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Experimental review on the Unitarity Triangle angles α (ϕ_2) and β (ϕ_1)

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The precise determination of the angles of the Unitarity Triangle can put tight constraints on the flavor sector of the Standard Model, by probing the validity of the Cabibbo-Kobayashi-Maskawa scheme. The measurement of time-dependent *CP* asymmetries provides the main experimental means to determine the angles α and β. The present status of the knowledge of these two angles and a summary of recent measurements performed by the *BABAR*, Belle and LHCb experiments are reported in this proceedings article.

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1. Introduction

The *BABAR* and Belle experiments, respectively at the PEPII and KEKB e^+e^- B-factories, observed the *CP* violation in the neutral B-meson system in 2001 [1, 2], 37 years after its discovery in the kaon system [3]. *CP* violation is explained within the three-generation Standard Model (SM) by the irreducible complex phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [4, 5].

From the unitarity of the CKM matrix, six vanishing relations among the matrix elements can be written and interpreted as triangles in the complex space of the parameters. Among these triangles, of most importance is that related to the neutral and charged B-mesons system, called the Unitarity Triangle (UT). It raises from the relation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0, \tag{1.1}$$

and has all sides of the same order $\mathscr{O}(\lambda_C^3)$, where $\lambda_C = \sin\theta_C$ is the CKM parameter of the Wolfenstein parametrization that corresponds to the sine of the Cabibbo angle θ_C. The primary goal of the B factories is to probe the validity of this picture by over-constraining the UT through precise measurements of both sides and angles. Any inconsistency would represent a hint for physics scenarios beyond the SM. The angles, being strictly related to the phase of the CKM matrix, are measured exploiting *CP* asymmetries. They are defined as[1]:

$$\beta \equiv \phi_1 = arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right);\ \alpha \equiv \phi_2 = arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right);\ \gamma \equiv \phi_3 = arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right). \tag{1.2}$$

In the following we present the experimental status about the first two angles β and α, reporting also some of the most recent results.

2. Time-dependent *CP* asymmetries

The angles β and α can be determined by measuring the decays of neutral B mesons to *CP* eigenstates f_{CP}. The final state f_{CP} is common to both the B^0 and $\bar{B}^0$ meson, and can be reached via a direct decay or a decay after oscillation in the opposite flavor. The interference between the two possible paths may result in different decay time distributions for B^0 and $\bar{B}^0$. A time-dependent (TD) *CP* asymmetry can then be defined as:

$$\mathscr{A}_{CP}(t) = \frac{\Gamma\left(B^0(t)\to f_{CP}\right) - \Gamma\left(\bar{B}^0(t)\to f_{CP}\right)}{\Gamma\left(B^0(t)\to f_{CP}\right) + \Gamma\left(\bar{B}^0(t)\to f_{CP}\right)} = S\,\sin(\Delta m_d\,t)\; - \;C\,\cos(\Delta m_d\,t). \tag{2.1}$$

The parameters S and C measure the mixing-induced and direct *CP* violation, respectively, and Δm_d is the parameter governing the $B^0-\bar{B}^0$ oscillation phenomenon. The $B^0-\bar{B}^0$ mixing process introduces the weak phase β. Therefore if the decay $B^0\to f_{CP}$ proceeds only via a single $b\to c$ transition amplitude that carry no additional weak phase, then no direct *CP* violation is present and the *CP* asymmetry provides a theoretically clean access to the angle, resulting:

$$S = -\eta_{f_{CP}}\sin 2\beta;\quad C = 0. \tag{2.2}$$

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[1] Different notations are commonly used: $(\alpha,\beta,\gamma) \leftrightarrow (\phi_2,\phi_1,\phi_3)$ for the UT angles, and $(S,C) \leftrightarrow (S,A)$, with $A = -C$, for the *CP*-violation parameters.

This is the case, to a good approximation, for the $b \to c\bar{c}s$ transitions such as the decays to a charmonium state plus a neutral kaon, like $B^0 \to J/\psi K^0_S$, for which the amplitudes carrying a different weak phase are related to strongly suppressed penguin processes.

3. Measurement of $\sin 2\beta$ from $b \to c\bar{c}s$ transitions

Since the *CP* asymmetry of Eq.(2.1) vanishes if integrated over time, time-dependent analysis are required to measure $\sin 2\beta$. In e^+e^- B factories the colliding beams of different energies produce the Υ(4S) with a boost along the z direction, providing an experimentally measurable spatial separation Δz between the two B-meson decay vertices. The asymmetry $\mathscr{A}_{CP}$ is therefore measured as a function of the decay-time difference of the two B mesons, which, neglecting the small momentum of the B mesons in the Υ(4S) reference frame, is obtained from the boost factor $\langle\beta\gamma\rangle$ (0.56 for BABAR and 0.43 for Belle) and the measured Δz as $\Delta t = \Delta z / \langle\beta\gamma\rangle c$. The vertex resolution is among the most critical aspects of the time-dependent measurements at a B factory, as the average decay length of the boosted B mesons is only a few hundred microns, and vertex and tracking systems with high performances are crucial. On the other hand, the $B^0 - \bar{B}^0$ pair from $e^+e^- \to \Upsilon(4S) \to B^0\bar{B}^0$ is produced in a coherent *P*-wave state, so that when one B meson decays to a known flavor eigenstate, B^0 or $\bar{B}^0$, at the same time the other B meson is projected onto the opposite eigenstate. As a consequence a clear tagging procedure is available with high efficiency and small wrong tag fraction (the figure of merit is an effective flavor tagging efficiency $Q > 30\%$).

