, \quad F_{B_d} \sqrt{\hat{B}_{B_d}}, \quad \hat{B}_K, \quad \tilde{\kappa}_\varepsilon. \quad (2.2)$$

The first three represent the hadronic matrix elements of the relevant operators, while the last one long distance effects in ε_K . During the last years these uncertainties decreased significantly. Of particular interest is the recent improved precision on $F_{B_s} \sqrt{\hat{B}_{B_s}}$ and $F_{B_d} \sqrt{\hat{B}_{B_d}}$, by the Fermilab Lattice and MILC Collaborations (Fermilab-MILC) [17] (see Table 1) as well as of their ratio

$$\xi = \frac{F_{B_s} \sqrt{\hat{B}_{B_s}}}{F_{B_d} \sqrt{\hat{B}_{B_d}}} = 1.206 \pm 0.019. \quad (2.3)$$

Its reduced uncertainty by a factor of three relative to previous results plays an important role in the analysis in [16]. An extensive list of references to other lattice determinations of these parameters can be found in [17]. In particular the ETM Collaboration [27] finds similar although less accurate results. Most importantly their result for ξ supports the one in (2.3).

Lattice QCD also made an impressive progress in the determination of the parameter \hat{B}_K which enters the evaluation of ε_K . The most recent preliminary world average from FLAG reads $\hat{B}_K = 0.7627(97)$ [28], very close to its large N value $\hat{B}_K = 0.75$ [29, 30]. In the dual QCD approach one finds $\hat{B}_K = 0.73 \pm 0.02$ [31] and arguments can be provided that \hat{B}_K cannot be larger than

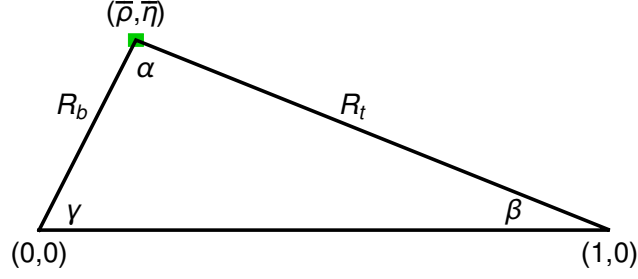


Figure 1: Universal Unitarity Triangle 2016. The green square at the apex of the UUT shows that the uncertainties in this triangle are very small. From [16].

0.75 [31, 32]. Thus presently I quote $\hat{B}_K = 0.750 \pm 0.015$ in order to encompass both large N and lattice QCD values but I expect that in the coming years \hat{B}_K from lattice QCD will move below 0.75 as already signalled by the results in [33, 34].

The long distance contributions to ϵ_K , represented by the departure of $\tilde{\kappa}_\epsilon$ from unity, as given in Table 1, are reasonably well known so that at present the theoretical uncertainty in ϵ_K is dominated by the parameter $\eta_{cc} = 1.87 \pm 0.76$ [25] summarising NLO and NNLO QCD corrections to the charm quark contribution. From present perspective the only way this uncertainty can be reduced is through joint effort of lattice and perturbative QCD experts which would hopefully lead to an improved matching between Wilson coefficients and hadronic matrix elements in the charm contribution and to the improved estimate of long distance contributions. Such efforts would also allow precise calculation of ΔM_K in the SM.

In order to demonstrate that the recent lattice results from [17] imply some tensions in the description of $\Delta F = 2$ processes within the SM, which are independent of some inconsistencies between inclusive and exclusive determinations of $|V_{ub}|$ and $|V_{cb}|$, the strategy of [16] is to leave the latter aside and use only results for $\Delta F = 2$ observables.

Now in the SM and more generally models with constrained minimal flavour violation (CMFV) [35–37] it is possible to construct the so-called *universal unitarity triangle* (UUT) [35] by using only

$$\frac{\Delta M_d}{\Delta M_s}, \quad S_{\psi K_S} \quad (2.4)$$

and this in turn allows to determine $|V_{ub}|/|V_{cb}|$ and the angle γ in the UUT in Fig. 1. As demonstrated in [16] this determination is rather precise after the improved precision on ξ in (2.3)

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.0864 \pm 0.0025 \quad \gamma = (63.0 \pm 2.1)^\circ. \quad (2.5)$$

We observe that the value of γ is significantly below its central value of about 70° from tree-level decays and has much smaller uncertainty.

The important virtue of the determinations in (2.5) and of the UUT shown in Fig. 1 is their universality within CMFV models. In the case of $\Delta F = 2$ transitions in the down-quark sector various CMFV models can only be distinguished by the value of a single flavour universal real one-loop function, the box diagram function $S(v)$, with v collectively denoting the parameters of a given CMFV model. This function enters universally ϵ_K , ΔM_s and ΔM_d and cancels out in the ratio

S_i	$ V_{ts} $	$ V_{td} $	$ V_{cb} $	$ V_{ub} $	$\text{Im}\lambda_t$	$\text{Re}\lambda_t$
S_1	39.0(13)	8.00(29)	39.7(13)	3.43(15)	1.21(8)	-2.88(19)
S_2	42.6(11)	8.73(26)	43.3(11)	3.74(14)	1.44(7)	-3.42(18)

Table 2: Upper bounds on CKM elements in units of 10^{-3} and of $\lambda_t = V_{ts}^*V_{td}$ in units of 10^{-4} obtained using strategies S_1 and S_2 as explained in the text. The bounds correspond to $S(v) = S_0(x_t)$.

in (2.4). Therefore the resulting UUT is the same in all CMFV models. Its apex is determined very precisely with $\bar{\rho} = 0.170 \pm 0.013$ and $\bar{\eta} = 0.333 \pm 0.011$. These values differ significantly from those obtained in global fits [38, 39], with the latter exhibiting smaller $\bar{\rho}$ and larger $\bar{\eta}$ values.

Moreover it can be shown that in these models $S(v)$ is bounded from below by its SM value [40]

$$S(v) \geq S_0(x_t) = 2.32. \quad (2.6)$$

The tension between ε_K and ΔM_s in the SM and CMFV can now be exhibited without the use of any tree-level determinations, except for $|V_{us}|$, by using the following two strategies [16]:

S_1 : ΔM_s **strategy** in which the experimental value of ΔM_s is used to determine $|V_{cb}|$ as a function of $S(v)$, and ε_K is then a derived quantity.

S_2 : ε_K **strategy** in which the experimental value of ε_K is used, while ΔM_s is then a derived quantity and ΔM_d follows then from the determined UUT.

