Calculation of ε'/ε on the lattice

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The calculation of ε' within the Standard Model has been a critical goal of particle physics since its definitive measurement sixteen years ago. The small size of this quantity makes it extremely sensitive to new Beyond the Standard Model sources of CP violation that may help to explain the origin of the matter/antimatter asymmetry in the Universe. Given the highly non-perturbative character of this quantity, lattice QCD is the favored approach. After more than six years of development, a successful lattice approach has emerged based on the Lellouch-Lüscher finite volume relation and G-parity boundary conditions. We discuss our first complete, realistic calculation of ε', published in PRL one year ago [1].

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One of the foremost goals of modern particle physics is understanding the origin of the matter-antimatter asymmetry in the Universe. A necessary ingredient [2] is CP-violation: the breaking of the combined fundamental charge conjugation and parity symmetries. In practice, the amount of CP-violation in the Standard Model is insufficient (by several orders of magnitude) to account for the observed asymmetry, hence there is a strong motivation to search for physics beyond the Standard Model (BSM) by studying CP-violating processes. In particular, the direct violation of CP in particle decays gains additional contributions in most BSM theories and offers a particularly attractive avenue for the search for new physics.

Historically, direct CP-violation was first observed in decays of a CP-odd kaon to the CP-even $\pi\pi$ state, where it is parametrized by the quantity ε'. From experiment, one measures

$$\mathrm{Re}(\varepsilon'/\varepsilon) \approx \frac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) \tag{1}$$

where

$$\eta_{q_1 q_2} = \frac{A(K_L^0 \to \pi^{q_1}\pi^{q_2})}{A(K_S^0 \to \pi^{q_1}\pi^{q_2})}. \tag{2}$$

Note $\varepsilon = (2\eta_{+-} + \eta_{00})/3 \approx 2.23 \times 10^{-3}$ is a measure of indirect CP-violation, and its magnitude is known experimentally to 0.5% precision. Indirect CP-violation in $K \to \pi\pi$ decays was first measured [3] at BNL in 1964, a result for which Cronin and Fitch were awarded the Nobel Prize in 1980. More recently (~ 2002), combined results from FNAL and CERN give [4]

$$\mathrm{Re}(\varepsilon'/\varepsilon) \approx 1.66(23) \times 10^{-3}. \tag{3}$$

In the underlying process, a neutral kaon can decay into either an isospin $I = 0$ ($\Delta I = 1/2$) $\pi\pi$ state or an $I = 2$ ($\Delta I = 3/2$) $\pi\pi$ state, for which the corresponding amplitudes are A_0 and A_2, respectively. Direct CP-violation manifests as a difference between the complex phases of these amplitudes:

$$\varepsilon' = \frac{i\omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2}}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right), \tag{4}$$

where δ_I are the s-wave $\pi\pi$-scattering phase shifts and $\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$.

Although the underlying weak interactions involved in these decays occur at very short distances, the hadronic interaction observed in experiments receives substantial corrections from non-perturbative QCD effects. For example, these contributions (as we discuss further below) are mainly responsible for the so-called $\Delta I = 1/2$ rule: an $\mathcal{O}(20\times)$ enhancement of A_0 over A_2. As the QCD coupling is large in this regime, the usual perturbative techniques for studying weakly coupled theories cannot be used. Lattice QCD offers the only known, systematically improvable technique - one for which all of the errors are controllable and can be improved with sufficient computational investment - for studying QCD in this regime.

It is only in recent years that theoretical and computational advances have opened the door to measuring $K \to \pi\pi$ decays directly on the lattice. Our collaboration has now calculated both the A_2 and A_0 amplitudes, and therefore ε'/ε. In this document we briefly summarize the salient details of these calculations, discuss our results and provide and the outlook for future work.