Both BABAR and Belle have completed the measurement of *CP* asymmetries in $B \to$ "$c\bar{c}$" $+ K^0$ ("$c\bar{c}$" indicates a charmonium vector state) on the full data sample, and obtained consistent results: $\sin 2\beta = 0.687 \pm 0.028 \pm 0.012$ by BABAR [6], and $\sin 2\beta = 0.668 \pm 0.023 \pm 0.013$ by Belle [7].

More recently, LHCb reported [8] a measurement of $\sin 2\beta$ from the time-dependent *CP* asymmetry in $B^0 \to J/\psi K^0_S$ decay, by using a data sample corresponding to 3 fb^{-1} of pp collisions at the center-of-mass energies of 7 and 8 TeV of the LHC. The lack of entanglement between the final b-hadrons and the harsh environment require the measurement of $\mathscr{A}_{CP}(t)$ as a function of the absolute decay time of the B^0 and result in a much lower effective tagging efficiency ($Q \sim 3\%$) compared to the e^+e^- B factories. This is however fully compensated by the much higher production rate (roughly a factor 1000 more signal events collected per fb^{-1}) and by the excellent decay-time resolution because of the large B^0-meson boost. The single-arm spectrometer of the LHCb experiment has reconstructed about $42k$ signal $B^0 \to J/\psi K^0_S$ events, with $J/\psi \to \mu^+\mu^-$ and $K^0_S \to \pi^+\pi^-$. The reconstructed mass distribution of the selected B^0 candidates and the measured *CP* asymmetry are shown in Fig. 1. The value of the *CP*-violation parameters resulting from the fit to the decay-time distribution are:

$$\begin{aligned} S &= +0.731 \pm 0.035(\text{stat.}) \pm 0.020(\text{syst.}), \\ C &= -0.038 \pm 0.032(\text{stat.}) \pm 0.005(\text{syst.}). \end{aligned} \tag{3.1}$$

These values are in agreement with those measured in $b \to c\bar{c}s$ transitions at the e^+e^- B factories, with a comparable uncertainty.

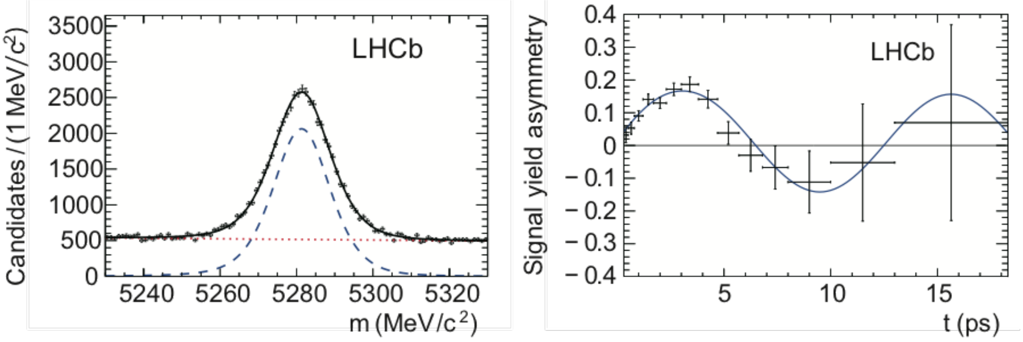
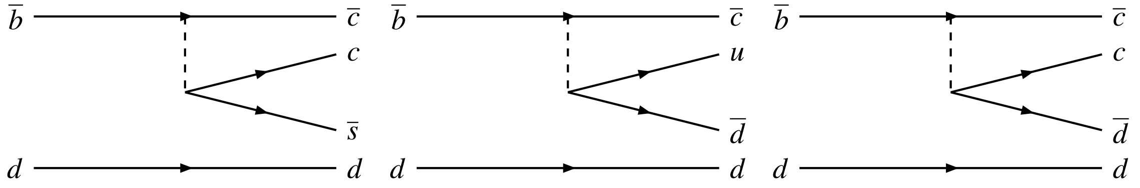


Figure 1: Distribution of the reconstructed mass (left) and time-dependent signal-yield asymmetry (right) of selected $B^0 \to J/\psi K^0_S$ candidates [8].