Both strategies use the determination of the UUT by means of (2.4) and allow to determine the whole CKM matrix, in particular $|V_{ts}|$, $|V_{td}|$, $|V_{ub}|$ and $|V_{cb}|$ as functions of $S(v)$. As seen in Table 2 their outcome is very different, which signals the tension between $\Delta M_{s,d}$ and ε_K in this framework. Indeed

- The lower bound in (2.6) implies in S_1 *upper bounds* on $|V_{ts}|$, $|V_{td}|$, $|V_{ub}|$ and $|V_{cb}|$ which are saturated in the SM, and in turn allows to derive an *upper bound* on ε_K in CMFV models that is saturated in the SM but turns out to be significantly *below* the data

$$|\varepsilon_K| \leq (1.64 \pm 0.25) \cdot 10^{-3}. \quad (2.7)$$

- The lower bound in (2.6) implies in S_2 also *upper bounds* on $|V_{ts}|$, $|V_{td}|$, $|V_{ub}|$ and $|V_{cb}|$ which are saturated in the SM. However the $S(v)$ dependence of these elements determined in this manner differs from the one obtained in S_1 , which in turn allows to derive *lower bounds* on $\Delta M_{s,d}$ in CMFV models that are reached in the SM but turn out to be significantly *above* the data

$$\Delta M_s \geq (21.1 \pm 1.8)\text{ps}^{-1}, \quad \Delta M_d \geq (0.600 \pm 0.064)\text{ps}^{-1}. \quad (2.8)$$

The resulting low value of ε_K in S_1 can be naturally raised in CMFV models by enhancing the value of $S(v)$ or/and increasing the value of $|V_{cb}|$. However, as pointed out in [41–43], this spoils the agreement with the data on $\Delta M_{s,d}$, signalling the tension between $\Delta M_{s,d}$ and ε_K in CMFV models. We conclude therefore, as already indicated by the analysis in [43], that it is impossible within CMFV models to obtain a simultaneous satisfactory agreement of $\Delta M_{s,d}$ and ε_K with the data. In

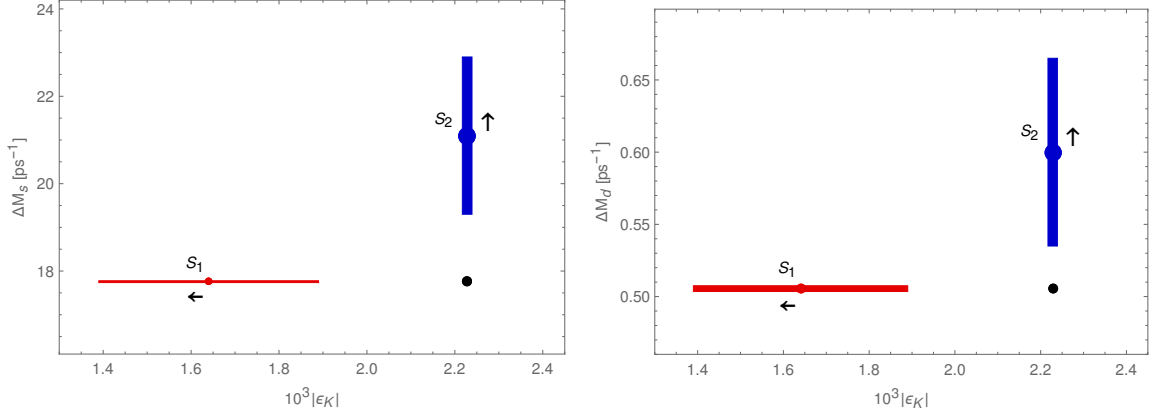


Figure 2: $\Delta M_{s,d}$ and ϵ_K obtained from the strategies S_1 and S_2 for $S(v) = S_0(x_t)$, at which the upper bound on ϵ_K in S_1 and lower bound on $\Delta M_{s,d}$ in S_2 are obtained. The arrows show how the red and blue regions move with increasing $S(v)$. The black dot represents the experimental values. From [16].

the context of the strategies S_1 and S_2 , the tension between $\Delta M_{d,s}$ and ϵ_K is summarized by the plots of $\Delta M_{s,d}$ vs. ϵ_K in Fig. 2.

This analysis will be generalized in the future through the measurements of the branching ratios $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$. As pointed out already in [44] precise measurements of both $K \rightarrow \pi \nu \bar{\nu}$ branching ratios would offer the determination of the unitarity triangle which could be compared with the one extracted these days dominantly from B physics. In particular as demonstrated in [45, 46] within the SM and models with MFV, rather precise determination of $\sin 2\beta$ without the usual QCD penguin pollution and almost independently of $|V_{cb}|$ can be obtained. Analytic expressions for the parameters $\bar{\rho}$ and $\bar{\eta}$ in terms of the branching ratios for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ can be found in these papers and numerical analyses have been presented by us in several papers since then. Recently also the authors of [47] have shown that the inclusion of ϵ_K and ϵ'/ϵ in addition to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ could help to determine such K-triangle.

3. ϵ'/ϵ , ϵ_K , $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and ΔM_K

3.1 ϵ'/ϵ Striking Back

The most recent estimate of ϵ'/ϵ in the SM reads [12]

$$\epsilon'/\epsilon = (1.9 \pm 4.5) \times 10^{-4}, \quad (3.1)$$

roughly 3σ away from the world average from NA48 [13] and KTeV [14, 15] collaborations

$$(\epsilon'/\epsilon)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}. \quad (3.2)$$

It is based on the RBC-UKQCD collaboration results on the relevant hadronic matrix elements of the QCD penguin operator Q_6 [10] and electroweak penguin operator Q_8 [9]. These results imply the following values for the known parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ [6, 12]

$$B_6^{(1/2)} = 0.57 \pm 0.19, \quad B_8^{(3/2)} = 0.76 \pm 0.05, \quad (\text{RBC-UKQCD}), \quad (3.3)$$

significantly below their values in the strict large N limit [30, 48, 49]

$$B_6^{(1/2)} = B_8^{(3/2)} = 1, \quad (\text{large } N \text{ Limit}). \quad (3.4)$$

This suppression of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ below the unity can be understood in the the dual QCD approach [11] as the effect of the meson evolution from scale $\mu = \mathcal{O}(m_\pi)$ at which (3.4) is valid to $\mu = \mathcal{O}(1 \text{ GeV})$ at which Wilson coefficients of Q_6 and Q_8 are evaluated. This evolution has to be matched to the perturbative quark evolution for scales higher than 1 GeV and in fact the suppressions in question and the property that $B_6^{(1/2)}$ is more strongly suppressed than $B_8^{(3/2)}$ are consistent with the perturbative evolution of these parameters above $\mu = \mathcal{O}(1 \text{ GeV})$. Thus we are rather confident that [11]

$$B_6^{(1/2)} \leq B_8^{(3/2)} < 1 \quad (\text{dual QCD}). \quad (3.5)$$

Explicit calculation in this approach gives $B_8^{(3/2)}(m_c) = 0.80 \pm 0.10$. The result for $B_6^{(1/2)}$ is less precise but in agreement with (3.3). For further details, see [11].