1. General strategy

1.1 Lattice simulations

Euclidean Green's functions of an operator $\mathcal{O}$ can computed using Feynman's path integral on a discretized space-time as

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}U \, \mathcal{O}(U) \exp(-S[U])}{\int \mathcal{D}U \, \exp(-S[U])} \tag{1.1}$$

where $\mathcal{D}U$ is the Haar measure for the QCD gauge fields, which on a discretized space-time are represented as SU(3) matrices residing between lattice sites (gauge links). Here S is the effective action obtained by integrating out the fermionic degrees of freedom from the path integral; this is done to avoid using anticommuting Grassman variables on a computer. Using Monte Carlo techniques, one can importance sample the probability distribution

$$P(U) = \frac{\exp(-S[U])}{\int \mathcal{D}U \, \exp(-S[U])}, \tag{1.2}$$

resulting in an ensemble of samples U_i of the gauge fields from which $\langle \mathcal{O} \rangle$ can be approximated as

$$\langle \mathcal{O} \rangle \approx \frac{1}{N} \sum_{i=1}^{N} O(U_i). \tag{1.3}$$

In this setup, the operator $\mathcal{O}(U)$ comprises quark propagators computed as the inverse of the fermionic Dirac matrix between the relevant sites. Given an operator comprising quark field operators and gauge links, the appropriate form can be obtained simply by Wick contracting the fermion fields.

In order to calculate a process involving particular particles, one must formulate operators that transform appropriately under the various lattice symmetries. For example, pion states can be created using pseudoscalar bilinear operators of the form $\bar{q}\gamma^5 q$ with appropriate quark fields. Such operators in fact generate all states with the corresponding quantum numbers, and the particles of interest can only be extracted by considering the Euclidean time dependence: The contribution to the Green's function of any eigenstate of the Hamiltonian decays exponentially in the energy of that state, and therefore interactions involving states close in energy to the ground state can be extracted by performing multi-exponential fits to the large time dependence. In practice the contributions of states with energy larger than the ground state are typically very noisy and therefore it is highly beneficial to work only with ground-state particles wherever possible.

Lattice calculations are necessarily performed in a finite box with some choice of boundary conditions on the fermion and gauge fields. This has the immediate effect of discretizing the allowed momenta of lattice states. For periodic boundary conditions, $\psi(x+L) = \psi(x)$ where L is the lattice spatial size, only momenta that are integer multiples of $2\pi/L$ are allowed. Similarly, with antiperiodic boundary conditions, $\psi(x+L) = -\psi(x)$, only odd-integer multiples of π/L are allowed. There are also secondary finite volume effects that occur because the pion cloud surrounding every particle is distorted by the finite volume (and its hypercubic geometry); these effects are exponentially suppressed in the lattice volume for the majority of cases, and can be ignored. For

interacting multi-particle states there are important finite-volume effects beyond naïve momentum quantization which require special treatment (cf. below).

1.2 $K \to \pi\pi$ on the lattice

Given the large difference in energy between the weak scale ($\mathscr{O}(80)$ GeV) and the hadronic scale (a few hundred MeV), the underlying weak process responsible for the decay can be very precisely described by the $\Delta S = 1$ weak effective Hamiltonian,

$$H_W^{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} [z_i(\mu) + \tau y_i(\mu)] Q_i(\mu), \qquad (1.4)$$

where z and y are c-number Wilson coefficients, $\tau = -V_{ts}^* V_{td}/V_{ud} V_{us}^*$, V_{ij} are CKM matrix elements, and Q_i are a set of 10 dimension-six four-quark operators. Both the Wilson coefficients and the operators are renormalization scheme dependent, as indicated by their dependence upon the renormalization scale μ, but their product is not. One important aspect of the lattice calculation is renormalizing the bare lattice operators into the same scheme as the perturbatively computed Wilson coefficients, typically the $\overline{\text{MS}}$ scheme. We accomplish this without resorting to the use of perturbative QCD at the hadronic scale by using an intermediate 'regularization-invariant MOM scheme' (RI-MOM) [5, 6] with which we can non-perturbatively run to a high energy scale where we can legitimately match to $\overline{\text{MS}}$ using continuum perturbation theory.

