

Solitonic Excitations In Collisions Of Superfluid Nuclei

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We investigate the role of the pairing field dynamics in low-energy heavy ion reactions within the nuclear time-dependent density functional theory extended to superfluid systems. Recently, we have reported on unexpectedly large effects associated with the relative phase of the pairing field of colliding nuclei on the reaction outcomes, such as the total kinetic energy and the fusion cross section [P. Magierski, K. Sekizawa, and G. Wlazłowski, arXiv:1611.10261 [nucl-th]]. We have elucidated that the effects are due to creation of a “domain wall” or a “solitonic structure” of the pairing field in the neck region, which hinders energy dissipation as well as the neck formation, leading to significant changes of the reaction dynamics. The situation nicely mimics the one extensively studied experimentally with ultracold atomic gases, where two clouds of superfluid atoms possessing different phases of the pairing field are forced to merge, creating various topological excitations, quantum vortices and solitons, as well as Josephson currents. In this paper, we present unpublished results for a lighter system, namely, $^{44}\text{Ca}+^{44}\text{Ca}$. It is shown that the pairing effects on the fusion hindrance are rather small in lighter systems, due to a strong tendency towards fusion, which is consistent with an earlier study.

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1. Introduction

Topological excitations are among the most peculiar properties of superfluid systems. A typical example is the existence of the quantum vortex, which was first predicted in superfluid ^4He by Onsager in 1949 [1] and was confirmed experimentally by Vinen in 1958 [2]. The quantum vortex is a manifestation of a “winding” of the phase of the pairing field (the order parameter), which generates rotating supercurrents having a normal (non-superfluid) core at the center of the vortex as a topological defect. Nowadays it is possible to experimentally study dynamics of topological excitations in superfluid systems with ultracold atomic gases. In the experiment of Ref. [3], for example, a cascade of solitonic excitations was observed after a merger of two clouds possessing different phases. Namely, at first a “domain wall” was created, in which the superfluidity is lost as the phase is changing rapidly, thus being a topological defect, which subsequently decays into a vortex ring and vortex lines. The study of dynamic excitation modes of superfluid systems is the forefront topic both experimentally and theoretically.

It is the common sense that nucleons in the majority of the atomic nuclei or in the neutron stars are in the superfluid phase. Indeed, it has been envisaged [4] that the pulsar glitch, a sudden spin-up of the rotational frequency, is caused by a catastrophic “unpinning” of a huge number of vortices which are “pinned” (immobilized) by the Coulomb lattice of neutron-rich nuclei immersed in neutron superfluid in the inner crust of neutron stars. Now, a naive question arises: does the topological excitations of the superfluid nucleons play any role in nuclear reactions? In the case of finite nuclei, the presence of a quantum vortex is hardly expected, since the pairing correlations are weak, in a sense that the ratio of the pairing gap to the Fermi energy is small, *i.e.* $\Delta/\varepsilon_F \lesssim 5\%$, and the coherence length, typical size of a quantum vortex, becomes significantly larger than the size of the system. Moreover, one would naively expect that the pairing in the nucleus is so fragile that it would only affect tunneling phenomena near and below the Coulomb barrier, like Josephson currents [5], and it would not play important role in dissipative collisions above the barrier.

Contrary to the naive expectations, in our recent work [6], we have found noticeably large effects of the pairing in low-energy heavy ion reactions. The effects are associated with the “phase” of the complex pairing field, $\Delta(\mathbf{r}) = |\Delta(\mathbf{r})|e^{i\varphi(\mathbf{r})}$, and more precisely, with the relative phase, $\Delta\varphi \equiv \varphi_1 - \varphi_2$, between two colliding nuclei, where φ_i denotes the phase of the pairing field of each nucleus, which is uniform in their ground state. The phase difference $\Delta\varphi$ triggers creation of a “solitonic excitation” of the pairing field in the neck region, where the pairing is vanishing due to the phase discontinuity, which hinders energy dissipation as well as the neck formation, leading to significant changes in the reaction dynamics: *e.g.*, total kinetic energy of the outgoing fragments in $^{240}\text{Pu}+^{240}\text{Pu}$ is changed up to 20 MeV and the energy necessary to fuse two nuclei in $^{90}\text{Zr}+^{90}\text{Zr}$ is changed by almost 30 MeV, depending on the phase difference $\Delta\varphi$ [6].

