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The role of SU(3)-flavour symmetry breaking in B-meson decay constants from Lattice QCD

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B-physics is currently facing a number of puzzles, and additional precision data - and precision calculations - will be required to pick these apart. In particular, as the Belle II detector approaches completion, there is increasing need for lattice calculations related to CKM matrix elements and to important *B* branching ratios such as $\mathscr{B}(B \to \tau\nu)$.

We present early results for f_B and f_{B_s} from the CSSM/QCDSF/UKQCD collaboration on a set of lattices with a fixed volume $32^3 \times 64$ and lattice spacing $a = 0.074$ fm. By varying the u,d,s quark masses while holding their average value constant, we are able to reliably control the SU(3)-flavour breaking effects.

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1. Introduction

Results from Belle and BABAR were crucial for our understanding of B-physics and the unitarity in the CKM matrix, but these results have also left us with a number of puzzles where further understanding of QCD (and of our detectors) is required. As the Belle II experiment [1] approaches its first science run and LHCb continues to increase statistics, the pressure is on to improve errors on theoretical and lattice calculations ahead of future improvements to experimental precision. We choose to focus on the B-meson decay constants f_B and f_{B_s} which are crucial to Standard Model calculations of the branching ratio $\mathscr{B}(B \to \tau\nu)$, though the f_B and f_{B_s} also appear in calculations of CKM matrix elements $|V_{tb}|$ and $|V_{ts}|$ from measurements of CP violation in B^0 and $\bar{B}^0$ mesons.

The Flavour Lattice Averaging Group (FLAG) reviews [2, 3] indicate that there are several groups working on f_B and related B meson observables with an eye toward improved precision [4–9]. As a companion to this body of existing work, we investigate f_B and f_{B_s} with a focus on SU(3) symmetry breaking effects by choosing light and strange quarks with a constant average mass $\overline{m}$. This methodology comes from the UKQCD/QCDSF group and we follow a similar method in our f_B/f_{B_s} study as in their related study of f_π and f_K. [10]

2. Simulation Details

2.1 Treatment of light and strange quarks

When extrapolating to the physical point using multiple lattice ensembles with different quark masses, many groups will choose the strange quark mass m_s to be constant. We instead follow the UKQCD/QCDSF process for choosing the masses of light and strange quarks in a 2+1 formalism [10]. The value of $\overline{m} = \frac{1}{3}(2m_l + m_s)$ is kept constant to control symmetry breaking and remove effects of $\mathscr{O}(\delta m)$. In fact, all flavour singlet quantities are only affected by SU(3)-flavour breaking effects at $\mathscr{O}((\delta m)^2)$, and have been shown to stay approximately constant from the SU(3) symmetric point to the physical point. [10]

For the ensembles of lattice configurations used in this work, there is a mixture of ensembles where $\overline{m}$ is equal to the physical value of $\overline{m}$ and ensembles where $\overline{m}$ has a slightly different value. The relationship between m_l and m_s (or equivalently, the relationship between the pion and kaon masses) for different ensembles at $a = 0.0074$ fm [11] is shown graphically in Figure 1 (inset) and also displayed in Table 1.

2.2 Bottom quarks

We generate bottom quarks using the anisotropic clover-improved action [7]

$$S_{lat} = a^4 \sum_{x,x'} \overline{\psi}(x') \left(m_0 + \gamma_0 D_0 + \zeta \vec{\gamma}\cdot\vec{D} - \frac{a}{2}(D^0)^2 - \frac{a}{2}\zeta(\vec{D})^2 + \sum_{\mu,\nu} \frac{ia}{4} c_P \sigma_{\mu\nu} F_{\mu\nu} \right)_{x,x'} \psi(x)$$

and tune m_0, c_P and ζ to specify the mass, hyperfine splitting, and dispersion relation of the generated $B(*)$ or $B(*)_s$ mesons. This is a variant of the 'Fermilab action' or 'RHQ action' [12, 13]. We choose the 'best' tuning by considering a flavour singlet B-meson $X_B = \frac{1}{3}(2B_l + B_s)$ and selecting the tuning parameters such that our calculated B and B_s mesons combine to create an X_B matching X_B for the physical B and B_s.