Note that the determination of the Wilson coefficients of this effective Hamiltonian utilizes perturbation theory to integrate out the contributions of the charm quark. While we believe the errors this introduces to be small, future calculations would benefit from the inclusion of a fully-dynamical charm quark. Note also that our calculations are performed with exact isospin and without electromagnetism; the effects of these phenomena are expected to enter at the percent level and are therefore significantly smaller than our present total errors. In future work we intend to investigate these contributions.

The kaon and $\pi\pi$ states are generated by the appropriate combinations of quark field operators as discussed above. For the $\pi\pi$ state we must take care to use operators that reside in the appropriate representation of SU(2) isospin and that project only onto the s-wave angular momentum state.

Ultimately then, we seek to compute matrix elements of the form:

$$\mathscr{M}_{\text{FV}}(\mu) = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} [z_i(\mu) + \tau y_i(\mu)] Z_{ij}(\mu) \langle \pi\pi | Q_j(\mu) | K \rangle, \qquad (1.5)$$

where Z_{ij} is the renormalization matrix relating the bare lattice operators to $\overline{\text{MS}}$, and the the subscript FV indicates that these are finite volume matrix elements. In practice these differ significantly from the infinite-volume decay amplitudes due to power-law finite-volume effects arising from the continuous interactions of the final state pions due to their close confinement by the typically small (in terms of the pion Compton wavelength) box. The appropriate correction is the Lellouch-Lüscher factor [7].

A key challenge to these calculations is ensuring physical kinematics: The ground-state of the $\pi\pi$ system comprises stationary pions (or the vacuum in the $I = 0$ case) and therefore has an energy substantially lower than the $\sim$500 MeV kaon mass. A correlation function describing a kaon

decaying to two stationary pions or the vacuum does not describe the physical, energy-conserving decay of interest. While the vacuum-state contribution can be isolated and explicitly subtracted, the state with two pions at rest cannot and will dominate the signal. As described above, one could attempt a multi-exponential fit to obtain the physical decay amplitude, which is an excited state contribution to the correlation function (for an appropriate L); such fits however typically suffer from large statistical noise, making a precise calculation difficult. This is particularly so for the $I = 0$ amplitude where additional noise arises from so-called 'disconnected' contributions (see below). We employ an alternative procedure, whereby we take advantage of the freedom to manipulate the quark boundary conditions and choose those that make the pions antiperiodic in space, such that their minimum momentum rises from zero to π/L where L can be tuned to match the kaon and $\pi\pi$ energies.

2. Calculation of A_2

The $\Delta I = 3/2$ amplitude A_2 can be computed directly via the following expression:

$$\langle (\pi^+\pi^0)^{I=2}|H_W|K^+\rangle = \sqrt{2}A_2 e^{i\delta_2} \tag{2.1}$$

where δ_2 is the $I = 2$ $\pi\pi$-scattering phase shift that we can compute directly on the lattice using the $I = 2$ $\pi\pi$ energy via Lüscher's formula [8]. With modern lattice techniques the computation of A_2 can be performed with percent-level statistical precision.

The imposition of physical kinematics is performed by applying antiperiodic spatial boundary conditions (APBC) to the *down* quarks in one or more directions. As a result, the charged pion states become antiperiodic also:

$$\begin{aligned}
\pi^+(x+L) &= [\bar{u}d](x+L) = -[\bar{u}d](x) = -\pi^+(x) \\
\pi^-(x+L) &= [\bar{d}u](x+L) = -[\bar{d}u](x) = -\pi^-(x).
\end{aligned} \tag{2.2}$$