4. Study of $b \to c$ transitions other than $b \to c\bar{c}s$

The sub-leading terms in $B^0 \to J/\psi K^0_S$ decays are $b \to s$ penguin transitions, and the magnitude of those with different weak phase are suppressed by a factor $|V_{ub}V^*_{us}/V_{cb}V^*_{cs}| \sim \mathcal{O}(\lambda_c^2)$. Therefore, their effect on β is expected to be very small. However, also new physics (NP) beyond the SM would occur in similar loop as the SM penguin diagrams and could modify the time-dependent *CP* asymmetry, leading to inconsistencies between β and other observables of the UT. It is therefore important to study processes that may measure $\sin 2\beta$ without the penguin contribution or constrain their effect. This can be obtained by studying other $b \to c$ transitions, such as the $b \to c\bar{u}d$ and $b \to c\bar{c}d$ processes whose diagrams are shown in Fig. 2, together with the leading tree diagram of the $b \to c\bar{c}s$ transitions.



(a) $\bar{b} \to \bar{c}c\bar{s}$ transitions.
(b) $\bar{b} \to \bar{c}u\bar{d}$ transitions.
(c) $\bar{b} \to \bar{c}c\bar{d}$ transitions.

Figure 2: Tree diagrams for the different $b \to c$ transitions.

4.1 Measurement of $\sin 2\beta$ in $B^0 \to \bar{D}^{(*)}_{CP}h^0$

The $b \to c\bar{u}d$ transition has no complex phase, and time-dependent *CP* asymmetries in $B^0 \to \bar{D}^{(*)}_{CP}h^0$ decays, where D^0 decays to a *CP* eigenstate and h^0 is a light non-strange neutral meson, allow a measurement of $\sin 2\beta$ free of penguin contributions. In fact, the sub-leading term in $B^0 \to \bar{D}^{(*)0}h^0$ is a doubly-Cabibbo-suppressed tree diagram $b \to u\bar{c}d$. This amplitude has the complex phase of the V_{ub} element and interferes with the leading term, but it is expected to be small and, being a tree diagram, its effect can be fully calculated within the SM. These processes can therefore provide a theoretically very clean way to extract β.

BABAR and Belle reported in 2015 [9] a measurement of $\sin 2\beta$ from TD *CP* asymmetries in $B^0 \to \bar{D}^{(*)}_{CP} h^0$ decays by a simultaneous analysis of their data sets (for a total of about 1.24×10^9 $B\bar{B}$ pairs). The light meson h^0 is a π^0, η or ω, the D^0 meson is reconstructed in two-body decays to the *CP* eigenstates K^+K^-, $K^0_S\pi^0$ and $K^0_S\omega$, and, if present, the D^{*0} is reconstructed in the decay $D^{*0} \to D^0\pi^0$. The experimental sensitivity to β of this process is limited by the small D^0-meson's branching fraction to *CP* eigenstates and by the presence of large background processes, the most important of which comes from $e^+e^- \to q\bar{q}$ $(q = u,d,s,c)$ continuum events. A yield of 508 ± 31 and 757 ± 44 signal events is obtained for *BABAR* and Belle, respectively. The reconstructed beam-constrained masses[2] of the B^0 candidates for both *BABAR* and Belle samples are shown in Fig. 3.

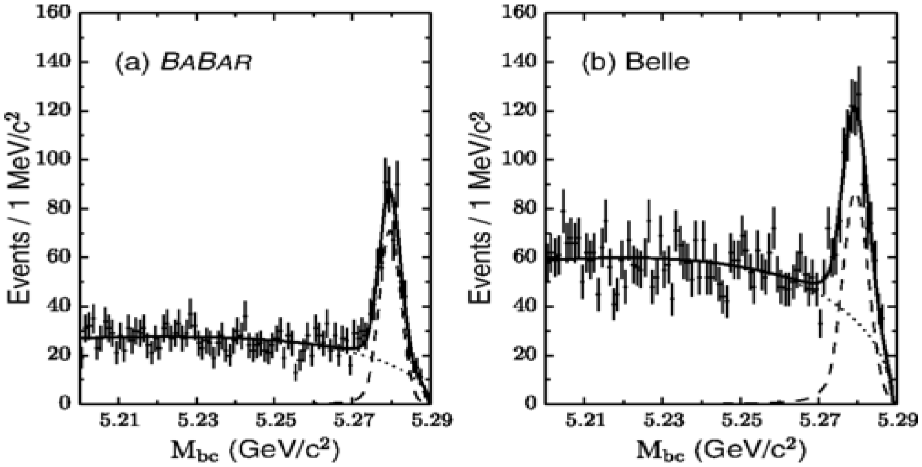


Figure 3: Beam-constrained mass distributions (data points with error bars) and fit projections (solid line) of $B^0 \to \bar{D}^{(*)}_{CP} h^0$ decays for (a) *BABAR* and (b) Belle. The dashed (dotted) lines represent projections of the signal (background) fit components [9].

The *CP*-violation parameters are extracted by a simultaneous extended unbinned maximum likelihood fit to the reconstructed flavor-tagged proper-decay-time distributions of both experiments:

$$\begin{aligned} -\eta_{f_{CP}} S &= +0.66 \pm 0.10(\text{stat.}) \pm 0.06(\text{syst.}), \\ C &= -0.02 \pm 0.07(\text{stat.}) \pm 0.03(\text{syst.}). \end{aligned} \tag{4.1}$$

This is the first observation of *CP* violation in $B^0 \to \bar{D}^{(*)}_{CP} h^0$ decays. The results are in good agreement with the world average value $\sin 2\beta = 0.69 \pm 0.02$ obtained from $b \to c\bar{c}s$ transitions [10], though the error is still much larger. With a data sample about 40 times larger planned for Belle II, the uncertainty is expected to scale down to $\Delta(\sin 2\beta) \leq 0.02$, that is at a similar level of the current world-average uncertainty.

