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Neutron star structure explored with a family of unified equations of state of neutron star matter

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Neutron star structure, particularly the star's mass, radius, and crust, is investigated systematically by using unified equations of state (EOSs) of neutron star matter, namely, EOSs that meet the following conditions: (1) Unified description of matter in the crust and core based on the same EOS of nuclear matter with specific values of the incompressibility K_0 of symmetric nuclear matter and the parameter L that characterizes the density dependence of the symmetry energy. (2) Consistency of the masses and radii of stable nuclei calculated within the same theoretical framework with the empirical values. We systematically construct a family of such unified EOSs within the framework of the Thomas-Fermi theory. In this framework, we are allowed to connect the poorly known high density behavior of the EOS of neutron star matter, which is mainly controlled by L and K_0, to uncertain three-nucleon forces. With this EOS family, we calculate the neutron star structure and discuss its (K_0, L) dependence. The star's mass-radius relation is presented with caution because the central density of the star is significantly higher than the nuclear density except for low mass neutron stars.

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1. Introduction

We have been investigating how nuclear compressibility affects the structure of nuclei and neutron stars. The equation of state (EOS) is the key to it. In this study, we adopt a simple macroscopic nuclear model [1], construct a EOS family consistent with empirical data for stable nuclei [2], and then examine the structure of neutron-rich nuclei [2, 3, 4] and neutron stars [5].

This EOS family is called Oyamatsu-Iida, or simply OI, EOS family [2, 5]. It is a unified EOS family which properly connects laboratory nuclei to neutron star matter. Each EOS is labeled with the empirical uncertain saturation parameters: the incompressibility K_0, and the density gradient of symmetry energy L. We adopt a simplified Thomas-Fermi model description of nuclei [1], which was also used to construct Shen EOS for supernova matter [6, 7].

2. Saturation parameters of nuclear matter

Energy per nucleon of nearly symmetric nuclear matter at density n with proton fraction x is approximately given by [8]

$$w(n,x) \approx w_0 + \frac{K_0}{18n_0^2}(n-n_0)^2 + (1-2x)^2[S_0 + \frac{L}{3n_0}(n-n_0)]. \tag{2.1}$$

Here, the important saturation parameters of symmetric nuclear matter are the nuclear density n_0, saturation energy w_0, and incompressibility K_0. We have two important parameters for the density dependent symmetry energy $S(n)$. They are the symmetry energy S_0 and its density gradient L.

We can also consider the saturation point of asymmetric matter with fixed proton fraction x. The saturation density n_s and energy w_s are, up to the order of $(1-2x)^2$, given by

$$n_s = n_0 - \frac{3n_0 L}{K_0}(1-2x)^2, \quad w_s = w_0 + S_0(1-2x)^2. \tag{2.2}$$

The saturation point moves along the saturation curve with the proton fraction x. The slope of the saturation curve, y, given by

$$y = -\frac{K_0 S_0}{3n_0 L} \tag{2.3}$$

is a good measure of the asymmetry property of the EOS [9].

3. Macroscopic nuclear model

The energy of a unit cell of matter in the neutron-star crust or a laboratory nucleus, W, is composed of the nuclear energy W_N, relativistic electron energy and Coulomb energy.

The nuclear energy W_N is written as the integral of the local energy density,

$$W_N = \int d^3r \left[\varepsilon_0\left(n_n(r), n_p(r)\right) + F_0|\nabla n(r)|^2 + m_n n_n(r) + m_p n_p(r)\right]. \tag{3.1}$$

Here, n_n (n_p) is the local neutron (proton) density, $n = n_n + n_p$ is the total nucleon density and m_n (m_p) is the neutron (proton) rest mass. The energy density ε_0 is the EOS of uniform nuclear matter of our interest. The coefficient F_0 characterizes the gradient term which contributes to the surface energy.