In practice, uncertainties on the measured mass, splitting, and dispersion relation also result in uncertainties in the values of m_0, c_P and ζ corresponding to the 'best' tuned B meson. We choose to always generate multiple b-quarks per lattice ensemble and interpolate to the 'best' B, rather than generating only one 'best' b-quark after completing the tuning process. This allows us to use the same set of seven b-quarks for each ensemble with the same lattice spacing and volume.

2.3 Additional Information

In this work, we use multiple ensembles of ~ 800 gauge field configurations with 2+1 flavours of non-perturbatively $\mathcal{O}(a)$ improved Wilson fermions. Further details of the configurations used in each ensemble are shown in Table 1 and Table 2.

(κ_l,κ_s)	m_π (MeV)	m_K (MeV)	# configs used	m_0	c_P	ζ
(0.12090,0.12090)	465	465	778	2.80 ± 0.13	3.60 ± 0.34	1.30 ± 0.11
(0.12104,0.12062)	360	505	758	2.65 ± 0.11	3.19 ± 0.29	1.37 ± 0.11
(0.121095,0.120512)	310	520	380	2.98 ± 0.22	4.03 ± 0.57	1.21 ± 0.16
(0.12095,0.12095)	400	400	400	2.69 ± 0.15	3.29 ± 0.39	1.50 ± 0.14
(0.12104,0.12077)	330	435	786	2.82 ± 0.13	3.59 ± 0.34	1.38 ± 0.10

Table 1: Lattice configurations and tuning results for ensembles with $V = 32^3 \times 64$, $a = 0.074$ fm, and $\beta = 5.5$ used in SU(3) breaking calculations

V	a (fm)	β	$\kappa_l = \kappa_s$	# configs used	m_0	c_P	ζ
$24^3 \times 48$	0.0818	5.4	0.11993	808	3.90 ± 0.32	4.64 ± 0.91	1.33 ± 0.24
$32^3 \times 64$	0.074	5.5	0.120900	778	2.80 ± 0.13	3.60 ± 0.34	1.30 ± 0.11
$32^3 \times 64$	0.0684	5.65	0.122005	410	2.74 ± 0.18	4.34 ± 0.48	1.13 ± 0.12

Table 2: Lattice configurations and tuning results for additional ensembles at the SU(3) symmetric point

The source locations for quarks are randomised to reduce correlations between neighbouring configurations in the ensemble. Future work will include either more configurations for each lattice ensemble where additional configurations are available, or additional source locations on the same configurations to increase statistics.

3. Calculating f_B on the lattice

The decay constant f_B is calculated from its lattice counterpart Φ_B via the equation

$$f_B = \frac{\hbar c}{a} Z_\Phi \Phi_B$$

where Φ_B is calculated from two-point correlators for Axial and Pseudoscalar operators:

$$\Phi_B = -\frac{\sqrt{2M_B}\mathscr{C}_{AP}}{\mathscr{C}_{PP}}, \qquad \mathscr{C}_{AP} = \frac{\langle\Omega|A_4|B\rangle\,\langle B|P|\Omega\rangle}{2M_B}, \qquad \mathscr{C}_{PP} = \frac{\langle\Omega|P|B\rangle\,\langle B|P|\Omega\rangle}{2M_B}$$

and Z_Φ is calculated:

$$Z_\Phi = \rho_A^{bl}\sqrt{Z_V^{bb}Z_V^{ll}}.$$

This formulation of Φ_B is equivalent to that used in [6], which does not include the factor $2m$ in the correlators.