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[2] The definition of the kinematic variables used in time-dependent analyses by *BABAR* and Belle can be found in [9]

4.2 Model-independent measurement of β in $B^0 \to \bar{D}^{(*)0}h^0$, $\bar{D}^0 \to K_S^0\pi^+\pi^-$

Belle reports on a model-independent measurement of the angle β from $B^0 \to \bar{D}^{(*)0}h^0$ decays with subsequent $\bar{D}^0 \to K_S^0\pi^+\pi^-$ decays. The method is based on a binned Dalitz-plot analysis applied to the time-dependent *CP*-asymmetry measurements.

Neglecting the small contribution from the suppressed $b \to u\bar{c}d$ transition, as well as charm mixing and possible *CP* violation in *D*-meson decays, the decay amplitude of the process $B^0 \to \bar{D}^{(*)0}h^0$ can be factorized as $A_f = \alpha_B \bar{A}_D$, and that of $\bar{B}^0 \to D^{(*)0}h^0$ as $\bar{A}_f = \alpha_B \xi_{h^0}(-1)^L A_D$, where ξ_{h^0} is the *CP* eigenvalue of the h^0 meson, *L* is the relative orbital angular momentum in the $\bar{D}^{(*)0}$ h^0 system, A_D ($\bar{A}_D$) is the $D^0 \to K_S^0\pi^+\pi^-$ ($\bar{D}^0 \to K_S^0\pi^+\pi^-$) decay amplitude, and α_B is a complex coefficient. The B^0 decay-time probability contains a term proportional to $\mathrm{Im}(\lambda_f)\sin(\Delta m \Delta t)$, with $\lambda_f = (q/p)(\bar{A}_f/A_f)$. As there is no *CP*-violating weak phase in the B^0 decay under study, it results:

$$\arg\left(\frac{\bar{A}_f}{A_f}\right) = \Delta\delta_f \implies Im(\lambda_f) = \left|\frac{\bar{A}_f}{A_f}\right| \sin(\Delta\delta_f - 2\beta); \tag{4.2}$$

here, $\Delta\delta_f$ is the strong-phase difference, which does not change sign under *CP* transformation. The three-body state $K_S^0\pi^+\pi^-$ is not a *CP* eigenstate; in this case the phase difference $\Delta\delta_f$ can assume any value and the time-dependent *CP* asymmetry is sensitive to both $\sin 2\beta$ and $\cos 2\beta$, allowing to resolve the ambiguity on β from the measurement of only $\sin 2\beta$.

The D^0 decay amplitude can be written as a function of the Dalitz-plot variables $m_\pm^2 = m^2(K_S^0\pi^\pm)$: $A_D(m_+^2, m_-^2)$, with $\bar{A}_D(m_+^2, m_-^2) = A_D(m_-^2, m_+^2)$ obtained by transposing the Dalitz variables. Therefore, $\Delta\delta_f$ is also a function of the Dalitz variables, and depends on the strong-phase difference between A_D and $\bar{A}_D$, $\Delta\delta_D(m_+^2, m_-^2)$.

Previous measurement from BABAR [11] and Belle [12] of this decay mode employed an approach based on an isobar model of the *D*-meson decay amplitude proposed in Ref. [13]. The new Belle measurement [14] is based instead on the binned Dalitz distribution approach [15] originally proposed to extract the CKM angle γ from $B \to DK$ decays. Here, the method is applied to the time-dependent analysis of $B^0 \to \bar{D}^{(*)0}h^0$ decays to measure the angle β. The Dalitz plot is divided in 16 bins symmetric with respect to $m_+^2 \leftrightarrow m_-^2$ exchange. The B^0 decay-time distribution is then written in terms of a set of parameters of the Dalitz plot for each bin. These parameters are K_i, the probability for the D^0 meson to decay into the phase space region of the Dalitz-plot bin "*i*", and C_i and S_i, the weighted averages over the bin "*i*" of $\cos\Delta\delta_D$ and $\sin\Delta\delta_D$. The binning and the $D^0 \to K_S^0\pi^+\pi^-$ decay amplitude model follow those reported in Ref. [16], see Fig. 4. The K_i parameters are extracted from a sample of $B^+ \to \bar{D}^0\pi^+$ followed by $D^0 \to K_S^0\pi^+\pi^-$, by measuring the signal yield in each bin. The C_i and S_i parameters are instead obtained from studies of coherent decays of $D^0\bar{D}^0$ pairs performed by CLEO-c [17].