The energy density ε_0 is the sum of the Fermi kinetic energy density and the potential energy density,

$$\varepsilon_0(n_n,n_p) = \frac{3}{5}(3\pi)^{2/3}\left(\frac{\hbar^2}{2m_n}n_n^{5/3} + \frac{\hbar^2}{2m_n}n_n^{5/3}\right) + \left[1-(1-2x)^2\right]v_n(n) + (1-2x)^2 v_p(n). \quad (3.2)$$

The first term in Eq. (3.2) is the kinetic energy density. The potential energy density is the weighted sum of the potential energy densities of symmetric nuclear matter and neutron matter, $v_s(n)$ and $v_s(n)$,

$$v_s(n) = a_1 n^2 + \frac{a_2 n^3}{1+a_3 n}, \qquad v_n(n) = b_1 n^2 + \frac{b_2 n^3}{1+b_3 n}. \quad (3.3)$$

The denominators of the potential energies soften the three body energy terms. The choice of $a_3 = 0$ and $b_3 = 0$ restores the simple three body energies. The b_3 value is hard to determine empirically. In the present study, this value is fixed $b_3 = 1.58632$ to give reasonable fit to Friedmann-Pandharipande EOS of neutron matter, i.e., a reasonable three body energy [1].

For simplicity, neutron and proton distributions are parametrized with radius parameter R_i and thickness parameter t_i $(i = n, p)$.

$$n_i(r) = \begin{cases} (n_i^{in} - n_i^{out})\left[1 - \left(\frac{r}{R_i}\right)^{t_i}\right]^3 + n_i^{out} & (r < R_i) \\ n_i^{out} & (r > R_i). \end{cases} \quad (3.4)$$

where n_i^{in} and n_i^{out} are the densities at $r = 0$ and $r > R_i$, respectively. The total energy is optimized with respect to these distribution parameters.

4. Model parameters, EOS saturation parameters and liquid drop mass formula

The parameters of the energy density ε, the EOS parameters and the coefficients of the liquid drop mass formula are summarized in Table 1.

For symmetric nuclear matter, the three model parameters $a_1 - a_3$ correspond to the three EOS parameters n_0, w_0, K_0, while the three model parameters $b_1 - b_3$ for neutron matter correspond to the three EOS parameters (energy, density derivative and compressibility at $n = n_0$), i.e., S_0, L, K_{sym0} for the symmetry energy, or w_{n0}, L, K_{n0} for neutron matter.

The EOS parameters K_0 and L govern the softness of neutron-rich nuclei and modify the value of the saturation density of asymmetric matter from n_0. The coefficient F_0 of the gradient energy giving about half of the surface energy [1] is also expected to correlate with K_0 and L.

In the liquid drop mass formula, the mass of a nucleus with proton number Z, neutron number N and mass number $A = Z + N$ is written as

$$M(Z,N) = m_p Z + m_n N + a_b A + a_I \frac{(N-Z)^2}{A} + a_C \frac{Z^2}{A^1/3} + a_s A^{2/3}. \quad (4.1)$$

Here, the bulk energy coefficient a_b corresponds to the saturation energy w_0 of symmetric nuclear matter, and the symmetry energy coefficient a_I roughly to the symmetry energy S_0 because the surface asymmetry effect is not treated in the liquid-drop formula. The Coulomb energy coefficient a_C

is related to nuclear size and hence to the the saturation density n_0. The surface energy coefficient a_s is primarily related to F_0.

From Table 1, we see that two parameters for symmetric matter and one for neutron matter are well constrained from nuclear masses. The uncertain EOS parameters are K_0 and L. Therefore, in the following, we construct a family of the EOS's such that a member EOS has an empirically reasonable (K_0, L) value.

Table 1: The parameter list of the local energy density ε, the EOS saturation parameters and the coefficients of the liquid drop mass formula.

category	ε	saturation	liquid drop mass formula
isoscaler symmetric matter	a_1, a_2, a_3	n_0, w_0, K_0	a_v, a_C
isovector neutron matter	$b_1, b_2(, b_3 = 1.59)$	$S_0, L(, K_{sym0})$ $w_{n0}, L(, K_{n0})$	a_I
inhomogeneous surface	F_0	F_0	a_s

5. Saturation parameters of OI EOS family

For a given (K_0, L) value, the six model parameters $a_1, a_2, a_3, b_1, b_2, F_0$ are optimized to fit masses and radii of stable nuclei. In practical computations, we utilize the slope of the saturation curve (Eq. (2.3)) in place of L. By taking TM1 EOS and SIII EOS as two extremes [9], we consider an auxiliary constraint $-1800 < y < -200$ $(\text{MeV} \cdot \text{fm}^3)$ [2]. There is clear correspondence between (K_0, L) and $(K_0, -y)$ (see Fig. 1). We specify an EOS with its (K_0, L) value because L is a better defined saturation parameter related to the symmetry energy than y.