In practice, Z_V^{bb} has been calculated using a light spectator quark in the three-point correlator, and we use $\rho_{\text{lat}}^{bl} = 1$ for these early calculations of f_B. f_B can also be improved by letting Φ_B go to $\Phi_B^0 + c_A\Phi_B^1$ using an improvement coefficient c_A, but this has not yet been explored at the time of this work.

4. Results

4.1 SU(3) symmetry breaking in f_B and f_{B_s}

Following the procedure used in the light quark sector [14], we plot f_B and f_{B_s} relative to the average decay constant $f_{B_X} = \frac{1}{3}(2 * f_B + f_{B_s})$ in order to cancel out some errors and look for SU(3)-flavour breaking behaviour. If SU(3) breaking of the lighter quarks is the main effect, we expect a linear fit for f_B/f_{B_X} against m_π^2/m_X^2 as was observed for f_π and f_K in [14]. These early results with a linear fit show good agreement with the FLAG2013 average of $N = 2+1$ flavour calculations. [2]

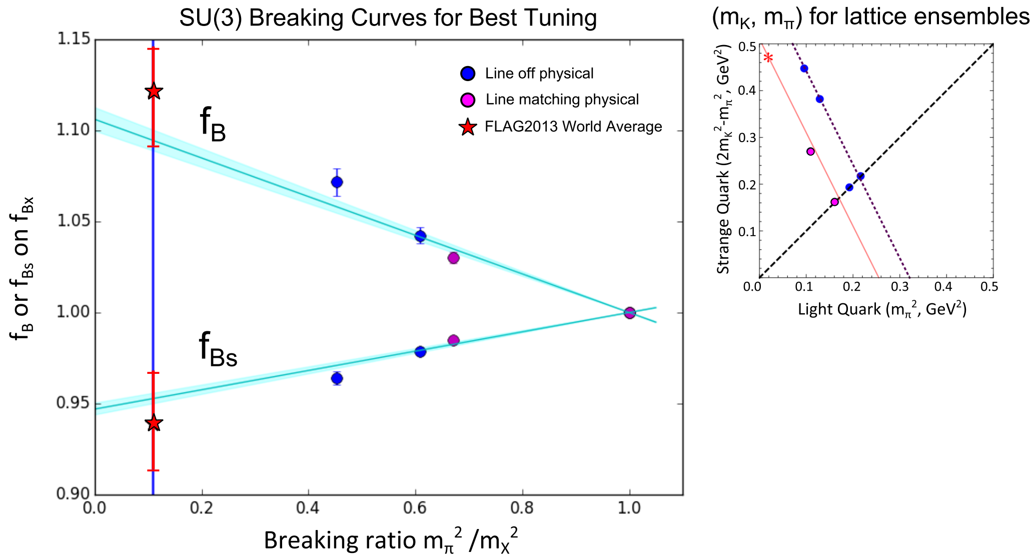

Figure 1: (Main Graph): Calculated values for f_B and f_{B_s} are plotted relative to the average decay constant $f_{B_X} = \frac{1}{3}(2 * f_B + f_{B_s})$. (Inset): Legend of location of lattice ensembles on a plane representing the strange and light quark masses relative to the physical point, plot taken from [15]

4.2 Toward predictions for f_B and f_{B_s}

It is crucial to control as many sources of uncertainty as possible in order to have competitive calculated values for f_B and f_{B_s}. As part of an investigation of lattice artefacts, we examine the way that the normalisation factor Z_Φ changes as the lattice volume and lattice spacing are varied. The effect of this normalisation on f_B is shown in Figure 2, while the changes in the components $\sqrt{Z_V^{bb}}$ and $\sqrt{Z_V^{ll}}$ of Z_Φ are plotted against lattice spacing a^2 in Figure 3.