In this context it should be emphasized that in the past values $B_6^{(1/2)} = B_8^{(3/2)} = 1.0$ have been combined in phenomenological applications with the Wilson coefficients evaluated at scales $\mu = \mathcal{O}(1 \text{ GeV})$. The discussion above shows that this is incorrect. The meson evolution from $\mu = \mathcal{O}(m_\pi)$ to $\mu = \mathcal{O}(1 \text{ GeV})$ has to be performed and this effect turns out to be stronger than the scale dependence of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ in the perturbative regime, where the scale dependence of these parameters is very weak.

Additional support for the small value of ε'/ε in the SM comes from the recent reconsideration of the role of final state interactions (FSI) in ε'/ε [50]. Already long time ago the chiral perturbation theory practitioners put forward the idea that both the amplitude $\text{Re}A_0$, governed by the current-current operator $Q_2 - Q_1$ and the Q_6 contribution to the ratio ε'/ε could be enhanced significantly through FSI in a correlated manner [51–57] bringing the values of ε'/ε close to its experimental value. However, as shown recently in [50] FSI are likely to be important for the $\Delta I = 1/2$ rule, in agreement with [51–58], but much less relevant for ε'/ε . It should also be emphasized that the latter authors did not include the meson evolution of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ in their analysis and already this effect would significantly lower their predictions for ε'/ε .

It should finally be noted that even without lattice results, varying all input parameters, the bound in (3.5) implies the upper bound on ε'/ε in the SM

$$(\varepsilon'/\varepsilon)_{\text{SM}} < (8.6 \pm 3.2) \times 10^{-4}, \quad (3.6)$$

still 2σ below the experimental data. On the other hand employing the lattice value for $B_8^{(3/2)}$ in (3.3) and $B_6^{(1/2)} = B_8^{(3/2)} = 0.76$, one obtains $(6.0 \pm 2.4) \times 10^{-4}$ instead of (3.6). We observe then that even for these values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ the SM predictions for ε'/ε are significantly below the data.

These results give strong motivation for searching for NP which could enhance ε'/ε above its SM value. On the other hand in the case of ε_K one can only claim, as discussed in the previous section, that there is a tension between ε_K and $\Delta M_{s,d}$ in the whole class of CMFV models. Moreover, there is some tendency that ε_K in the SM is below the data [38, 39, 59–61], but certainly one cannot talk presently about an anomaly in ε_K .

Still it is interesting to ask next what would be the implications of combined anomalies in ε'/ε and ε_K for rare decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$. This question can only be answered in a concrete NP scenario. Here we will summarize such implications in a number of simple scenarios with FCNCs appearing already at tree-level and being mediated by Z boson exchanges or Z' exchanges.

In this context it should be recalled that one of the reasons for a large uncertainty in the SM prediction for ε'/ε is the strong cancellation between QCDP and EWP contributions. As stressed in [18], beyond the SM, quite generally either EWP or QCDP dominate NP contributions to ε'/ε and theoretical uncertainties in these contributions are much smaller because no cancellations take place. We refer to [18] for the discussion of this point.

3.2 Strategy

In order to investigate the implications of anomalies in ε'/ε and ε_K for rare decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in a systematic fashion a strategy, consisting of four steps, has been proposed in [18]. We will describe now these steps. Subsequently we will collect the lessons from this study.

In this strategy the central role is played by ε'/ε and ε_K for which in the presence of NP contributions we have

$$\frac{\varepsilon'}{\varepsilon} = \left(\frac{\varepsilon'}{\varepsilon}\right)^{\text{SM}} + \left(\frac{\varepsilon'}{\varepsilon}\right)^{\text{NP}}, \quad \varepsilon_K \equiv e^{i\varphi_\varepsilon} [\varepsilon_K^{\text{SM}} + \varepsilon_K^{\text{NP}}]. \quad (3.7)$$

Therefore, it should be emphasized at this point that the anomalies in both quantities, if confirmed, would signal the presence of new sources of CP-violation beyond the CKM framework. This should be contrasted with the recent anomalies in $B \rightarrow D(D^*) \tau \nu_\tau$ and $B \rightarrow K(K^*) \ell^+ \ell^-$ that signal lepton flavour non-universalities and new sources of flavour violation in CP-conserving observables.

As in the case of ε'/ε and ε_K , in contrast to most B -physics observables, the interference between SM and NP contributions is totally negligible, one can fully concentrate the discussion on NP contributions. Therefore in order to identify the pattern of NP contributions to flavour observables implied by the ε'/ε anomaly in a transparent manner, we can proceed in a given NP model as follows:

Step 1: We assume that NP provides a positive shift in ε'/ε :

$$\left(\frac{\varepsilon'}{\varepsilon}\right)^{\text{NP}} = \kappa_{\varepsilon'} \cdot 10^{-3}, \quad 0.5 \leq \kappa_{\varepsilon'} \leq 1.5, \quad (3.8)$$

with the range for $\kappa_{\varepsilon'}$ indicating the required size of this contribution. This step determines the imaginary parts of flavour-violating Z or Z' couplings to quarks as functions of $\kappa_{\varepsilon'}$.

Step 2: Knowing that NP contribution to ε_K is proportional to the product of the real and imaginary parts of the couplings in question we assume that in addition to the ε'/ε anomaly, NP can also affect the parameter ε_K . This then allows to determine the relevant real parts of the couplings involved, in the presence of the imaginary part determined from ε'/ε . We describe this effect by the parameter κ_ε so that now in addition to (3.8) we study the implications of the shift in ε_K due to NP

$$(\varepsilon_K)^{\text{NP}} = \kappa_\varepsilon \cdot 10^{-3}, \quad 0.1 \leq \kappa_\varepsilon \leq 0.4. \quad (3.9)$$

The positive sign of κ_ε is motivated by the discussions in the previous section but we cannot exclude that κ_ε is smaller and even slightly negative. But one should note that in the case of $\kappa_\varepsilon = 0$ but $\kappa_{\varepsilon'} \neq 0$, the flavour-violating Z and Z' couplings must be imaginary implying a strong correlation between the branching ratios for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and *negative* shift in ΔM_K from NP [18].

Step 3: In view of the uncertainty in $\kappa_{\varepsilon'}$ one can first set several input parameters to their central values. In particular for the SM contributions to rare decays the CKM factors have been set in [18] to

$$\text{Re}\lambda_t = -3.0 \cdot 10^{-4}, \quad \text{Im}\lambda_t = 1.4 \cdot 10^{-4}. \quad (3.10)$$

These values are in the ballpark of present estimates obtained by UTfit [39] and CKMfitter [38] collaborations. For this choice of CKM parameters the central value of the resulting $\varepsilon_K^{\text{SM}}$ is $1.96 \cdot 10^{-3}$. With the experimental value of ε_K this implies $\kappa_\varepsilon = 0.26$. But it is useful to vary κ_ε while keeping the values in (3.10) fixed as NP contributions do not depend on them but are sensitive functions of κ_ε .