In the present study we impose the following four constraints for the model parameters.

1. $180 < K_0 < 360$(MeV) and $-1800 < y < -200$ $(\text{MeV} \cdot \text{fm}^3)$
2. consistent with empirical masses and radii of stable nuclei
3. $b_3 = 1.59$ fm^3 (reasonable three body energy for pure neutron matter)
4. $K_{n0} > 0$

The last constraint is added in this study. It is a necessary condition to support a neutron star. For a given (K_0, L), the values of the six model parameters $a_1, a_2, a_3, b_1, b_2, F_0$ are optimized to best reproduce empirical masses and radii of stable nuclei [2]. From the above constraints 1-3, we systematically obtain more than 200 EOS's. The values of (K_0, L) are shown in Fig. 1, where typical EOS's A-I, B' and E' are shown with filled circles. Figure 2 shows quite interesting strong correlations among S_0, L and K_{n0}. From this strong $K_{n0} - L$ correlation and constraint 4 above, we obtain an important constraint $L > 20$ (MeV).

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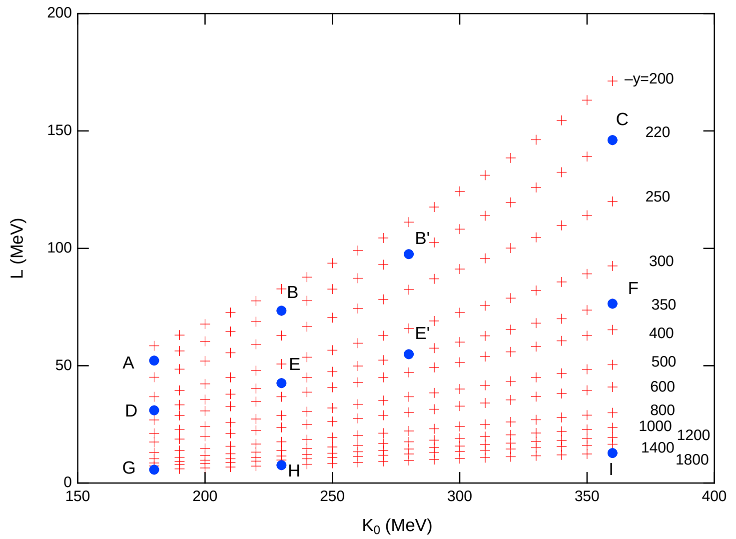

Figure 1: Values of (K_0, L) of OI EOS family (crosses). Typical EOS's are marked with filled circles. The values of y (MeV·fm^3) are also shown on the right.

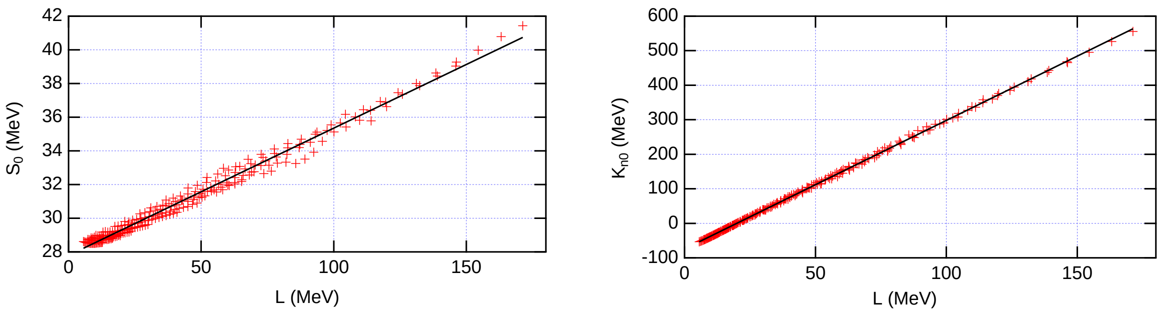

Figure 2: The strong correlations among S_0, L and K_{n0} of OI EOS family.