It is worth noting here that although the situation nicely mimics the one studied with ultracold atomic gases the physics of interest is quite different. In the ultracold atomic gases, the pairing is so strong that the coherence length is on the same order as the mean inter-particle distance, which is much smaller than the size of the system. Due to this fact the manifestations of topological excitations, like, *e.g.*, creation and decay of a vortex ring and vortex lines, and the dynamics of Josephson currents are better pronounced. On the other hand, in the case of nuclear reactions, the main concern would not be dynamics of topological excitations itself, but the possible influence

on reaction mechanisms, such as dynamics of fusion, (quasi)fission, transfer reactions, energy dissipation, collective and single-particle excitations, quantum tunneling, and so on. Especially, the fact that the system may split after collision, due to the interplay between nuclear forces and the Coulomb repulsion, is the unique property of the nuclear system, which has not been studied with ultracold atomic gases.

The pairing effects in nuclear reactions have rarely been investigated to date. The most satisfactory description is based on time-dependent density functional theory (TDDFT) [7, 8, 9]. In the field of nuclear physics, it has been developed as time-dependent mean-field theories, such as time-dependent Hartree-Fock(-Bogoliubov) theory [TDHF(B)], with effective two- and three-body nuclear interactions (for recent reviews, see Refs. [10, 11]). To perform a full TDHFB calculation is still computationally challenging. Thus, the possible pairing effects on the reaction dynamics were investigated with simplified approaches [12, 13]. Very recently, the first attempt has been reported in Ref. [14], where the effects of the phase difference in head-on collisions of $^{20}\text{O}+^{20}\text{O}$ were investigated based on TDHFB. From the results, a vestige of the repulsive effect of the phase difference was indeed seen in collision trajectories, although the magnitude is very small, as compared to our results for Zr+Zr and Pu+Pu systems. In order to clarify this issue, in this article we examine the effects in collisions of relatively light nuclei, $^{44}\text{Ca}+^{44}\text{Ca}$, at energies around the Coulomb barrier.

This article is organized as follows. In Sec. 2, we briefly summarize our theoretical framework. In Sec. 3, we show results of TDSLDA calculations for the $^{44}\text{Ca}+^{44}\text{Ca}$ reaction. In Sec. 4, a short summary is given.

2. Theoretical Framework

We use a microscopic framework based on TDDFT, which is capable of describing reaction dynamics, taking explicitly into account nucleonic degrees of freedom. We utilize a local treatment of superfluid TDDFT known as time-dependent superfluid local density approximation (TDSLDA). The feasibility of the approach has been tested for describing the dynamics of strongly correlated Fermionic systems in both ultracold atomic gases [15, 16, 17, 18, 19, 20, 21] and in nuclear systems [22, 23, 24, 25]. To simulate heavy ion reactions, we have extended a computational code that we used to study dynamics of a quantum vortex in the presence of a nuclear impurity in the inner crust of neutron stars [26]. Here we briefly summarize the theoretical and computational aspects of the framework. (We refer readers to Refs. [6, 26], for more details.)

We numerically solve the TDSLDA equations

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} h(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h(\mathbf{r}) \end{pmatrix} \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix}, \quad (2.1)$$

where $u_i(\mathbf{r})$ and $v_i(\mathbf{r})$ are the quasiparticle wave functions. $h(\mathbf{r})$ is the single-particle Hamiltonian and $\Delta(\mathbf{r})$ is the pairing field, which are derived from appropriate functional derivatives of an energy density functional (EDF), $\mathcal{E}(\mathbf{r}) = \mathcal{E}_0(\mathbf{r}) + \mathcal{E}_{\text{pair}}(\mathbf{r})$. For the normal part, $\mathcal{E}_0(\mathbf{r})$, we use FaNDF⁰ functional proposed by Fayans [27]. In the present study we neglect the spin-orbit term in the functional, which allows to construct a highly efficient TDSLDA solver [26] which works on hundreds of GPUs with almost perfect scalability. We supplement the Fayans EDF with a local

pairing functional,

$$\mathcal{E}_{\text{pair}}(\mathbf{r}) = -g[|v_n(\mathbf{r})|^2 + |v_p(\mathbf{r})|^2], \quad (2.2)$$