Six $B^0 \to \bar{D}^{(*)0}h^0$ decay modes are used in Belle analysis: $\bar{D}^0\pi^0$, $\bar{D}^0\eta$, $\bar{D}^0\eta'$, $\bar{D}^0\omega$, $\bar{D}^{*0}\pi^0$ and $\bar{D}^{*0}\eta$. For each mode, the signal yield and the background composition of the selected samples are obtained by an extended unbinned maximum likelihood fit to the $(\Delta E, M_{bc})$ two-dimensional distribution. The signal fractions range between 44% and 72%. The selected events with a B^0-candidate within a 2D $(\Delta E, M_{bc})$ signal region optimized for each mode are then used for the

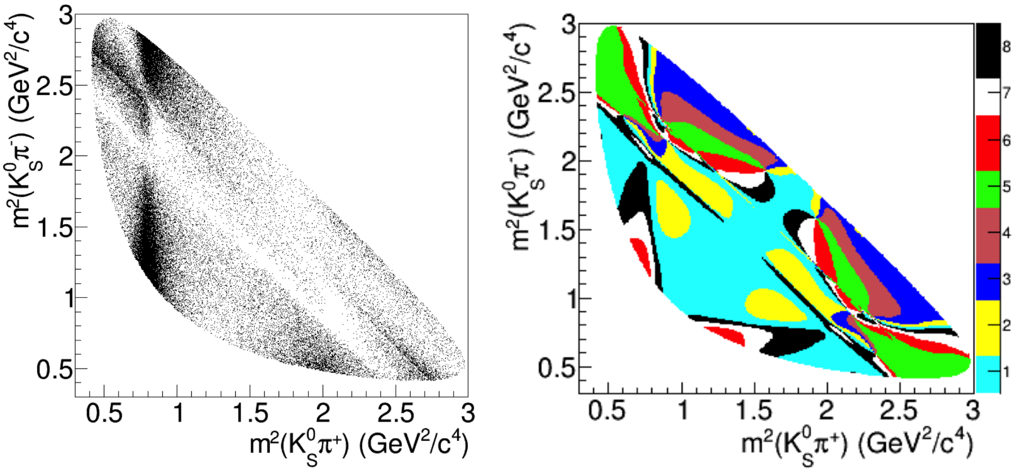

Figure 4: Dalitz plot distribution (left) and equal-phase binning (right) obtained with the amplitude model of $D^0 \to K^0_S\pi^+\pi^-$ decay from Ref. [16].

time-dependent fit over the Dalitz plot. The results of the combined fit for all modes are:

$$\begin{aligned} \sin 2\beta &= +0.43 \pm 0.27(\text{stat.}) \pm 0.08(\text{syst.}), \\ \cos 2\beta &= +1.06 \pm 0.33(\text{stat.}) \,^{+0.21}_{-0.15}(\text{syst.}), \\ \beta &= (11.7 \pm 7.8(\text{stat.}) \pm 2.1(\text{syst.}))^\circ. \end{aligned} \tag{4.3}$$

The value $\sin 2\beta = 0.69 \pm 0.02$ measured in $b \to c\bar{c}s$ decays corresponds to two possible solutions for β in the angular range $[0^\circ, 180^\circ]$. The result of this analysis excludes at 5.1 standard deviations the solution $\beta = 68.1^\circ$, while it agrees with the solution $\beta = 21.9^\circ$ (that is for positive $\cos 2\beta$), at about 1.3σ.

The measurement is statistically limited, and therefore it is expected to be greatly improved at Belle II. The largest systematic uncertainty comes from the knowledge of the parameters C_i and S_i, which could be improved by analyzing the large sample of coherently-produced $D^0\bar{D}^0$ pairs collected by the BES-III experiment.

4.3 Constraints on penguin contribution from $b \to c\bar{c}d$ transitions

The $b \to c\bar{c}d$ transitions are equivalent to $b \to c\bar{c}s$ transitions in terms of the weak phase of the decay. However, unlike $b \to c\bar{c}s$, here the CKM factors of the sub-leading penguin diagrams with different weak phase are of the same order $\mathcal{O}(\lambda_c^3)$ as the tree diagram. The measurement of time-dependent *CP* asymmetries may therefore result in an altered value of $\sin 2\beta$, already within the SM framework. Applying SU(3) flavor-symmetry arguments certain $b \to c\bar{c}d$ decays can be used to constrain the shift of the phase β caused by possible penguin contributions in the corresponding $b \to c\bar{c}s$ decays. For example, from the measured asymmetries in $B^0 \to J/\psi\pi^0$ decay performed by the BABAR and Belle collaborations, constraints on the effect of penguin contributions on $\sin 2\beta$ measured in $B^0 \to J/\psi K^0_S$ have been derived by the authors of Refs. [18, 19].

