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Estimating transport coefficients in hot and dense quark matter

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We compute the transport coefficients, namely, the coefficients of shear and bulk viscosity as well as thermal conductivity for hot and dense quark matter within the Nambu- Jona Lasinio (NJL) model. The estimation of the transport coefficients is made by solving the Boltzmann kinetic equation within the relaxation time approximation. The transition rates are calculated in a manifestly covariant manner to estimate the thermal-averaged cross sections for quark-quark and quark-antiquark scattering. The calculations are performed for finite chemical potential also. Within the parameters of the model, the ratio of shear viscosity to entropy density has a minimum at the Mott transition temperature. At vanishing chemical potential, the ratio of bulk viscosity to entropy density, on the other hand, decreases with temperature with a sharp decrease near the critical temperature, and vanishes beyond it. The coefficient of thermal conductivity also shows a minimum at the critical temperature.

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1. Introduction

Transport properties of hot and dense matter have attracted a lot of attention recently, in the context of relativistic heavy-ion collisions [1], as well as in astrophysical situations such as the early Universe [2] and perhaps neutron stars [3]. The transport coefficients like shear and bulk viscosity as well as thermal conductivity enter as theoretical inputs for hydrodynamic simulation which is critical for interpretation of heavy ion collision data. The coefficients of viscosity influence various observables like the flow coefficients and the transverse momentum spectra. Further, the temperature and chemical potential dependences of these coefficients may reveal the location of phase transition. In many physical sytems, the shear viscosity is minimum at the phase transition point while the bulk viscosity is a maximum [4, 5]. In deed, the smallness of η/s to explain heavy ion flow data is significant in connection with the conjectured lower bound $\eta/s = 1/4\pi$, the 'Kovtun-Son-Starinets'(KSS) bound obtained in the context of AdS/CFT correspondence [6]. Large bulk viscosity near the phase transition related to large values of the trace of energy momentum tensor as indicated by lattice QCD simulation can affect the P_T spectrum [7] as well as can give rise to interesting phenomena like cavitation [8]. The other transport coefficient that also plays an important role for hydrodynamic evolution at finite chemical potential is the thermal conductivity [9]. The effects of thermal conductivity in the relativistic hydrodynamics in the context of quark gluon plasma have only recently been studied[9, 10]. Although, in principle, these coefficients can be directly estimated directly within QCD using Kubo formalism [11], QCD is strongly coupled for the energies accessible in heavy-ion collision experiments making such estimates unreliable. First-principle lattice simulations at finite chemical potential is also challenging and limited only to the equilibrium properties at small baryon chemical potential [12, 13]. We attempt here to estimate the transport coefficients using Boltzmann kinetic equation within relaxation time approximation. The nontrivial factor that enters here is the relativistic generalization of the Boltzmann equation with a mass and mean fields with medium dependence. Further, the effects of chemical potential is also taken into account. The actual calculations are performed within the ambit of Nambu Jona Lasinio model.

2. Thermodynamics of two-flavor NJL model and meson masses

We summarize here the thermodynamics of the simplest NJL model with two flavors with a four-point interaction in the scalar and pseudo scalar channels, with the thermodynamic potential given as[14, 15]

$$\Omega(\beta,\mu) = -\frac{2N_cN_f}{(2\pi)^3}\int\sqrt{\mathbf{k}^2+M^2}d\mathbf{k} - \frac{2N_cN_f}{(2\pi)^3\beta}\int d\mathbf{k}\,(\ln(1+\exp(-\beta(E-\mu))+\ln(1+\exp(-\beta(E+\mu)))+\frac{(M-m_0)^2}{4G}, \tag{2.1}$$

where, β is the inverse of temperature, μ is the quark chemical potential, and, $E(\mathbf{k}) = \sqrt{\mathbf{k}^2+M^2}$ is the on-shell single-particle energy with 'constituent' quark mass M. The constituent quark mass

satisfies the self-consistent gap equation

$$M = m_0 + \frac{2N_cN_f}{(2\pi)^3}\int \frac{M}{E(\mathbf{k})}\left(1 - f^0_-(\mathbf{k},\beta,\mu) - f^0_+(\mathbf{k},\beta,\mu)\right)d\mathbf{k}, \tag{2.2}$$