Step 4: Having fixed the flavour violating couplings of Z or Z' in this manner, one can express NP contributions to the branching ratios for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K_L \rightarrow \mu^+ \mu^-$ and to ΔM_K in terms of $\kappa_{\varepsilon'}$ and κ_ε . Explicit formulae can be found in [18]. In this manner one can directly study the impact of ε'/ε and ε_K anomalies in Z and Z' scenarios on these four observables. The pattern of flavour violation depends in a given NP scenario on the relative size of real and imaginary parts of the couplings involved and we will see this explicitly in the lessons below.

The present strategy above assumes that the progress in the evaluation of ε'/ε in the SM will be faster than experimental information on $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. If in 2018 the situation will be reverse, it will be better to choose as variables κ_ε and $R_+^{\nu\bar{\nu}}$ defined by

$$R_+^{\nu\bar{\nu}} \equiv \frac{\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}}}, \quad R_0^{\nu\bar{\nu}} \equiv \frac{\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}}} \quad (3.11)$$

with $R_0^{\nu\bar{\nu}}$ to be determined in the next decade. Presently one can only provide $R_+^{\nu\bar{\nu}}$ as a function of $\kappa_{\varepsilon'}$ for fixed values of κ_ε using the strategy above. But knowing $R_+^{\nu\bar{\nu}}$ better than ε'/ε in the SM will allow us to read off from the plots presented in [18] and here the favourite range for $\kappa_{\varepsilon'}$ in a given NP scenario for a given κ_ε and the diagonal lepton or quark couplings of Z' . The dictionary for doing this can be found in [18]. Note that knowing $R_+^{\nu\bar{\nu}}$ will allow us to obtain $R_0^{\nu\bar{\nu}}$ directly from the plots in [18], using the value of $\kappa_{\varepsilon'}$ extracted from $R_+^{\nu\bar{\nu}}$ and κ_ε . Clearly, when $R_0^{\nu\bar{\nu}}$ will also be known the analysis will be rather constrained.

3.3 Lessons on NP Patterns in Z Scenarios

We will now summarize the lessons in Z and Z' scenarios obtained in [18]. To this end we define flavour violating couplings $\Delta_{L,R}^{sd}(Z)$ by [62]

$$i\mathcal{L}(Z) = i \left[\Delta_L^{sd}(Z) (\bar{s} \gamma^\mu P_L d) + \Delta_R^{sd}(Z) (\bar{s} \gamma^\mu P_R d) \right] Z_\mu, \quad P_{L,R} = \frac{1}{2}(1 \mp \gamma_5) \quad (3.12)$$

with analogous definitions for Z' couplings.

The flavour violating couplings of Z in a given model can be left-handed (LH), right-handed (RH) or can be an arbitrary linear combination of LH and RH couplings, to be termed general

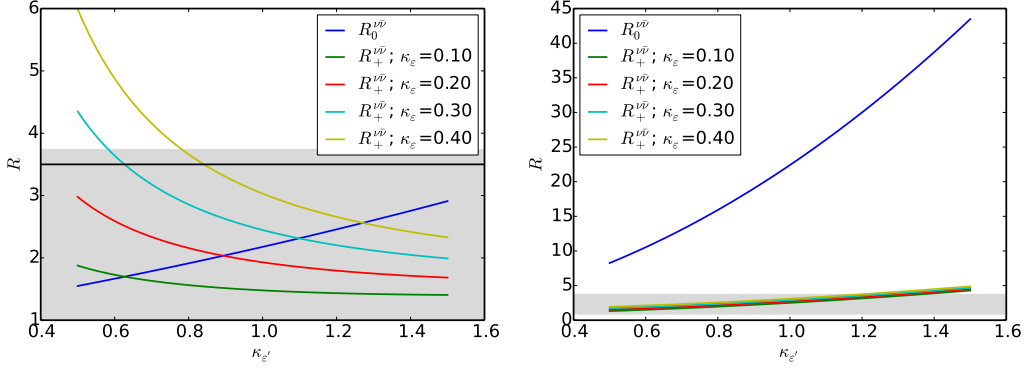


Figure 3: $R_0^{v\bar{v}}$ and $R_{+}^{v\bar{v}}$, as functions of $\kappa_{\epsilon'}$ for $\kappa_{\epsilon} = 0.1, 0.2, 0.3, 0.4$ in the case of the dominance of real parts of the flavour-violating Z couplings (example 1, left panel) and for the case of the dominance of imaginary parts of the flavour-violating Z couplings (example 2, right panel). The horizontal black line in the left panel corresponds to the upper bound from $K_L \rightarrow \mu^+ \mu^-$. The experimental 1σ range for $R_{+}^{v\bar{v}}$ is displayed by the grey band. From [18].

scenario. In [18] all these possibilities have been considered. In what follows we will use for the first scenarios the abbreviations:

$$\text{LHS} \equiv \text{left - handed scenario}, \quad \text{RHS} \equiv \text{right - handed scenario}. \quad (3.13)$$

The summary of the lessons is rather brief. On the other hand the presentation in [18] is very detailed with numerous analytic expressions.

Lesson 1: In the LHS, a given request for the enhancement of ϵ'/ϵ determines the coupling $\text{Im}\Delta_L^{sd}(Z)$.

Lesson 2: In LHS there is a direct unique implication of an enhanced ϵ'/ϵ on $K_L \rightarrow \pi^0 v\bar{v}$: suppression of $\mathcal{B}(K_L \rightarrow \pi^0 v\bar{v})$. This property is known from NP scenarios in which NP to $K_L \rightarrow \pi^0 v\bar{v}$ and ϵ'/ϵ enters dominantly through the modification of Z -penguins. The known flavour diagonal Z couplings to quarks and leptons and the sign of the matrix element $\langle Q_8 \rangle_2$ determines this anticorrelation which has been verified in all models with only LH flavour-violating Z couplings. Fig. 1 in [18] shows this anticorrelation.

Lesson 3: The imposition of the $K_L \rightarrow \mu^+ \mu^-$ constraint in LHS determines the range for $\text{Re}\Delta_L^{sd}(Z)$ which with the already fixed $\text{Im}\Delta_L^{sd}(Z)$ allows to calculate the shifts in ϵ_K and ΔM_K . These shifts turn out to be very small for ϵ_K and negligible for ΔM_K . Therefore unless loop contributions from physics generating $\Delta_L^{sd}(Z)$ play a significant role in both quantities, the SM predictions for ϵ_K and ΔM_K must agree well with data for this NP scenario to survive. In models with vector-like quarks (VLQs) box diagrams with VLQs can indeed provide contributions to ϵ_K and ΔM_K that are larger than coming from tree-level Z -exchange when the masses of VLQs are far above 1 TeV [64].