This approach has two drawbacks: for the neutral pions, which enter the matrix element of interest, the down-quark momenta cancel and the ground-state remains stationary; and the formulation explicitly breaks the isospin symmetry by applying different boundary conditions to the two light quark flavors. Both of these issues can be circumvented by applying the Wigner-Eckart theorem to relate the above matrix element to an unphysical one,

$$\langle (\pi^+\pi^0)^{I=2}|Q^{\Delta I_z=\frac{1}{2}}|K^+\rangle = \frac{\sqrt{3}}{2}\langle (\pi^+\pi^+)^{I=2}|Q^{\Delta I_z=\frac{3}{2}}|K^+\rangle, \tag{2.3}$$

which contains only charged pions in a charge-2 final state which cannot mix with other representations by virtue of being the only charge-2 state in the system.

As the $I = 2$ final state cannot mix with the vacuum, it can be shown [9] that one can apply these APBC only in the valence sector (i.e during measurements and not when generating the ensemble of gauge fields) and incur only errors exponentially suppressed in the lattice volume. This significantly reduces the cost of employing this technique.

Our most recent calculation [10] was performed on two lattices with large physical volumes $\mathcal{O}((5\ \text{fm})^3)$, physical pion masses, and two different lattice spacings. This enables us to take
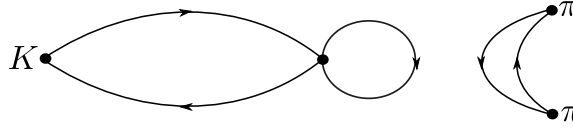
Figure 1: An example of a disconnected (type 4) diagram in the $K \to (\pi\pi)_{I=0}$ decay. The four-quark operator is in the center, and the self-contracted $\pi\pi$ "bubble" is on the right.

the continuum limit, improving on our earlier calculation [11]. Specifically we used the RBC & UKQCD $48^3 \times 96$ (48I) and $64^3 \times 128$ (64I) Möbius domain wall fermion ensembles with the Iwasaki gauge action at $\beta = 2.13$ and 2.25 respectively, corresponding to inverse lattice spacings of $a^{-1} = 1.730(4)$ GeV and $a^{-1} = 2.359(7)$ GeV. The full properties of these ensembles can be found in Ref. [12].

The simulated pion masses are $139.1(2)$ MeV and $139.2(3)$ MeV for the 48I and 64I respectively. We applied APBC to the down quark in all three spatial directions, resulting in $\pi\pi$ energies of $E_{\pi\pi}^{I=2} = 496.5(1.6)$ MeV and $507.0(1.6)$ MeV; comparing to the kaon masses of $m_K = 498.82(26)$ MeV and $507.4(4)$ MeV we observe excellent agreement indicating the decays are energy conserving. The slight mismatch between the the simulated pion and kaon masses and their physical values is included as a 4.5% systematic error on $\text{Re}(A_2)$ and 1.1% on $\text{Im}(A_2)$.

For the amplitude we obtained

$$\begin{aligned} \text{Re}(A_2) &= 1.50(4)(14) \times 10^{-8} \text{ GeV} \\ \text{Im}(A_2) &= -6.99(20)(84) \times 10^{-13} \text{ GeV}, \end{aligned} \tag{2.4}$$

where the errors are statistical and systematic respectively. The real part of this amplitude agrees well with the experimental value of $1.479(3) \times 10^{-8}$ GeV, whereas the imaginary part represents an entirely new Standard Model prediction.

For the $I = 2$ $\pi\pi$ scattering phase shift we obtain $\delta_2 = -11.6(2.5)(1.2)°$ (a corrected version of our continuum result [10]).

The systematic error in this calculation is dominated by the truncation at NLO of the perturbative matching between the intermediate RI-MOM schemes and $\overline{\text{MS}}$, and also the truncation error on the perturbative Wilson coefficients. We intend to reduce these errors in the future through the use of the step-scaling procedure to raise the renormalization scale in order to reduce the size of the missing NNLO and higher corrections.

