

Semileptonic B_c decays from full lattice QCD

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We present first lattice QCD results for semileptonic form factors of the decays $B_c \rightarrow \eta_c l \nu$ and $B_c \rightarrow J/\psi l \nu$ over the full q^2 range, using both improved non-relativistic QCD (NRQCD) and fully relativistic (HISQ) formalisms. These can be viewed as prototype calculations for pseudoscalar to pseudoscalar and pseudoscalar to vector decays involving a $b \rightarrow c$ transition. In particular we can use information from the relativistic computations to fix the NRQCD current normalisations, which can then be used in improved computations of decays such as $B \rightarrow D l \nu$ and $B \rightarrow D^* l \nu$.

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1. Introduction

Semileptonic decays of B -mesons provide the main inputs for exclusive determinations of $|V_{ub}|$ and $|V_{cb}|$ (a status update of lattice QCD's impact on these quantities was presented at this conference [1]). The precise determination of $|V_{cb}|$ requires precision in both theoretical computation and measurement of $b \rightarrow c$ processes. Treatment of c and especially b quarks is a challenge for lattice simulations due to lattice artifacts which grow as $(am_q)^n$ where a is the lattice spacing and m_q is a quark mass. We have two complementary approaches to the treatment of b quarks: using a highly improved relativistic action at small lattice spacings to simulate masses approaching m_b , and working directly at m_b with an improved non-relativistic (NRQCD) effective theory formalism.

Methodology

We use a highly improved staggered quark (HISQ) action [2] which systematically removes lattice artifacts, allowing for simulation of charm quarks with small discretisation effects. We can even simulate quarks with mass significantly larger than m_c , especially on the ensembles with very fine lattice spacings of $a \approx 0.045$ fm [3]. This motivates one of our approaches for doing b -physics. By working in a regime $am_h \lesssim 0.8$, say, but on finer and finer lattice spacings, we calculate the physics of interest over a range in m_h and then extrapolate that data to m_b .

We are also able to work directly at the b mass, without the need for extrapolation, using an improved non-relativistic (NRQCD) formalism [4]. This approach is complementary to the fully relativistic approach described above. The NRQCD Hamiltonian is expressed as an expansion in the velocity of the heavy quark. In addition the current operators have a relativistic expansion, e.g. the temporal axial-vector current

$$A_0^{\text{nrqcd}} = (1 + \alpha_s z_0^{(0)}) \left[A_0^{(0)} + (1 + \alpha_s z_0^{(1)}) A_0^{(1)} + \alpha_s z_0^{(2)} A_0^{(2)} \right] + \dots,$$

where $A_0^{(1)}$, $A_0^{(2)}$, \dots are higher order current corrections, with matrix elements proportional to $1/m_b$. The matching to the continuum current above is only known in QCD perturbation theory to $\mathcal{O}(\alpha_s)$, and so has a systematic uncertainty from missing α_s^2 terms. One output of the present work will be to improve the normalisation of the currents using the fully relativistic calculation where the normalisation is simpler and nonperturbative.

We use gauge ensembles generated by the MILC collaboration, which include the effects of u/d , s , and c quarks in the sea, and with lattice spacings of $a \approx 0.09, 0.06$, and 0.045 fm. Although we have ensembles with physical u/d quark mass, all results presented here use $m_s/m_{u/d} = 5$, i.e. unphysically heavy pions.

Obtaining the form factors

In both formalisms we compute the matrix element of the $V - A$ operator between the states of interest. We work in the frame where the B_c is at rest. The matrix elements are parametrised in terms of form factors which are functions of q^2 , where $q = P - p$ is the difference in four-momentum between the B_c and outgoing particle. The kinematic endpoint q_{max}^2 is where the outgoing hadron is at rest, whereas the energy of the outgoing hadron is a maximum at $q^2 = 0$.

For the $B_c \rightarrow \eta_c$ decay there are two form factors to determine, f_+ and f_0 .

$$\langle \eta_c(p) | V^\mu | B_c(P) \rangle = f_+(q^2) \left[P^\mu + p^\mu - \frac{M^2 - m^2}{q^2} q^\mu \right] + f_0(q^2) \frac{M^2 - m^2}{q^2} q^\mu$$

For the relativistic case we can determine f_0 using a scalar current, which is absolutely normalised.

$$\langle \eta_c(p) | S | B_c(P) \rangle = \frac{M^2 - m^2}{m_{b0} - m_{c0}} f_0(q^2)$$

There are five form factors to determine for the $B_c \rightarrow J/\psi$ decay, one from the vector current and four from the axial-vector current (three of which are independent).

$$\begin{aligned} \langle J/\psi(p, \varepsilon) | V^\mu - A^\mu | B_c(P) \rangle = & \frac{2i\varepsilon^{\mu\nu\rho\sigma}}{M+m} \varepsilon_\nu^* p_\rho P_\sigma V(q^2) - (M+m) \varepsilon^{*\mu} A_1(q^2) + \\ & \frac{\varepsilon^* \cdot q}{M+m} (p+P)^\mu A_2(q^2) + 2m \frac{\varepsilon^* \cdot q}{q^2} q^\mu A_3(q^2) - 2m \frac{\varepsilon^* \cdot q}{q^2} q^\mu A_0(q^2) \end{aligned}$$

Results

Figure 1 (left) shows our results for the $B_c \rightarrow \eta_c$ form factors $f_+(q^2)$ and $f_0(q^2)$ computed using improved NRQCD on the $a \approx 0.09$ fm ensemble.