where $v_{n(p)}(\mathbf{r})$ are neutron (proton) anomalous densities. The local treatment of the pairing requires a regularization, since the anomalous density is divergent, $v(\mathbf{r}, \mathbf{r}') \propto \frac{1}{|\mathbf{r}-\mathbf{r}'|} \rightarrow \infty$ for the limit $|\mathbf{r}-\mathbf{r}'| \rightarrow 0$ [28]. We apply a regularization procedure of Ref. [29]. Densities of neutrons ($q = n$) and protons ($q = p$) are then evaluated as $\rho_q(\mathbf{r}) = 2\tilde{\sum}_{i \in q} |v_i(\mathbf{r})|^2$, $\tau_q(\mathbf{r}) = 2\tilde{\sum}_{i \in q} |\nabla v_i(\mathbf{r})|^2$, $v_q(\mathbf{r}) = \tilde{\sum}_{i \in q} v_i^*(\mathbf{r}) u_i(\mathbf{r})$ (the factor of two stands for the spin degree of freedom), where $\tilde{\sum}_{i \in q}$ takes summation over positive quasiparticle energy states defined at $t = 0$ smaller than a cutoff, $0 \leq E_i \leq E_c$. We note that the pairing field $\Delta_q(\mathbf{r})$ does not depend on the regularization procedure.

The quasiparticle wave functions are represented on three-dimensional Cartesian spatial lattice (without symmetry restrictions) with periodic boundary conditions. A box of 80 fm \times 25 fm \times 25 fm with lattice spacing of 1.25 fm was used to simulate head-on collisions. The spatial derivatives and the Coulomb potential are computed employing Fourier transforms. The initial wave function of projectile and target nuclei placed with a certain distance within the computational box is prepared using Shifted Conjugate Orthogonal Conjugate Gradient (COCG) method [30], combined with a direct diagonalization of the Hamiltonian matrix. We used an external potential to keep two nuclei at rest during the self-consistent iterations (to compensate the Coulomb repulsion). After getting a convergent solution, the phase difference was dynamically imprinted with a constant external potential for half of the box. We also used an external potential to boost two nuclei. For time evolution, the Trotter-Suzuki decomposition was used with a single predictor-corrector step. The time step was set to $\Delta t \simeq 0.038$ fm/c. The cutoff energy for the pairing regularization was set to $E_c = 100$ MeV. The corresponding numbers of quasiparticle wave functions for neutrons and protons within the initial cutoff energy were 8,740 and 7,723, respectively. These settings ensured the stable time evolution within the intervals exceeding 12,000 fm/c.

We have neglected the spin-orbit interaction which, although crucial for a proper description of nuclear structure (*i.e.*, shell structure and deformation) and energy dissipation in low-energy heavy ion reactions [31], does not influence the mechanism of described effect. It is worth emphasizing here that the impact of the pairing phase difference on the reaction dynamics may be captured even without the spin-orbit interaction. The energy cost to build the ‘‘domain wall’’ is given by (derived from Ginzburg-Landau theory) [6]

$$E_j = \frac{S}{L} \frac{\hbar^2}{2m} n_s \sin^2 \frac{\Delta\phi}{2}, \quad (2.3)$$

where S is the surface area of the wall, L is the length over which the phase varies, m is the nucleon mass, and n_s is the superfluid density. The main ingredients to have reasonable values of S, L, n_s , *i.e.* the radius of the nucleus, the Fermi energy and the pairing strength, are correctly described even without the spin-orbit interaction. Thus, nevertheless the interaction is not fully realistic, our framework is enough to study the possible impact of the pairing field dynamics in low-energy heavy ion reactions. Moreover, apart from the inevitable increase of enormous computation costs, inclusion of the spin-orbit coupling will also increase the complexity of the reaction mechanism (see, *e.g.*, Refs. [32, 33]). Therefore, to clearly underpin the possible effects of the pairing, to neglect the spin-orbit coupling would be rational, as a first step.