In the above $f^0_\mp(\mathbf{k},\beta,\mu) = (\exp(\beta(E\mp\mu))+1)^{-1}$ is the equilibrium fermion distribution function for quarks and antiquarks, respectively, with a constituent mass M and, are related to the quark number density in the standard way:

$$\rho = \frac{2N_cN_f}{(2\pi)^3}\int d\mathbf{k}\left[f^0_-(\mathbf{k},\beta,\mu) - f^0_+(\mathbf{k},\beta,\mu)\right]. \tag{2.3}$$

Within random phase approximation (RPA), the meson propagator can be calculated as [20]

$$D_M(\omega,\mathbf{p}) = \frac{2iG}{1-2G\Pi_M(\omega,\mathbf{p})} \tag{2.4}$$

where, –$M = \sigma,\pi$ for scalar and pseudo scalar channel mesons, respectively, and Π_M is the polarization function in the corresponding mesonic channel. The mass of the meson is extracted from the pole position of the meson propagator at zero momentum specified by the equation

$$1 - 2GRe\Pi_M(m_M,\mathbf{0}) = 0 \tag{2.5}$$

Here, we have chosen to define the mass of the unbound resonance by the real part of Π_M. For bound state solutions, i.e. for $\omega = m_M < 2M$, the polarization function is always real . For $m_M > 2M$, Π_M has an imaginary part that is related to the decay width of the resonance as $\Gamma_M = Im\Pi_M(m_M,\mathbf{0})/m_M$.

3. Transport coefficients within relaxation time approximation.

In the relativistic kinetic theory, the shear (η) and bulk (ζ) viscosity coefficients are defined as coefficients of the space-space component of the energy momentum tensor away from the equilibrium. Similarly, the thermal conductivity (λ) is defined in terms of the non equilibrium part of the conserved current. For a fluid composed of quasi particles with medium dependent mass and under relaxation time approximation for the Boltzmann kinetic equation, these coefficients are given as [18, 15]

$$\eta = \frac{1}{15T}\sum_a \int d\Gamma_a \frac{\mathbf{p}_a^4}{E_a^2}\left(\tau_a f^0_a(1-f^0_a))\right), \tag{3.1}$$

$$\begin{aligned}\zeta \;=\; & \frac{1}{9T}\sum_a \int d\Gamma^a \tau^a f^0_a(1-f^0_a) \\ & \times \left[\frac{\mathbf{p}^2}{E^a} - 3v_n^2\left(E^a - T\frac{\partial E^a}{\partial T} - \mu\frac{\partial E^a}{\partial \mu}\right) + 3\left(\frac{\partial P}{\partial n}\right)_\varepsilon\left(\frac{\partial E^a}{\partial \mu} - t^a\right)\right]^2\end{aligned} \tag{3.2}$$

$$\lambda = \frac{1}{3}\left(\frac{w}{nT}\right)^2 \sum_a \int d\Gamma \frac{\mathbf{p}^2}{E_a^2}\tau^a \left(t^a - \frac{nE^a}{w}\right)^2 f_a^0(1-f_a^0) \tag{3.3}$$

In the above, 'a' is the species' index; $d\Gamma^a = \nu^a d\mathbf{p}^a/(2\pi)^3$, ν_a being the degeneracy factor like color and flavor and $w = \varepsilon + p$ is the enthalpy density. Further, the single particle energy $E^a(\mathbf{p}) = \sqrt{\mathbf{p}^2 + M^2(T,\mu)}$ which is medium dependent and $t^a = \pm 1$ for particles and antiparticles respectively.The relaxation time $-\tau^a$, the relaxation time for particle 'a', in general is a function of energy. For the two body scattering process $a,b \to c,d$, is given as

$$\tau_a^{-1}(E_a) \equiv \omega_a(E_a) = \int d\pi^b d\pi^c d\pi^d f_b^0 W(a,b|c,d)(1-f_c^0)(1-f_d^0), \tag{3.4}$$

with

$$W(a,b:c,d) = \frac{(2\pi)^4\delta^4(p_a+p_b-p_c-p_d)}{2E_a 2E_b 2E_c 2E_d}|M|^2,$$