Very recently, Belle measured the branching fraction of the $B^0 \to \psi(2S)\pi^0$ decay [20]. If the $b \to c\bar{c}s$ decays are experimentally very clean, the corresponding $b \to c\bar{c}d$ decays are instead affected by a significant background, consisting of continuum $e^+e^- \to q\bar{q}$ production and of other B decays with a J/ψ in the final state. The $B^0 \to \psi(2S)\pi^0$ branching fraction is extracted from an unbinned extended maximum likelihood to the M_{bc} and ΔE distributions. A total of 85 ± 12 signal events are found for a branching fraction $\mathscr{B}(B^0 \to \psi(2S)\pi^0) = (1.17 \pm 0.17(\text{stat.}) \pm 0.08(\text{syst.})) \times 10^{-5}$. It is the first observation, with a significance of 7.2σ, for this decay mode. The data sample available at the present B factories is not large enough to perform a significant time dependent analysis of this decay mode and use it to constrain the penguin contribution in $B^0 \to \psi(2S)K^0_S$. These modes with a π^0 in the final state are not accessible to LHCb. Instead, not only $B^0 \to J/\psi\pi^0$ but also $B^0 \to \psi(2S)\pi^0$ looks very promising for Belle II.

LHCb reported in 2015 a study of $B^0 \to J/\psi\pi^+\pi^-$ decays [21] based on the 3 fb^{-1} data previously described. The composition of the $\pi^+\pi^-$ system in this final state has been studied in a previous work [22]. The dominant channel is $B^0 \to J/\psi\rho^0$, which is mainly a *CP*-even final state. Because of SU(3) symmetry among light vector mesons, *CP*-violation measurements in $B^0 \to J/\psi\rho^0$ can put stringent constraints on the penguin contribution to the $B_s - \bar{B}_s$ mixing phase ϕ_s, measured in $B_s \to J/\psi\phi$ decays, whose leading amplitude is a $b \to c\bar{c}s$ tree diagram.

From the time-dependent *CP* analysis, the phase measured for the $B^0 \to J/\psi\rho^0$ component is $2\beta^{\text{eff}}_{J/\psi\rho} = (41.7 \pm 9.6((\text{stat.})^{+2.8}_{-6.3}(\text{syst.}))^\circ$. This value is consistent with the 2β value measured in $b \to c\bar{c}s$ transitions. As the latter is, in first approximation, unaffected by penguin amplitudes, their difference is also a measurement of the phase shift in $B^0 \to J/\psi\rho^0$ due to the penguin contribution. It results:

$$\Delta(2\beta_{J/\psi\rho}) = 2\beta^{\text{eff}}_{J/\psi\rho} - 2\beta_{c\bar{c}s} = (-0.9 \pm 9.7(\text{stat.})^{+2.8}_{-6.3}(\text{syst.}))^\circ. \tag{4.4}$$

The penguin amplitude in $b \to c\bar{c}s$ transitions is Cabibbo-suppressed by a factor $\varepsilon = \lambda_C^2/(1-\lambda_C^2) \simeq 0.0534$ with respect to the tree amplitude, while such suppression is missing in the $b \to c\bar{c}d$ transitions. Therefore, even if the above estimate of $\Delta(2\beta_{J/\psi\rho})$ has a quite large uncertainty, the constraint derived for the SU(3) related process is much more stringent. LHCb estimates a limit on the shift of the phase ϕ_s measured in $B_s \to J/\psi\phi$ of about $\pm 1^\circ$ with 95% confidence level [21]. This limit is consistent with the theoretical predictions [23]. Even if there is no strict flavor symmetry relating $B^0 \to J/\psi\rho^0$ to $B^0 \to J/\psi K^0_S$, similar limits may be expected also for this channel.

5. Measurement of the angle α

The angle α can be extracted from the measurement of time-dependent *CP* asymmetries of B^0 decays to *CP* eigenstates governed by $b \to u$ transitions, with a procedure analogous to that for the measurement of $\sin 2\beta$. Neutral B decays to a pair of light mesons are suitable to this purpose. However, in those decays there might be a significant contribution to the final amplitude from penguin diagrams with different weak phase. As a consequence, the decay could be subjected to direct *CP* violation, that is $C \neq 0$ in Eq.(2.1), and the measured angle could be shifted to $\alpha^{\text{eff}} = \alpha + \Delta\alpha$, so that the observed CP-violation parameters would be related by $S = \sqrt{1-C^2}\,\sin 2\alpha^{\text{eff}}$.

Several complementary methods have been developed to estimate the phase shift $\Delta\alpha$. In particular, a method based on isospin analysis of $B^0 \to \pi\pi$ and $B^0 \to \rho\rho$ decays. The following isospin relations can be written among the amplitudes of B-meson and $\bar{B}$-meson decays to the different $\pi\pi$ or $\rho\rho$ charge combinations [24]:

$$\begin{aligned} A^{+-}/\sqrt{2} + A^{00} &= A^{+0} \\ \bar{A}^{+-}/\sqrt{2} + \bar{A}^{00} &= \bar{A}^{+0}, \end{aligned} \tag{5.1}$$

where $A^{ij} = A(B \to h^i h^j)$, $i, j = +$ or $-$, and $h = \pi, \rho$. Each relation can be represented as a triangle in a complex plane, as shown in Fig. 5. The relative sizes and phases of each amplitude can be extracted from the complete isospin analysis of the three decay rates and corresponding CP asymmetries. The angle between the sides of length proportional to A^{+-} and $\bar{A}^{+-}$ is the phase shift $2\Delta\alpha$. Experimentally, the complete isospin analysis of the $B \to \pi\pi$ is complicated by the need to measure the time-dependent asymmetry of the decay $B^0 \to \pi^0\pi^0$, which is not feasible with the statistic of the present B factories. This analysis is one of the challenges for the Belle II physics program.