3. Calculation of A_0

The $I = 0$ amplitude is determined by measuring the neutral kaon decays $K^0 \to \pi^+ \pi^-$ and $K^0 \to \pi^0 \pi^0$. Due to the vacuum quantum numbers of the final state, this calculation perforce involves the calculation of so-called "disconnected diagrams" where the $\pi\pi$ state annihilates into gluons and is recreated at a later time; an example of such a "type 4" diagram in this calculation is shown in Fig. 1. Such diagrams are typically very noisy as the fluctuations are independent of the separation of the disconnected parts.

A significant amount of the $\pi\pi$ vacuum coupling can be eliminated by separating the pion sources in Euclidean time, a refinement first introduced in Ref. [13]. Further suppression can be

achieved by combining the quark and anti-quark fields into "meson wavefunctions" constructed in our case from hydrogen atom wavefunctions. The remaining statistical error can be reduced by averaging over all spatial locations of the two-pion wavefunctions. These two steps are made possible by employing the all-to-all propagator technique of Ref. [14] whereby an approximation to the propagator from any site to any other is obtained by first computing a subset of exact eigenmodes of the Dirac matrix (900 in our case) using, for example, the Lanczos algorithm, and 'patching up' the remaining, typically less important, high-mode contributions using a stochastic technique. In this approach one can optionally 'dilute' the stochastic sources in the spin, color, and in our case flavor indices, in order to reduce the reliance on stochastic cancellation to correctly describe the index structure of the propagator on a given site, at the cost of increasing the number of inversions and the memory footprint. In practice we dilute in all three of these indices, and additionally dilute in the source time coordinate to improve the temporal resolution.

Unfortunately the strategy of using antiperiodic boundary conditions on the down-quark propagator in order to induce momentum in the ground-state pion and therefore assure an energy-conserving decay cannot be employed for the $\Delta I = 1/2$ calculation for two reasons: first, there is no Wigner-Eckart relation that can be used to remove the neutral pions in the second of the two decay channels; and second the breaking of the isospin symmetry cannot be avoided, making it difficult to extract the $\Delta I = 1/2$ contribution. As a result we chose to employ G-parity spatial boundary conditions [15, 16, 17] (GPBC) for this calculation.

G-parity is a symmetry of the QCD Lagrangian comprised of a charge conjugation followed by an isospin rotation of π radians about the y-axis. The charged and neutral pions are G-parity odd eigenstates hence the application of this operation at the lattice boundary is equivalent to imposing APBC on the pion states, giving them a ground-state momentum of π/L. At the quark level, G-parity transforms an up quark to an anti-down quark and the down quark to an anti-up quark. This flavor mixing at the boundary introduces a number of difficulties [18, 19], the most significant of which is the need to generate new ensembles in order to ensure that the sea pions that enter as intermediate states in the disconnected diagrams have the same behavior as their valence counterparts. An additional difficulty is a naïve factor of two in the computational cost of applying the Dirac operator, which translates to a large increase in the time to generate configurations and perform measurements.

We have recently published the first complete Standard Model calculation [1] of A_0. We used a $32^3 \times 64$ Möbius domain wall ensemble with the Iwasaki+DSDR gauge action at $\beta = 1.75$, corresponding to a somewhat coarse lattice spacing of $a^{-1} = 1.378(7)$ GeV but a large $(4.6 \text{ fm})^3$ spatial volume. This compromise resulted in reduced finite-volume errors at the cost of a larger discretization systematic. We employed G-parity boundary conditions in all three spatial directions, resulting in an $I = 0$ $\pi\pi$ energy of $E_{\pi\pi}^{I=0} = 498(11)$ MeV that closely matches the kaon mass $m_K = 490.6(2.4)$ MeV. The pion mass can be obtained by applying the continuum dispersion relation to the ground-state (moving) pion energy, giving $m_\pi = 143.1(2)$ MeV. We show the effective energies of the kaon and $\pi\pi$ state in Fig. 2.