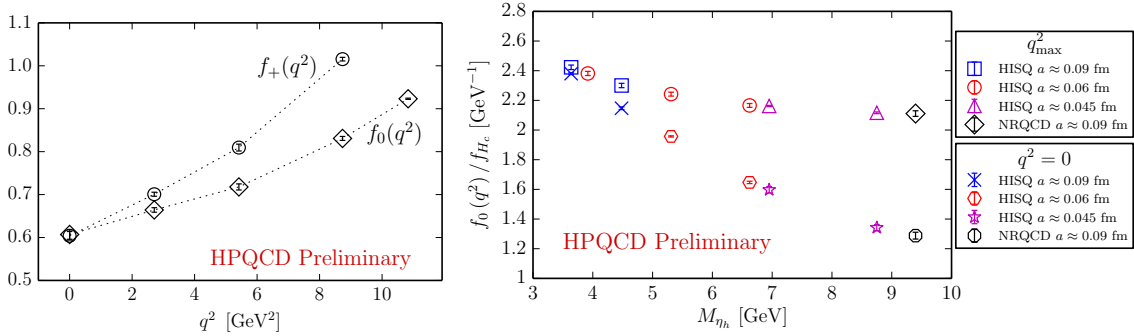


Figure 1: (Left) Results for $B_c \rightarrow \eta_c$ form factors f_0 and f_+ from lattice NRQCD, determined over the full q^2 range. (Right) Extrapolation results in heavy quark mass m_h for $f_0(q_{\max}^2)/f_{H_c}$ and $f_0(0)/f_{H_c}$ using the fully relativistic (HISQ) formalism. The rightmost points are the corresponding NRQCD results with physical b mass.

In Figure 1 (right) we show results for $f_0(q_{\max}^2)/f_{H_c}$ and $f_0(0)/f_{H_c}$ using the relativistic formalism on ensembles with lattice spacings of $a \approx 0.09, 0.06$, and 0.045 fm. For each ensemble we use valence masses m_h such that $am_h \leq 0.8$, which correspond to heavier physical masses as we go to smaller lattice spacings. As m_h approaches m_b the results join smoothly with results computed using NRQCD (rightmost points).

Figure 2 is another extrapolation plot, this time showing the $B_c \rightarrow J/\psi$ form factor $A_1(q_{\max}^2)$. This is the only form factor that contributes to the decay rate at zero recoil. Furthermore Luke's theorem tells us the $1/m_b$ current corrections vanish there so that the comparison between NRQCD and relativistic data is particularly simple.

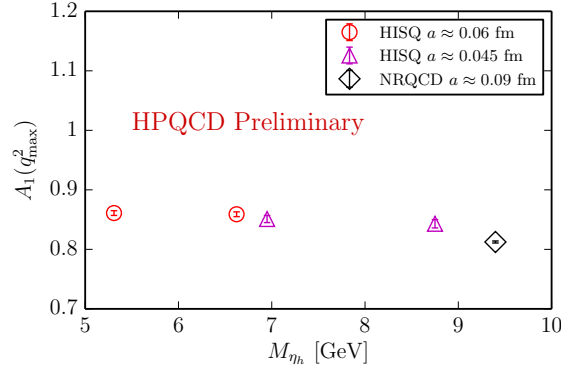


Figure 2: Extrapolation in heavy quark mass m_h for the $B_c \rightarrow J/\psi$ form factor $A_1(q_{\max}^2)$ using the relativistic HISQ formalism. The rightmost point is the NRQCD result at physical b mass.

Conclusions

We have presented results for the B_c semileptonic decays to η_c and J/ψ using two complementary approaches.

- The $B_c \rightarrow \eta_c$ results provide proof-of-principle for our strategy; we are able to control the calculation over the full q^2 range and find good agreement between the NRQCD and fully relativistic approaches.
- Our first results for the $B_c \rightarrow J/\psi$ decay also appear promising. The full lattice calculation of this decay will allow the extraction of $|V_{cb}|$ if the decay is measured at LHCb.
- The NRQCD $b \rightarrow c$ currents also mediate the decays $B \rightarrow D$ and $B \rightarrow D^*$. Using information from the relativistic calculation we will improve the normalisations of the currents, which will lead to improvements in theoretical precision for these decays.

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