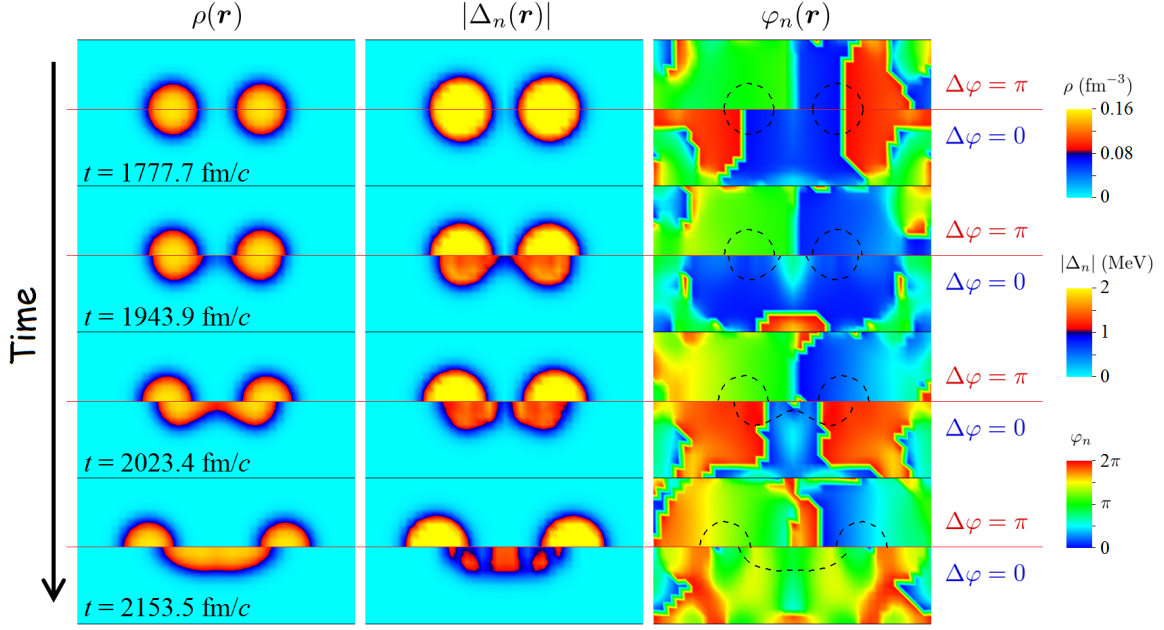


Figure 1: Results of the TDSLDA calculations for the $^{44}\text{Ca}+^{44}\text{Ca}$ reaction at $E \simeq 1.09V_{\text{Bass}}$. $V_{\text{Bass}} (\simeq 48 \text{ MeV})$ is the phenomenological fusion barrier [34]. In the left, middle and right columns, respectively, the total density $\rho(\mathbf{r})$, the absolute value and the phase of the neutron pairing field, $\Delta_n(\mathbf{r}) = |\Delta_n(\mathbf{r})|e^{i\varphi_n(\mathbf{r})}$, are shown, as a cross section on the reaction plane. In the upper (lower) half of each panel, the $\Delta\varphi = \pi$ (0) case is presented. Each row corresponds to a different time. In the right column, a contour of density, $\rho_0/2 = 0.08 \text{ fm}^{-3}$, is depicted by dashed lines. Note that blue and red colors in the right column, corresponding to $\varphi_n(\mathbf{r}) = 0$ and 2π , respectively, are equivalent.

3. Pairing Effects in a Lighter System— $^{44}\text{Ca}+^{44}\text{Ca}$ case

In order to investigate the pairing effects in a lighter system, we selected $^{44}\text{Ca}+^{44}\text{Ca}$ system. Since the proton number $Z = 20$ is a magic number even without the spin-orbit interaction, only neutrons are in superfluid phase. Here, let us first analyze the main effects of the pairing phase difference on the reaction dynamics. In Fig. 1, an illustrative example is shown for two cases, $\Delta\varphi = 0$ and π . At this collision energy ($E = 1.09V_{\text{Bass}}$), the $\Delta\varphi = 0$ case resulted in fusion, whereas in the $\Delta\varphi = \pi$ case binary fragments were observed.

The observed difference of the reaction dynamics is essentially caused by dynamic effects. The crucial difference can be seen in the second row of the figure ($t = 1943.9 \text{ fm}/c$). The density distribution $\rho(\mathbf{r})$ (left) exhibits a subtle neck between two nuclei in the $\Delta\varphi = 0$ case (lower part). This fact can be understood as follows. The “precursor” of the neck is expected to be mainly formed by the neutrons near the Fermi level, which also play a predominant role in the pairing phenomena. On the other hand, any spatial change of the phase of the pairing field induces a supercurrent, as its velocity is proportional to the gradient of the phase, *i.e.*, $\mathbf{v}_s(\mathbf{r}) = (\hbar/2m)\nabla\varphi(\mathbf{r})$. If the phase exhibits large variations in space, the supercurrent would be very large, which is clearly unfavorable. The system thus chooses to become normal in such a region, where the phase is changing steeply. This is the reason why we have observed a “domain wall”, where the pairing field is vanishing in the neck region, especially in the $\Delta\varphi = \pi$ case [6] (see also Fig. 1). Therefore,