The results presented in this section were obtained using 216 independent measurements, the generation of which required 200M Blue Gene/Q core hours.
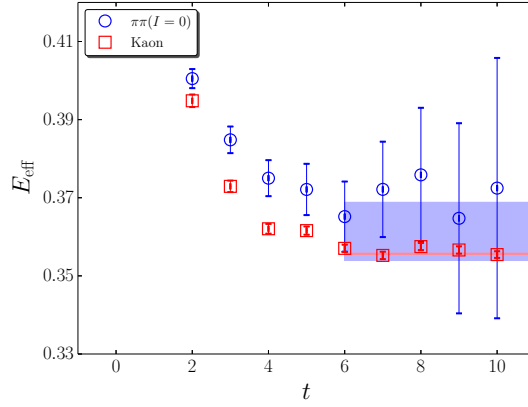
Figure 2: Effective energies of the kaon (squares) and two-pion (circles) states deduced from the corresponding two-point functions by equating the results from two time separations to the function $A \cosh E_{\text{eff}}(T/2 - t)$ where $T = 64$ is the temporal lattice size, plotted as a function of the smallest of those two separations. (We replace T by $T - 8$ for the $\pi\pi$ case.) These are overlaid by the errorbands corresponding to the fitted values of $E_{\pi\pi}$ (light blue) and m_K (pink).

We obtained the following results for the real and imaginary parts of A_0:

$$
\begin{aligned}
\text{Re}(A_0) &= 4.66(1.00)(1.21) \times 10^{-7} \text{ GeV} \\
\text{Im}(A_0) &= -1.90(1.23)(1.04) \times 10^{-11} \text{ GeV},
\end{aligned}
\tag{3.1}
$$

where the errors are again statistical and systematic, respectively. The real part of this amplitude agrees with the experimental value of $3.3201(18) \times 10^{-7}$ GeV, serving as a test of the method. The imaginary part once again represents an entirely new Standard Model prediction.

The 85% relative total error on our value for $\text{Im}(A_0)$ is significantly larger than that of the real part due to a 50%-level numerical cancellation between the dominant operator contributions:

$$
\begin{aligned}
\Delta[\text{Im}(A_0), Q_4] &= 1.82(0.62)(0.32) \times 10^{-11} \text{ GeV} \\
\Delta[\text{Im}(A_0), Q_6] &= -3.57(0.91)(0.24) \times 10^{-11} \text{ GeV}.
\end{aligned}
\tag{3.2}
$$

As part of this calculation we apply the Lüscher method [8] to obtain the $I = 0$ $\pi\pi$ scattering phase shift, for which we obtain

$$
\delta_0 = 23.8(4.9)(1.2)^\circ.
\tag{3.3}
$$

This value is $\sim 2.7\sigma$ lower than the value obtained from phenomenology via the Roy equations coupled with ChPT and/or experimental data [20, 21]. The origin of this discrepancy is presently not known; one possibility is that the rapidly degenerating signal-to-noise ratio of our $\pi\pi$ correlation function (Fig. 2) is masking excited state contamination. Another possibility is that the errors associated with the Roy equation procedure or of the high-energy $\pi\pi$-scattering data that enters the calculation are underestimated. Future higher-precision lattice measurements should shed light on this issue and also help us reduce the estimated 11% systematic error on the energy-dependence of this quantity which enters into the determination of the Lellouch-Lüscher factor.