Lesson 4: With fixed $\text{Im}\Delta_L^{sd}(Z)$ and the allowed range for $\text{Re}\Delta_L^{sd}(Z)$, the range for $\mathcal{B}(K^+ \rightarrow \pi^+ v\bar{v})$ can be obtained. But in view of uncertainties in the $K_L \rightarrow \mu^+ \mu^-$ constraint [65] both an enhancement and a suppression of $\mathcal{B}(K^+ \rightarrow \pi^+ v\bar{v})$ are possible and no specific pattern of correlation between $\mathcal{B}(K_L \rightarrow \pi^0 v\bar{v})$ and $\mathcal{B}(K^+ \rightarrow \pi^+ v\bar{v})$ is found. In the absence of a relevant

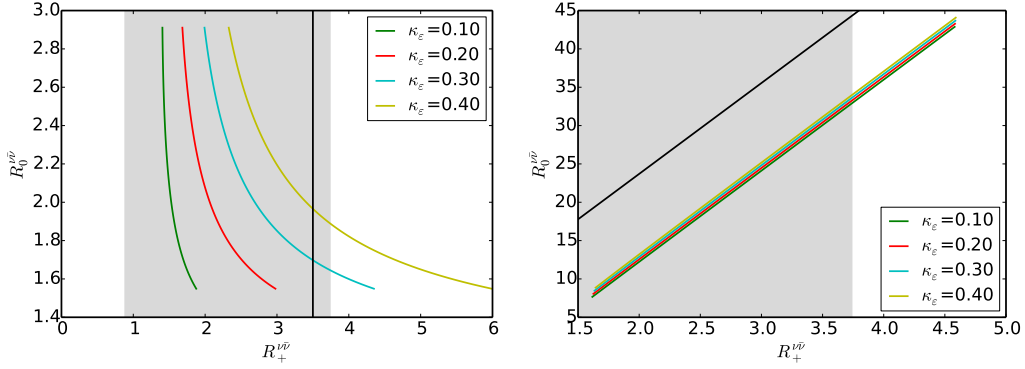


Figure 4: $R_0^{v\bar{v}}$ vs $R_+^{v\bar{v}}$ for $\kappa_\varepsilon = 0.1, 0.2, 0.3, 0.4$ for the example 1 (left panel) and the example 2 (right panel) varying $0.5 \leq \kappa_{\varepsilon'} \leq 1.5$. The vertical black line in the left panel corresponds to the upper bound from $K_L \rightarrow \mu^+ \mu^-$. The dependence on κ_ε in the right panel is negligible and the black line represents the Grossman-Nir (GN) bound [63]. The experimental 1σ range for $R_+^{v\bar{v}}$ is displayed by the grey band. From [18].

ε_K constraint this is consistent with the general analysis in [66]. $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ can be enhanced by a factor of 2 at most.

Lesson 5: Analogous pattern is found in RHS, although the numerics is different. See Fig. 1 in [18]. First due the increased value of the Wilson coefficients of the relevant Q'_8 operator relative to the one of Q_8 operator, the suppression of $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ for a given $\kappa_{\varepsilon'}$ is smaller. Moreover, the flip of the sign in NP contribution to $K_L \rightarrow \mu^+ \mu^-$ allows for larger enhancement of $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, a property first pointed out in [67]. An enhancement of $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ up to a factor of 5.7 is possible.

Lesson 6: In a general Z scenario with LH and RH flavour-violating couplings the pattern of NP effects changes because of the appearance of LR operators dominating NP contributions to ε_K and ΔM_K . Consequently for a large range of parameters these two quantities, in particular ε_K , provide stronger constraint on $\text{Re}\Delta_{L,R}^{sd}(Z)$ than $K_L \rightarrow \mu^+ \mu^-$. But the main virtue of the general scenario is the possibility of enhancing simultaneously ε'/ε , ε_K , $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ which is not possible in LHS and RHS. Thus the presence of both LH and RH flavour-violating currents is essential for obtaining simultaneously the enhancements in question. The correlations between ε'/ε and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ depend sensitively on the ratio of real and imaginary parts of the flavour-violating couplings involved. In the left panel of Fig. 3 we show $R_0^{v\bar{v}}$ and $R_+^{v\bar{v}}$, as functions of $\kappa_{\varepsilon'}$ for the case of the dominance of real parts (example 1) and in the right panel for the case of the dominance of imaginary parts (example 2). In Fig. 4 we show the correlations between $R_0^{v\bar{v}}$ and $R_+^{v\bar{v}}$ for these two examples. We observe that in the example 2 the correlation takes place along the line parallel to the line representing Grossman-Nir bound [63]. Otherwise, as is the case of example 1, the correlation is rather different.

But the main message from this analysis is that in the presence of both LH and RH flavour-violating couplings of Z to quarks, large departures from SM predictions for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ are possible.