6. Neutron rich nuclei in laboratory

First, let us take a brief look at predictions of an extremely neutron rich nuclei with OI EOS family. The radius, neutron skin thickness, mass of a neutron rich nucleus are sensitive to L and but less sensitive to K_0 [3]. Typical examples are shown in Fig. 3. The left panel shows the calculated neutron skin of neutron rich ^{78}Ni with OI EOS family. The neutron drip line (right panel) departs farther away from the stability line with L [4]. It is noteworthy that the EOS uncertainty is comparable with shell and pairing effects of KTUY mass formula [10].

7. Nuclei in the neutron star crust

The neutron drip point (NDP) is the boundary density between the outer and inner crust. The calculated NDP is almost constant about 4×10^{11} g/cm^3 but slightly increases with L (see the left panel of Fig. 4).

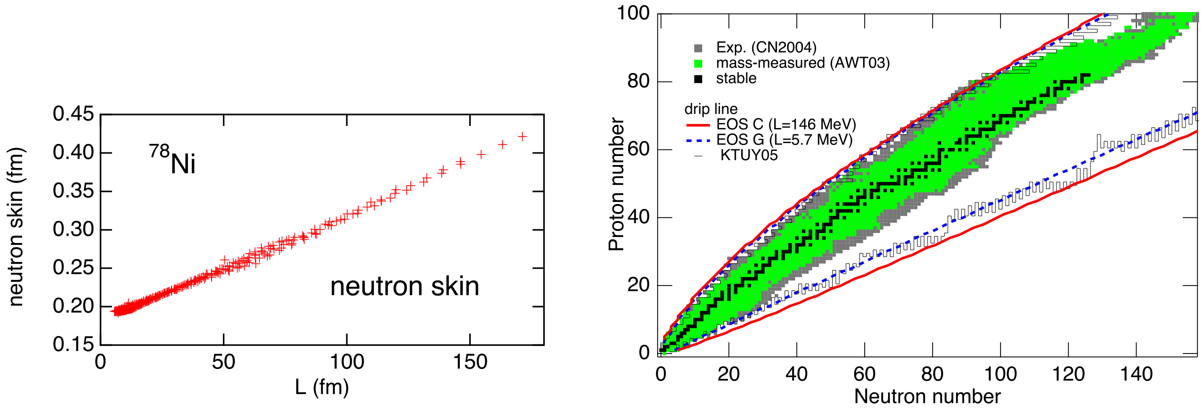

Figure 3: The ^{78}Ni neutron skin thickness (left), and the neutron and proton drip lines (right, extracted from Ref. [4]) calculated with OI EOS family.

The spherical nuclei could become unstable below the core-crust boundary density and there could be a density region for pasta nuclei. The density at the core-crust boundary estimated from stability against proton clustering decreases with L. The upper density for the spherical nucleus region is estimated from fission instability. From the right panel of Fig. 4 where the stabilities against the proton clustering and the fission are shown respectively with crosses and circles [5], we see that we have a room for pasta nuclei for $L < 100$ MeV.

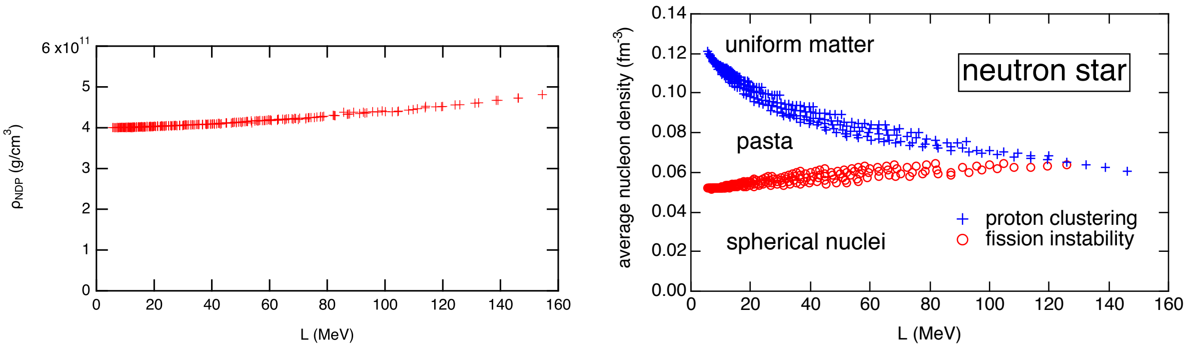

Figure 4: The neutron drip point (left) and the core-crust boundary and the upper density for the spherical nucleus region (right, extracted from Ref. [5]).