Our result for A_0 is statistical error dominated; it is therefore of the highest priority to in-
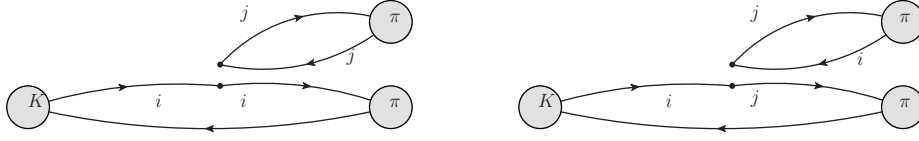
Figure 3: Dominant contractions contributing to Re(A_2) - C_1 (left) and C_2 (right). Here i and j are color indices and the associated four-quark operator is the current-current operator transforming as $(27,1)$ under $SU(3)_L \times SU(3)_R$.

crease the number of measurements, for which we aim to achieve a factor of 4 increase within the timescale of a year. As part of this effort we intend to replace the existing data, which are affected by a recently discovered error [22] in the generation of the ensembles whereby duplicate random number generator (RNG) seeds were used for the two light quark flavors separated by 12 sites in the y-direction, resulting in a persistent, unphysical correlation between gauge links separated by this distance. Through consideration of a number of observables we have concluded that the effects of this error are too small to affect our calculation; nevertheless the error damages our ability to claim an *ab initio* calculation, justifying our intention to replace the data.

Among the dominant systematic errors on this calculation are once again those associated with the truncation of the perturbative series in the computation of the Wilson coefficients and the $\overline{\text{MS}}$ matching in the renormalization factors, which are estimated to be 12% and 15% respectively. These errors are exacerbated here by the low, 1.53 GeV scale at which the underlying perturbative calculations are performed. We expect significant improvements through the use of step-scaling to a higher scale of $\mu = 2.3$ GeV, a calculation which is also currently underway.

We estimate the discretization error arising due to our use of a single lattice spacing to be 12%. Unfortunately the computational cost of generating an additional lattice spacing will likely require the next generation of supercomputers.

4. Result for ε'/ε

Combining our lattice determinations of Im(A_2), Im(A_0) and the $\pi\pi$ phase shifts with the precise experimental values for Re(A_2), Re(A_0) and ω we obtain

$$\text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \text{Re}\left\{\frac{i\omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0}\right]\right\} \tag{4.1}$$

$$= 1.38(5.15)(4.59) \times 10^{-4}, \tag{4.2}$$

This is 2.1σ below the experimental value $16.6(2.3) \times 10^{-4}$. While statistically insignificant, the difference between the lattice and experimental values offers a tantalizing hint of a tension that certainly justifies further study.

5. $\Delta I = 1/2$ **rule**

Another interesting result that can be obtained from these calculations is a non-perturbative
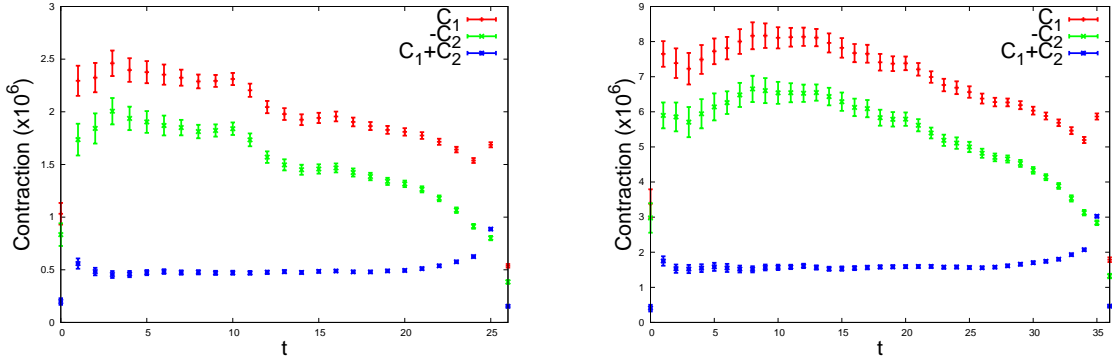
Figure 4: Cancellation of dominant contributions to $\text{Re}(A_2)$ on the 48^3 ensembles with a K - $\pi\pi$ separation of 27 and the 64^3 ensembles with separation 36.