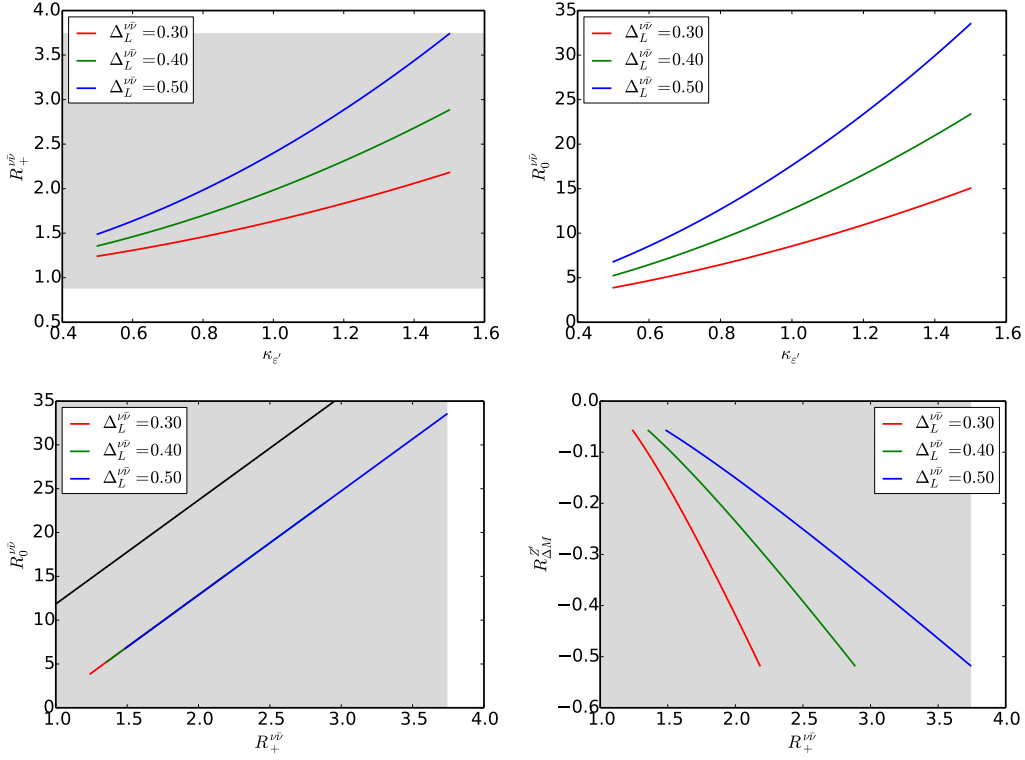


Figure 5: $R_+^{V\bar{V}}$ and $R_0^{V\bar{V}}$, as functions of $\kappa_{\epsilon'}$ for $\Delta_L^{V\bar{V}}(Z') = 0.3, 0.4, 0.5$ for QCDP scenario. $M_{Z'} = 3\text{TeV}$. The dependence on κ_ϵ is negligible. The upper black line in the lower left panel is the GN bound [63]. In the fourth panel correlation of $R_{\Delta M}^{Z'}$ with $R_+^{V\bar{V}}$ is given. The experimental 1σ range for $R_+^{V\bar{V}}$ is displayed by the grey band. From [18].

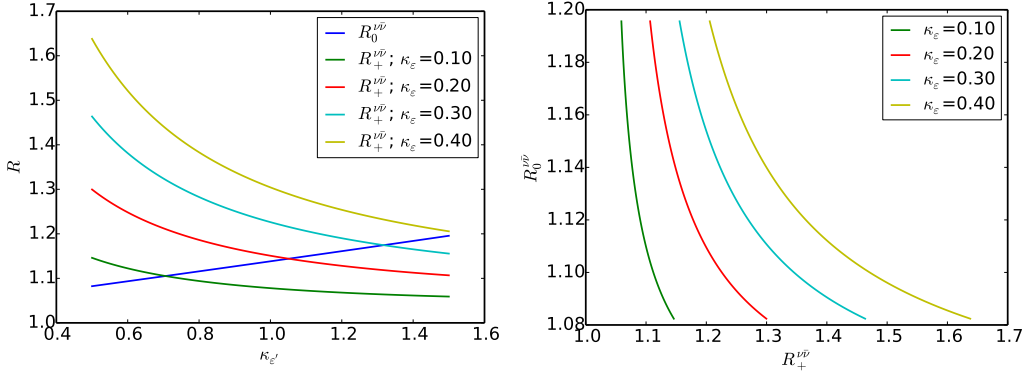


Figure 6: $R_0^{V\bar{V}}$ and $R_+^{V\bar{V}}$, as functions of $\kappa_{\epsilon'}$ for $\kappa_\epsilon = 0.1, 0.2, 0.3, 0.4$ for EWP scenario. $\Delta_L^{V\bar{V}}(Z') = 0.5$. From [18].

3.4 Lessons on NP Patterns in Z' Scenarios

Z' models exhibit quite different pattern of NP effects in the K meson system than the LH and RH Z scenarios. In Z scenarios only electroweak penguin (EWP) Q_8 and Q'_8 operators can

contribute in an important manner because of flavour dependent diagonal Z coupling to quarks. But in Z' models the diagonal quark couplings can be flavour universal so that QCD penguin operators (QCDP) (Q_6, Q'_6) can dominate NP contributions to ε'/ε . Interestingly, the pattern of NP in rare K decays depends on whether NP in ε'/ε is dominated by QCDP or EWP operators. Moreover, the striking difference from Z scenarios, known already from previous studies, is the increased importance of the constraints from $\Delta F = 2$ observables. This has two virtues in the presence of the ε'/ε constraint:

- The real parts of the couplings are determined for not too a large κ_ε from the ε_K constraint even for LH and RH scenarios.
- There is a large hierarchy between real and imaginary parts of the flavour violating couplings implied by anomalies in QCDP and EWP scenarios. As shown in [18] in the case of QCDP imaginary parts dominate over the real ones, while in the case of EWP this hierarchy is opposite unless the ε_K anomaly is absent.

Because of a significant difference in the manner QCDP and EWP enter ε'/ε , there are striking differences in the implications for the correlation between $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in these two NP scenarios if significant NP contributions to ε'/ε are required.

The plots in Figs. 5 and 6 show clearly the differences between QCDP and EWP scenarios. We refer to [18] for more details, in particular analytic derivation of all these results. We extract from these results the following lessons:

Lesson 7: In the case of QCDP scenario the correlation between $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ takes place along the branch parallel to the GN bound. See lower left panel in Fig. 5. Moreover this feature is independent of $M_{Z'}$. Also the dependence on κ_ε is negligible and we show therefore the dependence on $\Delta_L^{\nu\bar{\nu}}(Z')$.

Lesson 8: In the EWP scenario this correlation between $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ proceeds away from this branch for diagonal quark couplings $\mathcal{O}(1)$ if NP in ε_K is present and it is very different from the one of the QCDP case as seen in Fig. 6. As NP effects turn out to be modest in this case we set $\Delta_L^{\nu\bar{\nu}}(Z') = 0.5$. Only for the diagonal quark couplings $\mathcal{O}(10^{-2})$ the requirement of shifting upwards ε'/ε implies large effects in $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in EWP scenario. See [18] for a detail discussion of this point.

Lesson 9: For fixed values of the neutrino and diagonal quark couplings in ε'/ε the predicted enhancements of $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ are much larger when NP in QCDP is required to remove the ε'/ε anomaly than it is the case of EWP. This is simply related to the fact that the $\Delta I = 1/2$ rule suppresses QCDP contributions to ε'/ε so that QCDP operators are less effective in enhancing ε'/ε than EWP operators and consequently the imaginary parts of the flavour violating Z' couplings are required to be larger.

Lesson 10: In QCDP scenario ΔM_K is *suppressed* and this effect increases with increasing $M_{Z'}$ whereas in the EWP scenario ΔM_K is *enhanced* and this effect decreases with increasing $M_{Z'}$ as long as real couplings dominate. Already on the basis of this property one could differentiate between these two scenarios when the SM prediction for ΔM_K improves.