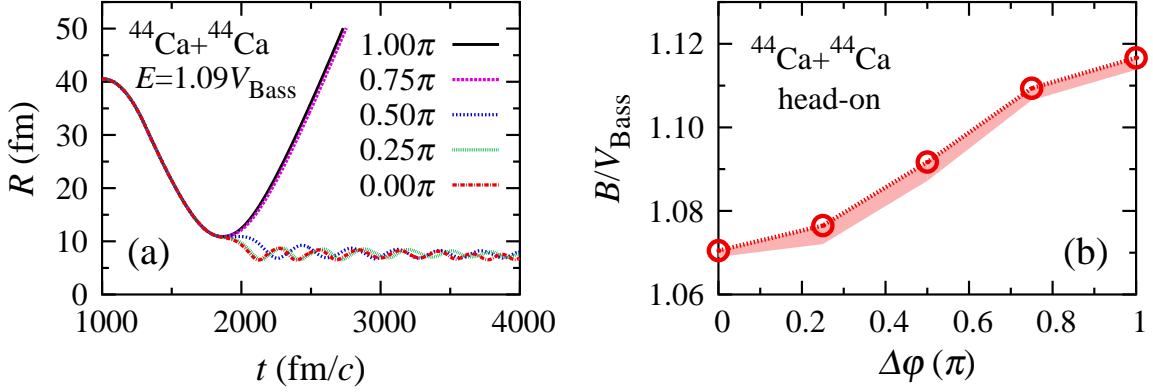


Figure 2: Results of the TDSLDA calculations for the $^{44}\text{Ca} + ^{44}\text{Ca}$ reaction. (a) Relative distance $R(t)$ in the $^{44}\text{Ca} + ^{44}\text{Ca}$ reaction at $E \simeq 1.09V_{\text{Bass}}$ for various relative phase $\Delta\phi$ is shown as a function of time. V_{Bass} ($\simeq 48$ MeV) is the phenomenological fusion barrier [34]. (b) The fusion threshold energy B/V_{Bass} is shown as a function of the relative phase $\Delta\phi$.

the phase difference prevents the superfluid neutrons to take part in the formation of the precursor of the neck, which resulted in the dramatic change of the reaction dynamics, as shown in Fig. 1.

In Fig. 2 (a), we show the relative distance $R(t)$ between the two colliding nuclei in the $^{44}\text{Ca} + ^{44}\text{Ca}$ reaction at $E = 1.09V_{\text{Bass}}$ as a function of time for various phase differences. In this case, we observe fusion for $\Delta\phi \leq \pi/2$. The fusion reaction does not occur for $\Delta\phi = 3\pi/4$ and π due to the hindered neck formation. In this way, the system requires additional energy to fuse two colliding nuclei, which increases with the phase difference. By repeating the simulations with different collision energies, we searched for the minimum energy at which the fusion reaction takes place. In Fig. 2 (b), the obtained fusion threshold energy B is shown as a function of the phase difference $\Delta\phi$. The filled area indicates the uncertainty due to finite collision energy steps ($\lesssim 220$ keV). From the figure, we find a change of the fusion threshold energy, up to about 5% of the barrier (2.3 MeV). Taking an average over the phase difference, we obtain an effective barrier increase of $E_{\text{extra}} = \frac{1}{\pi} \int_0^\pi [B(\Delta\phi) - B(0)] d(\Delta\phi) \approx 1.3$ MeV. Interestingly, we find a $(\sin^2 \frac{\Delta\phi}{2})$ -like pattern in Fig. 2 (b), the same dependence as the energy of the domain wall [cf. Eq. (2.3)], which was not present in a heavier system, $\text{Zr} + \text{Zr}$ [6]. It is worth emphasizing here that physics of the observed effect cannot be explained as the nuclear Josephson effect, since the Josephson current is proportional to $\sin \Delta\phi$, which clearly fails to explain observed $\sin^2 \frac{\Delta\phi}{2}$ pattern [6].

4. Summary

We have performed three-dimensional, microscopic, dynamic simulations of low-energy heavy ion reactions based on time-dependent density functional theory extended to superfluid systems. We have investigated the effects of the relative phase of the complex pairing field of colliding nuclei on the reaction dynamics in a relatively light system, $^{44}\text{Ca} + ^{44}\text{Ca}$. We have found that the fusion reaction is hindered by the phase difference, due to the suppressed neck formation, as was observed in a heavier system, $\text{Zr} + \text{Zr}$ [6]. However, the magnitude of the effective barrier increase does not exceed several percent of the Coulomb barrier, consistent with the earlier study [14]. In

order to make a quantitative prediction, realistic simulations including the spin-orbit coupling are mandatory.

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