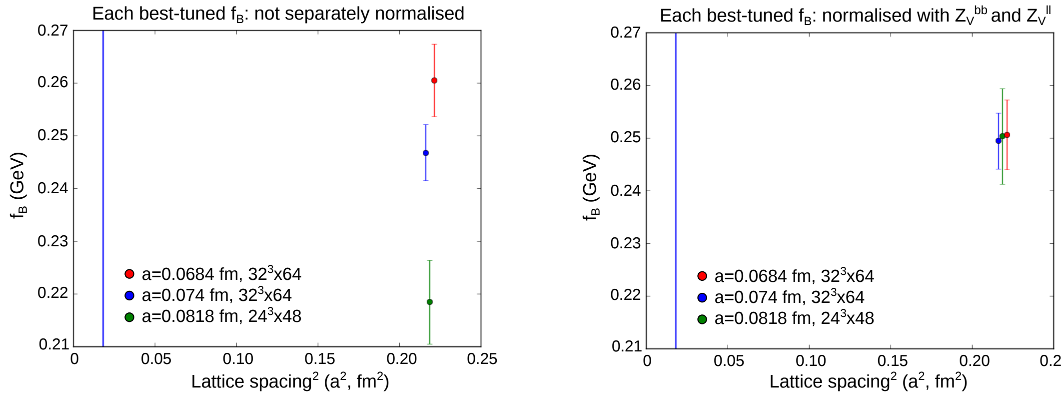

Figure 2: (Left) f_B, where the same Z_Φ value is used regardless of lattice spacing/lattice volume (Right) f_B, where Z_Φ is calculated for each lattice ensemble individually.

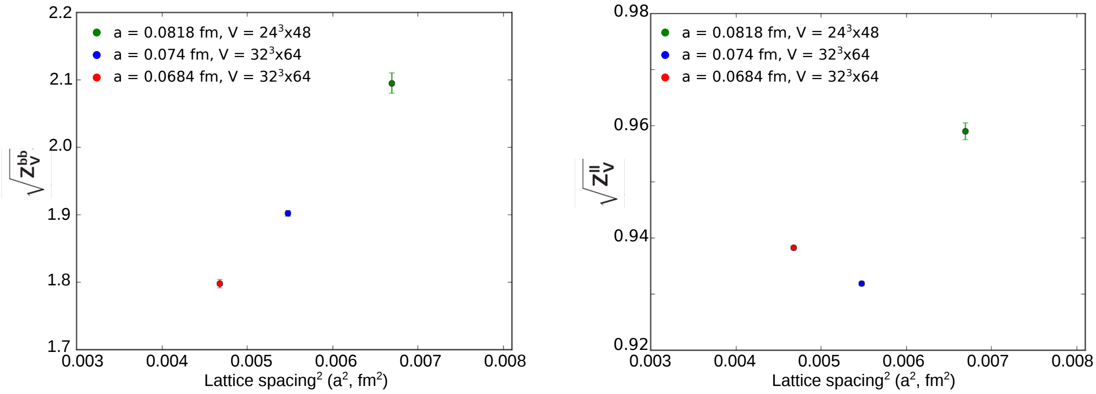

Figure 3: (Left) Normalisation factor $\sqrt{Z_V^{bb}}$ against lattice spacing squared. The relationship between the two is approximately linear. (Right) Normalisation factor $\sqrt{Z_V^{ll}}$ against lattice spacing squared. Z_V^{ll} appears to be affected most strongly by the lattice volume.

While $\sqrt{Z_V^{bb}}$ changes linearly with a^2 and appears to be unaffected by the lattice volume, the value of Z_V^{ll} is significantly different for the $24^3 \times 48$ and $32^3 \times 64$ lattices. We suspect that this difference is due to discretisation errors on the small $24^3 \times 48$ lattices, but this remains to be checked in future work. Future work will also involve $48^3 \times 96$ lattices for additional comparison at $\beta = 5.65$ as well as at finer lattice spacings (β =5.8,5.95).

5. Conclusion

We have presented preliminary results for f_B and f_{B_s} with controlled SU(3) symmetry breaking by controlling the way light and strange quark masses are chosen. Investigation of systematic lattice discretisation effects is underway, and we look forward to further progress toward calculations of f_B and f_{B_s} with competitive errors.

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