MI	scen.	β	$\tan \bar{\beta}$	MI	scen.	β	$\tan \bar{\beta}$	MI	scen.	β	$\tan \bar{\beta}$
M1	F_1	$-2/\sqrt{3}$	1	M9	F_2	$-2/\sqrt{3}$	1	M17	F_1	$-2/\sqrt{3}$	0.2
M2	F_1	$-2/\sqrt{3}$	5	M10	F_2	$-2/\sqrt{3}$	5	M18	F_2	$-2/\sqrt{3}$	0.2
M3	F_1	$-1/\sqrt{3}$	1	M11	F_2	$-1/\sqrt{3}$	1	M19	F_1	$-1/\sqrt{3}$	0.2
M4	F_1	$-1/\sqrt{3}$	5	M12	F_2	$-1/\sqrt{3}$	5	M20	F_2	$-1/\sqrt{3}$	0.2
M5	F_1	$1/\sqrt{3}$	1	M13	F_2	$1/\sqrt{3}$	1	M21	F_1	$1/\sqrt{3}$	0.2
M6	F_1	$1/\sqrt{3}$	5	M14	F_2	$1/\sqrt{3}$	5	M22	F_2	$1/\sqrt{3}$	0.2
M7	F_1	$2/\sqrt{3}$	1	M15	F_2	$2/\sqrt{3}$	1	M23	F_1	$2/\sqrt{3}$	0.2
M8	F_1	$2/\sqrt{3}$	5	M16	F_2	$2/\sqrt{3}$	5	M24	F_2	$2/\sqrt{3}$	0.2

Table 3: Definition of the various 331 models. From [72].

4. 331 Flavour News

The 331 models are based on the gauge group $SU(3)_C \times SU(3)_L \times U(1)_X$ [68, 69]. In these models new contributions to ϵ'/ϵ and other flavour observables are dominated by tree-level exchanges of a Z' with non-negligible contributions from tree-level Z exchanges generated through the $Z - Z'$ mixing. The size of these NP effects depends not only on $M_{Z'}$ but in particular on a parameter β , which distinguishes between various 331 models, on fermion representations under the gauge group and a parameter $\tan \bar{\beta}$ present in the $Z - Z'$ mixing. Extensive recent analyses in these models can be found in [19, 20, 70–72]. References to earlier analysis of flavour physics in 331 models can be found there and in [73, 74].

A detailed analysis of 331 models with different values of β , $\tan \bar{\beta}$ for two fermion representations F_1 and F_2 , with the third SM quark generation belonging respectively to an antitriplet and a triplet under the $SU(3)_L$, has been presented in [72]. They are collected in Table 3. Requiring that these 24 models perform at least as well as the SM as far as electroweak tests are concerned, seven models have been selected for a more detailed study of FCNC processes. These are

$$M9, M8, M6, M11, M3, M16, M14, \quad (\text{favoured}) \quad (4.1)$$

with the first five performing better than the SM while the last two basically as the SM.

A recent updated analyses have been presented in [19, 20] and we summarize the main results of these two papers putting the emphasize on the last analysis in [20] which could take into account new lattice QCD results from Fermilab Lattice and MILC Collaborations [17] on $B_{s,d}^0 - \bar{B}_{s,d}^0$ hadronic matrix elements.

The new analyses in [19, 20] show that the impact of a required enhancement of ϵ'/ϵ on other flavour observables is significant. The one in [20] also shows that the results are rather sensitive to the value of $|V_{cb}|$ which has been illustrated there by choosing two values: $|V_{cb}| = 0.040$ and $|V_{cb}| = 0.042$.

The main findings of [19, 20] for $M_{Z'} = 3 \text{ TeV}$ are as follows:

- Among seven 331 models in (4.1) singled out through electroweak precision study only three (M8, M9, M16) can provide for both choices of $|V_{cb}|$, significant shift of ε'/ε but not larger than 6×10^{-4} , that is $\kappa_{\varepsilon'} \leq 0.6$.
- The tensions between $\Delta M_{s,d}$ and ε_K , discussed in Section 2, can be removed in these models (M8, M9, M16) for both values of $|V_{cb}|$.
- Two of them (M8 and M9) can simultaneously suppress $B_s \rightarrow \mu^+ \mu^-$ by at most 10% and 20% for $|V_{cb}| = 0.042$ and $|V_{cb}| = 0.040$, respectively. This can still bring the theory within 1σ range of the combined result from CMS and LHCb and for $|V_{cb}| = 0.040$ one can even reach the present central experimental value of this rate. The most recent result from ATLAS [75], while not accurate, appears to confirm this picture. On the other hand the maximal shifts in the Wilson coefficient C_9 are $C_9^{\text{NP}} = -0.1$ and $C_9^{\text{NP}} = -0.2$ for these two $|V_{cb}|$ values, respectively. This is only a moderate shift and these models do not really help in the case of $B_d \rightarrow K^* \mu^+ \mu^-$ anomalies that require shifts as high as $C_9^{\text{NP}} = -1.0$ [76, 77].
- In M16 the situation is opposite. The rate for $B_s \rightarrow \mu^+ \mu^-$ can be reduced for $M_{Z'} = 3 \text{ TeV}$ for the two $|V_{cb}|$ values by at most 3% and 10%, respectively but with the corresponding values $C_9^{\text{NP}} = -0.3$ and -0.5 the anomaly in $B_d \rightarrow K^* \mu^+ \mu^-$ can be significantly reduced.
- The maximal shifts in ε'/ε decrease fast with increasing $M_{Z'}$ in the case of $|V_{cb}| = 0.042$ but are practically unchanged for $M_{Z'} = 10 \text{ TeV}$ when $|V_{cb}| = 0.040$ is used.
- On the other hand for higher values of $M_{Z'}$ the effects in $B_s \rightarrow \mu^+ \mu^-$ and $B_d \rightarrow K^* \mu^+ \mu^-$ are much smaller. We recall that NP effects in rare K decays and $B \rightarrow K(K^*) \nu \bar{\nu}$ remain small in all 331 models even for $M_{Z'}$ of few TeV. This could be challenged by NA62, KOTO and Belle II experiments in this decade.

We show these correlations for $M_{Z'} = 3 \text{ TeV}$ and $|V_{cb}| = 0.040$ in Fig. 7. Thus the main message from [19, 20] is that NP contributions in 331 models can simultaneously solve $\Delta F = 2$ tensions, enhance ε'/ε and suppress either the rate for $B_s \rightarrow \mu^+ \mu^-$ or C_9 Wilson coefficient without any significant effect on $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $b \rightarrow s \nu \bar{\nu}$ transitions. While sizable NP effects in $\Delta F = 2$ observables and ε'/ε can persist for $M_{Z'}$ outside the reach of the LHC, such effects in $B_s \rightarrow \mu^+ \mu^-$ will only be detectable provided Z' will be discovered soon.