In the inner crust, the proton number of a nucleus Z and the average proton fraction of matter Y_p are shown in Fig. 5 as functions of matter density [5]. Both Z and Y_p decrease with L. The proton fraction decreases with density but the proton number shows a behavior dependent on L.

8. Neutron star mass and radius

We calculate neutron stars with typical OI EOS, A-I, B', E' in Fig. 1. Figure 6 show the gravitational mass and radius as functions of the central density. The (K_0, L) dependence of the results are not very simple. The neutron star mass increases with L, and also with K_0. EOS G and H which have smallest L values have negative K_{n0} values and can not support a neutron star.

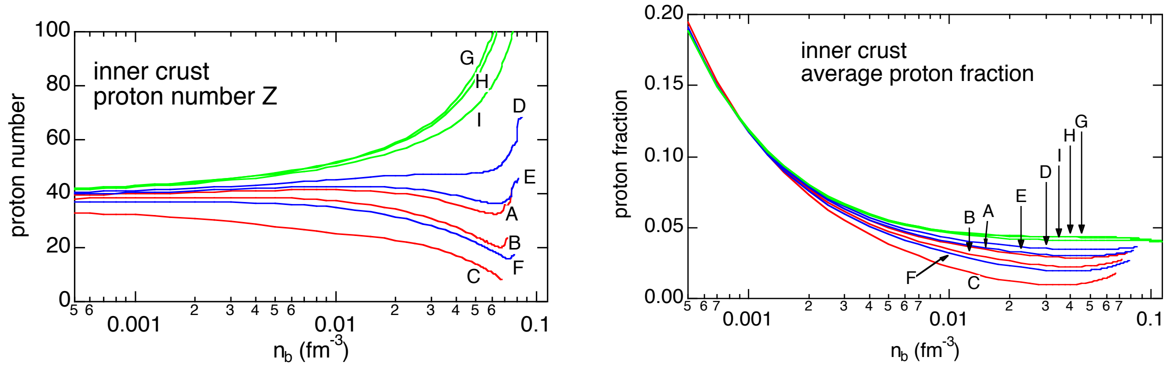

Figure 5: The proton number of the inner crust nucleus (left) and the average proton fraction of the inner crust matter (right) as functions of average nucleon density extracted from Ref. [5].

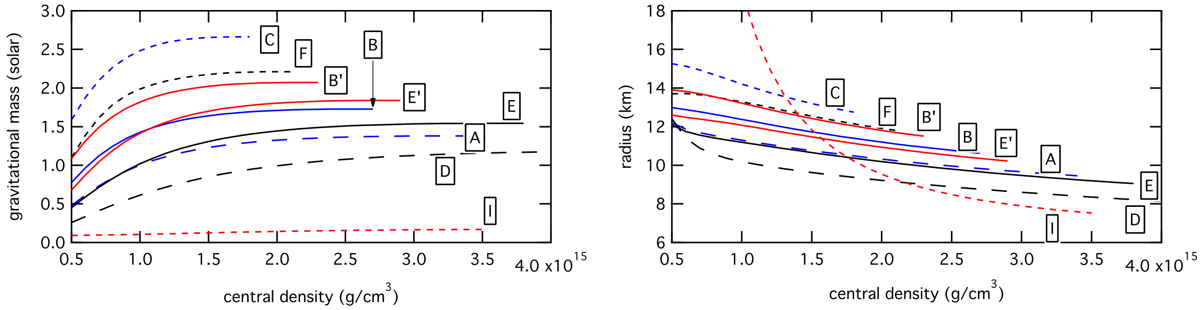

Figure 6: The (K_0, L) dependence of the neutron star's gravitational mass ($M_\odot$) and radius (km) as functions of the central density.