QCD determination of the $\Delta I = 1/2$ rule. The experimental value for the ratio of the real parts of A_0 and A_2 is

$$\frac{\text{Re}(A_0)}{\text{Re}(A_2)} \simeq 22.5 \,. \tag{5.1}$$

The origin of the large difference between the two amplitudes has been a long-standing mystery. The perturbative Wilson coefficients by themselves suggest a factor two enhancement of $\text{Re}(A_0)$ [23, 24], but it was not known whether the remaining factor of ~ 10 arises due to non-perturbative QCD or from new physics. In our earlier calculations, and also in our most recent work, we observed a strong, 70% cancellation between the two main contractions (Fig. 3) of the dominant $(27, 1)$ operator contribution to $\text{Re}(A_2)$, giving rise to a strong suppression of this quantity relative to $\text{Re}(A_0)$. This cancellation is demonstrated graphically in Fig. 4 (reproduced from Ref. [10]). However, until a complete Standard Model determination of $\text{Re}(A_0)$ was performed, we were unable to demonstrate that this cancellation explains the $\Delta I = 1/2$ rule.

With our recent calculation of $\text{Re}(A_0)$ we can now compute the $\Delta I = 1/2$ rule directly from the lattice, for which we obtain

$$\frac{\text{Re}(A_0)}{\text{Re}(A_2)} \simeq 31.1(11.2) \,. \tag{5.2}$$

While the errors are still presently quite large, this result strongly suggests that the $\Delta I = 1/2$ rule can be explained by low-energy QCD.

For recent discussions of earlier lattice and phenomenological estimates concerning the $\Delta I = 1/2$ rule see Refs. [25, 26, 27].

6. Conclusions

We have detailed the first lattice determination of the direct CP violation parameter ε', computed via the $K \to \pi\pi$ amplitudes with $I = 2$ (A_2) and $I = 0$ (A_0) final states.

The calculation of A_2 has been performed with high statistical precision on two lattice spacings. The $\mathcal{O}(10\%)$ total error is dominated by the truncation of the perturbative series used to convert our lattice non-perturbative renormalization scheme to $\overline{\text{MS}}$ and also to compute the $\overline{\text{MS}}$

Wilson coefficients. These errors can be decreased in the future through the application of the step-scaling procedure to go to higher renormalization scales and to cross the charm threshold.

We have also performed the much more difficult calculation of A_0, albeit on a single, somewhat coarse lattice. The errors are presently dominated by statistics due to a combination of the presence of noisy disconnected diagrams and also, for the crucial $\text{Im}(A_0)$, a 50% cancellation between the dominant Q_4 and Q_6 contributions. Our main focus in the near term is to improve the statistical error, and we hope to achieve a factor of 4 increase in the number of measurements within a year. In addition to the systematic discretization error resulting from the coarse lattice spacing, we estimate that a comparable systematic error arises from the truncations of the perturbative series used for the renormalization and Wilson coefficients, here exacerbated by the low, 1.53 GeV scale at which we perform the RI $\rightarrow$ $\overline{\text{MS}}$ matching. We are already in the late stages of computing the non-perturbative step-scaling factors that will raise this scale to 2.3 GeV. Going to finer lattice spacings to reduce the discretization error remains a long-term goal.

Combining these results we obtain a value for $\text{Re}(\varepsilon'/\varepsilon)$ that is broadly in agreement (2.1σ) with the experimental number but may, with continued effort, reveal a discrepancy indicating new physics. We have also computed $\text{Re}(A_0)/\text{Re}(A_2)$, which is experimentally determined to be significantly larger than one (the $\Delta I = 1/2$ rule) due to a formerly unknown mechanism. We obtain a value consistent with the experimental number, from which we conclude that the $\Delta I = 1/2$ rule arises due to an observed strong cancellation between the dominant contributions to $\text{Re}(A_2)$ and not due to new physics.

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