5. Outlook

5.1 2018 Visions

Let us begin the final section with a dream about the discovery of NP in $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ by the NA62 experiment through

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (18.0 \pm 2.0) \cdot 10^{-11}, \quad (\text{NA62, 2018}). \quad (5.1)$$

It should be emphasized that such a result would be without any doubt a clear signal for NP. Moreover, looking at the grey bands in several figures shown by us, such a result would be truly

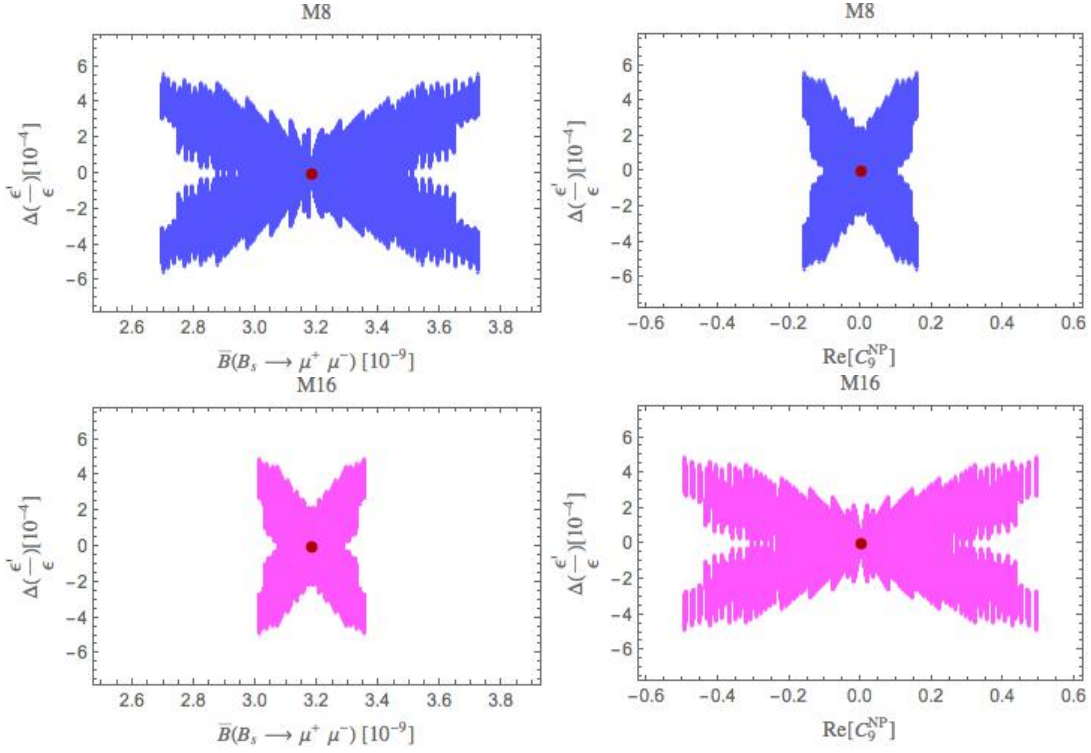


Figure 7: Correlations of $\Delta(\epsilon'/\epsilon)$ with $B_s \rightarrow \mu^+\mu^-$ (left panels) and with C_9^{NP} (right panels) for M8 and M16. Red dots represent central SM values. $M_{Z'} = 3 \text{ TeV}$ and $|V_{cb}| = 0.040$.

tantalizing with a big impact on our field. Assuming then that the lattice values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ will not be modified significantly and the ϵ'/ϵ anomaly will stay with us with $\kappa_{\epsilon'} = 1.0$ this measurement will allow to exclude certain scenarios and favour other ones. But this will also depend on the allowed size of NP in ϵ_K , ΔM_K and rare $B_{s,d}$ decays.

5.2 Open Questions

There is no doubt that in the coming years K meson physics will strike back, in particular through improved estimates of SM predictions for ϵ'/ϵ , ϵ_K , ΔM_K and $K_L \rightarrow \mu^+\mu^-$ and through crucial measurements of the branching ratios for $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K_L \rightarrow \pi^0\nu\bar{\nu}$. Correlations with other meson systems, lepton flavour physics, electric dipole moments and other rare processes should allow us to identify NP at very short distance scales [1] and we should hope that this physics will also be directly seen at the LHC. Let us then close this talk by listing most pressing questions in kaon flavour physics for the coming years. On the theoretical side we have:

- **What is the precise value of $\kappa_{\epsilon'}$?** Here the answer will come not only from lattice QCD but also through improved values of the CKM parameters, NNLO QCD corrections and an improved understanding of FSI and isospin breaking effects. The NNLO QCD corrections should be available soon. The recent analysis in the large N approach in [50] indicates that FSI are likely to be important for the $\Delta I = 1/2$ rule in agreement with previous studies

[51–58], but much less relevant for ε'/ε . But it is important that other lattice groups beyond RBC-UKQCD collaboration make efforts to calculate ε'/ε in the SM.

- **What is the precise value of κ_ε ?** Here the reduction of CKM uncertainties is most important. Also the large error in the charm contribution η_{cc} should be decreased. In spite of these uncertainties the most recent analysis in [16] indicates that if no NP is present in ε_K , it is expected to be found in $\Delta M_{s,d}$.
- **What is the precise value of ΔM_K in the SM?** The present calculations from dual QCD approach [31, 78] and lattice QCD [79, 80] give values of ΔM_K in the ballpark of its experimental value but with uncertainties as high as $\pm 30\%$ which do not allow to conclude whether NP is required to contribute here or not. Even the sign of possible NP contributions is unknown. Let us hope this will be found out in this decade.
- **What are the precise values of $\text{Re}A_2$ and $\text{Re}A_0$?** Again lattice QCD will play the crucial role here. The study of NP contributions to $\text{Re}A_0$ can be found in [81].

On the experimental side we have:

- **What is $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ from NA62?** We should know it in 2018.
- **What is $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ from KOTO?** We should know it around the year 2020.
- **Do Z' or other new particles with masses in the reach of the LHC exist?** We could know it already this year.

There are clearly other topics in kaon physics which we did not mention here. In particular the study of $K^+ \rightarrow \pi^+ \ell^+ \ell^-$ and $K_L \rightarrow \pi^0 \ell^+ \ell^-$ will become more important when the theory improves. For recent analyses in the dual QCD approach and lattice QCD see [82] and [83], respectively. Lattice QCD could also reduce the uncertainty in the charm contribution to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ [84]. Another interesting issue is the violation of lepton flavour and lepton flavour universality in rare kaon decays [85].

I also expect that the interest in searching for NP behind the ε'/ε anomaly will increase. Beyond the papers discussed by us there is an interesting recent analysis in [86], where it is shown that ε'/ε anomaly can be explained in the MSSM with squark masses above 3 TeV while satisfying ε_K constraint without fine-tuning of CP-phases or other parameters. More papers will appear soon.

Definitely there are exciting times ahead of us!

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