In Fig. 7, the mass and radius of the maximum gravitational mass neutron star with OI EOS are drawn for $L > 20$ MeV (corresponding to $K_{n0} > 0$) as contour lines in the $K_0 - L$ plane.

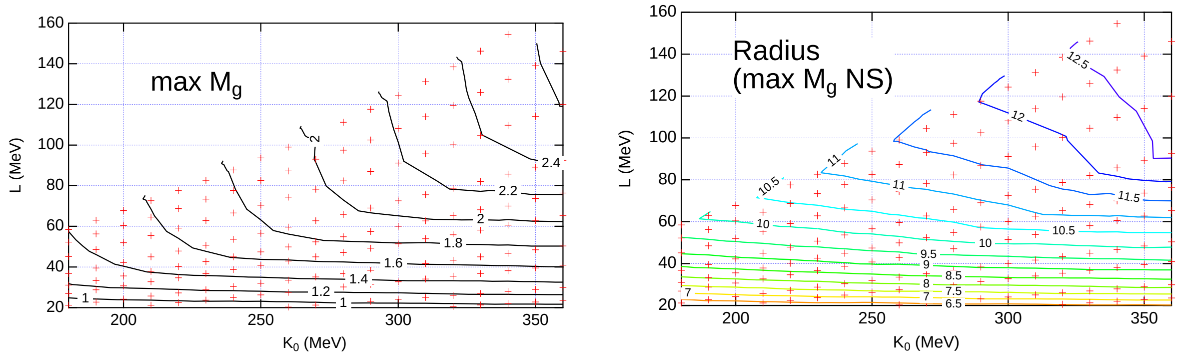

Figure 7: The (K_0, L) dependence of the maximum neutron star's gravitational mass ($M_\odot$) and radius (km).

9. Summary and outlook

With OI EOS family we describe the structure of neutron rich nuclei and neutron stars as functions of (K_0, L) related to the softness of the EOS of nuclear matter.

The structures of neutron rich nuclei and neutron star crusts are mainly dominated by L. The neutron star mass and radius increase with L and also with K_0. (For low mass neutron stars, see Ref. [11].) This reflects the behavior of high density EOS controlled by the three body energy parameter b_3.

Now we have just started a high density EOS project, namely, Oyamatsu-Sotani-Iida EOS family. We choose typical values of the three body energy parameter b_3 of the potential density of neutron matter. They are $b_3 = 0$ (hard) and $b_3 = 5$ (very soft). Even with these extreme b_3 values, we obtain quite similar correlations among S_0, L and K_{n0} as shown in Fig. 8. It is interesting that $L > 20$ MeV from the positive K_{n0} constraint independently of b_3. The choice of b_3 value would not affect nuclei and crusts very much, but will probably alter the core structure.

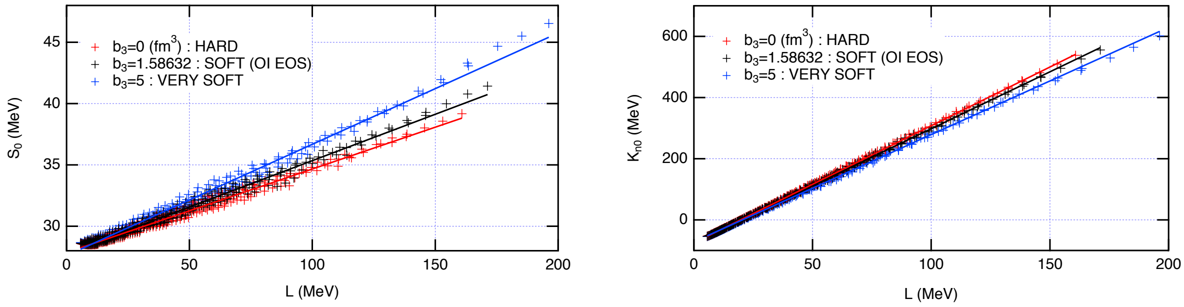

Figure 8: The correlations among S_0, L and K_{n0} for different values of the three body energy parameter b_3.

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