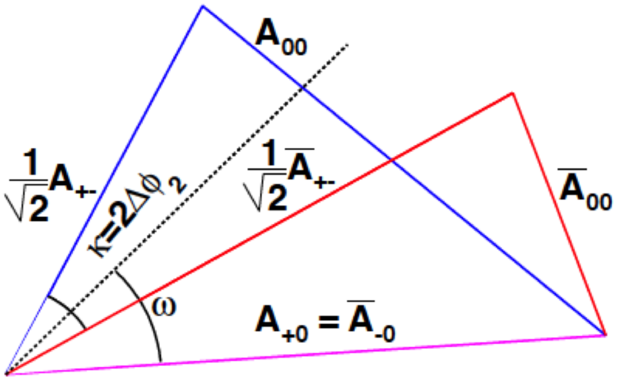

Figure 5: Sketch of the isospin triangles for B and $\bar{B}$ decays into light-hadron final states with isospin $I = 1$.

The isospin analysis of the $B \to \rho\rho$ system is instead complicated by the vector-vector nature of the final state. It is a superposition of three helicity amplitudes: one longitudinal and two transverse amplitudes, with CP-even and CP-odd components. However, the longitudinal (CP-even) component is largely dominant, as confirmed by previous measurements [25, 26], and the isospin analysis can be limited to the fraction of longitudinally polarized events.

As none of the methods and the decay modes investigated can yet provide a precise determination of α, the information from all of them are combined in global fits. The BaBar and Belle collaborations have combined their results on $B \to \pi\pi$, $B \to \rho\rho$ and $B \to \pi^+\pi^-\pi^0$ to obtain [27]:

$$\alpha = (88 \pm 5)^\circ, \tag{5.2}$$

which is consistent with the SM predictions.

5.1 Measurement of α in $B^0 \to \rho^+\rho^-$ decays by Belle

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Belle has recently performed a new study of $B^0 \to \rho^+\rho^-$ decays analyzing the full data set of about 772×10^6 $B\bar{B}$ pairs [28]. They use the helicity basis to separate the different helicity amplitudes, and perform an extended unbinned maximum likelihood fit to the distributions of the discriminating kinematic and angular variables and to the Δt distribution, to extract simultaneously the branching fraction $\mathscr{B}(B^0 \to \rho^+\rho^-)$, the longitudinal polarization of the ρ mesons f_L, and the CP-violation parameters S and C. The latter are measured only for decays into longitudinally polarized ρ mesons. A total of 1754 ± 94 and 21 ± 22 signal $B^0 \to \rho^+\rho^-$ events are selected with longitudinal and transverse polarization, respectively, from which the following results are obtained:

$$\begin{aligned}
\mathscr{B}(B^0 \to \rho^+\rho^-) &= (28.3 \pm 1.5(\text{stat.}) \pm 1.5(\text{syst.})) \times 10^{-6}, \\
f_L &= 0.988 \pm 0.012(\text{stat.}) \pm 0.023(\text{syst.}), \\
C &= +0.00 \pm 0.10(\text{stat.}) \pm 0.06(\text{syst.}), \\
S &= -0.13 \pm 0.15(\text{syst.}) \pm 0.05(\text{syst.}).
\end{aligned} \tag{5.3}$$

The isospin analysis is then performed using previous Belle measurement of the branching fractions and longitudinal polarization of $B^+ \to \rho^+\rho^0$ and $B^0 \to \rho^0\rho^0$ decays. The χ^2 distributions from the individual constraints are combined to determine the $1 - \text{C.L.}$ value as a function of α (Fig. 6(a)).

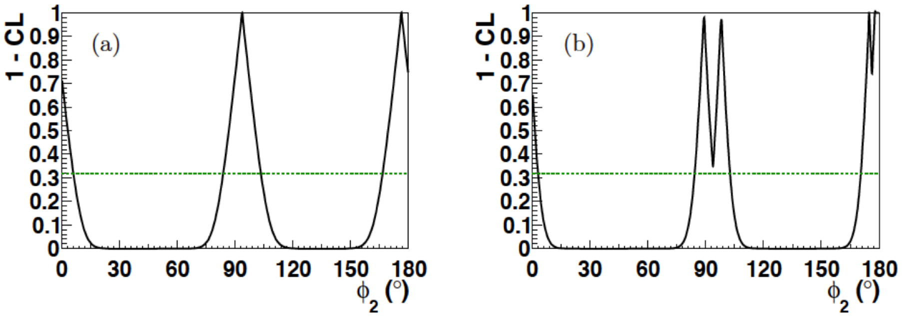

Figure 6: $1 - \text{C.L.}$ versus $\phi_2 = \alpha$ obtained from $B^0 \to \rho^+\rho^-$ decays [28]: (a) isospin analysis, (b) SU(3) flavor analysis. The horizontal line shows the 68% C.L.

Two solutions for α are found, the one that is consistent with other SM-based constraints yields:

$$\alpha = (93.7 \pm 10.6)^\circ. \tag{5.4}$$

It is worth to notice that the shift on α due to penguin contributions is consistent with zero: $\Delta\alpha = (0.0 \pm 9.6)^\circ$. An alternative method based on SU(3) symmetry relating $B^0 \to \rho^+\rho^-$ with $B^+ \to K^*\rho^+$ yield consistent results, shown in Fig. 6(b). The solution most compatible with other SM-based constraints is

$$\alpha = (89.3 \pm 4.8\ ^{+1.0}_{-3.4})^\circ, \tag{5.5}$$

where the first error comes from the scan procedure, and the second is due to SU(3)-breaking uncertainties.

5.2 Measurement of $B^0 \to \rho^0\rho^0$ decays by LHCb

One open issue regarding the extraction of α from $B \to \rho\rho$ is the inconsistency between *BABAR* [29] and Belle [30] measurement of the longitudinal polarization in $B^0 \to \rho^0\rho^0$ at about 2σ level, as reported in Table 1.

	BABAR	Belle
$\mathscr{B}(B^0 \to \rho^0\rho^0) \times 10^6$	$0.92 \pm 0.32 \pm 0.14$	$1.02 \pm 0.30 \pm 0.15$
f_L^{00}	$0.75^{+0.11}_{-0.14} \pm 0.05$	$0.21^{+0.18}_{-0.22} \pm 0.15$

Table 1: Branching fraction and longitudinal polarization measured by *BABAR* [29] and Belle [30] for $B^0 \to \rho^0\rho^0$ decay mode.

It would be advisable to have additional measurements to improve the knowledge of these parameters. LHCb used the 3.0 fb^{-1} of collected data to study the $B^0 \to \rho^0\rho^0$ decay mode [31]. More than 600 B^0 signal decays are selected and used to perform an amplitude analysis, under the assumption of no *CP* violation in the decay, from which the $B^0 \to \rho^0\rho^0$ decay is observed with a significance of 7.1 standard deviations. The measured longitudinal polarization and branching fraction are:

$$\begin{aligned}\mathscr{B}(B^0 \to \rho^0\rho^0) &= (0.94 \pm 0.17(\text{stat.}) \pm 0.09(\text{syst.}) \pm 0.17(BF)) \times 10^{-6},\\ f_L^{00} &= 0.745^{+0.048}_{-0.058}(\text{stat.}) \pm 0.034(\text{syst.}).\end{aligned} \tag{5.6}$$

The last error (indicated as BF) of the branching fraction determination accounts for the uncertainty on the branching fraction of the reference decay mode $B^0 \to K^*\phi$, used for normalization. The measured value of f_L^{00} is in very good agreement with the *BABAR* value, while it differs by 2.3σ from the value obtained by Belle.

6. Summary

The precision reached by the e^+e^- B factories experiments, *BABAR* and Belle, have reached a precision of about 3% on $\sin 2\beta$, in the Cabibbo-favored $B \to$ charmonium decays. LHCb also measured $\sin 2\beta$ from time-dependent analysis of $B^0 \to J/\psi K^0_S$, reaching a competitive precision. It is therefore predictable a significant improvement with the future data by LHCb at the LHC, as well as by the Belle II experiment at the SuperKEKB collider under construction. At a precision of less than 1% on $\sin 2\beta$ the measurement could be sensitive to possible contributions from loop processes, either from the SM or beyond it. It is therefore fundamental to perform complementary measurements to establish additional firm references for the SM value of $\sin 2\beta$ and to constrain the effect of the loop contributions on $b \to c\bar{c}s$ transitions. We presented here some recent measurements of the *CP*-violation parameters in $b \to c\bar{c}d$ and $b \to c\bar{u}d$ processes. Important results have already been obtained, showing the effectiveness of the developed analysis techniques, which could be fully exploited with the data sets expected for Belle II and LHCb.

A precision of better than 5° on the determination of the angle α has already been obtained at the B factories, despite the much higher theoretical and experimental difficulties with respect to measure β. The most effective method to extract α is the isospin analysis of $B \to \rho\rho$ decays. Belle has recently performed a new study of the $B^0 \to \rho^+\rho^-$ decay with a simultaneous measurement of the *CP*-violation parameters, the longitudinal polarization, and the branching fraction. LHCb instead obtained the first observation of the more rare decay $B^0 \to \rho^0\rho^0$ and measured the longitudinal polarization of the final state. Also for α significant improvements are expected in the near future at LHCb, and in particular at Belle II, which can precisely measure all the interesting channels, while decays as $B^0 \to \pi^0\pi^0$ and $B^0 \to \rho^+\rho^-$ are hardly measurable at LHCb because of the presence of neutral pions in the final state. An uncertainty from global fits of all experimental inputs of $\delta\alpha \sim 1^\circ$ should be under reach.

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