and M being the dimensionless amplitude for the corresponding scattering and $d\pi^i = d\mathbf{p}^i/(2\pi)^3$. One can also define an energy independent relaxation time as

$$\tau_a^{-1} = \frac{1}{n_a}\sum_b \int d\pi^a \omega_a(E_a) f(E_a) \tag{3.5}$$

Few comments may be in order. All the expressions for the transport coefficients are positive definite as they should be. The expression for the transport coefficients reduce to already known expression for these coefficients in limit of vanishing density [16, 17]. Although there are 12 possible scattering processes contributing to the relaxation time, using symmetry arguments (i-spin, charge conjugation and crossing) one can evaluate only two independent matrix elements. In the present case of two flavors there are 12 possible scattering processes that contribute to the relaxation time. We can choose these, as in Ref. [20, 15], to be $u\bar{u} \to u\bar{u}$ and $ud \to u\bar{d}$ and use the symmetry conditions to calculate the rest. These quark scattering amplitudes are calculated within the NJL model through exchange of mesons whose masses are also medium dependent. The details of the matrix elements for the scattering with exchange of mesons are given in Ref.[20, 15] and we do not repeat here.

4. Results and Discussions

For the two flavor NJL model, there are three parameters– the three momentum cut off Λ, the four Fermi coupling G and the current quark mass m. We take them as $m = 5.6$ MeV, Λ=587.9 MeV and $G\Lambda^2 = 2.44$. These are fixed so as to give $m_\pi = 135$ MeV, $f_\pi = 92.4$ MeV and the quark condensate $-\langle\bar{u}u\rangle^{1/3} = 241$ MeV. by fitting the pion mass, the pion decay constant and the quark condensate. This leads to the vacuum value of the constituent quark mass to be $M \simeq 400$MeV. In Fig. 1-a we have plotted the constituent masses M, and the meson masses m_σ, m_π as a function of temperature. The chiral cross over transition for $\mu = 0$ defined by the maximum of dM/dT turns out to be $T_\chi = 188$ MeV while the Mott temperature T_M defined my $m_\pi(T_M) = 2M(T_M)$ turns out to be about 197 MeV.

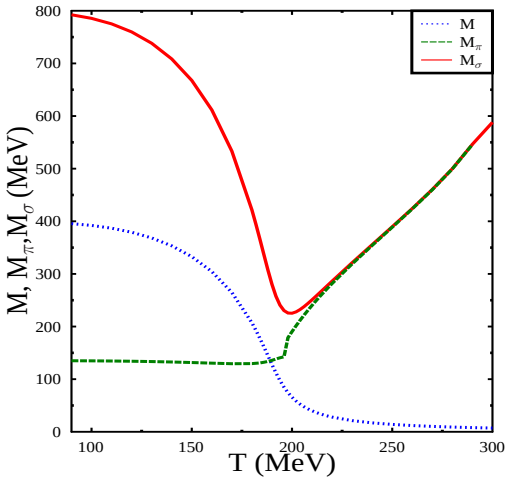

Fig. 1-a

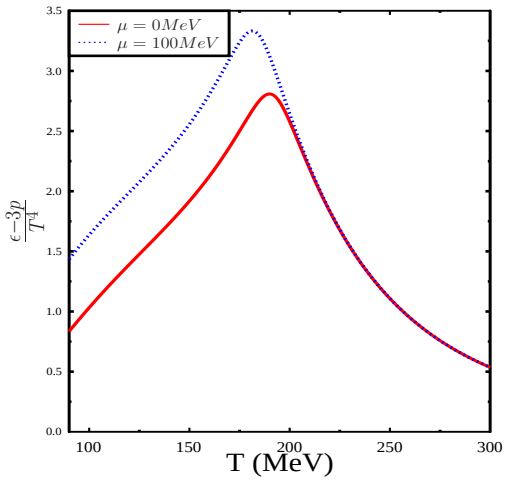

Fig. 1-b

Figure 1: (Fig 1 a) Temperature dependence of the masses of constituent quarks (M), and pions (M_π) and sigma mesons (M_σ) for $\mu = 0$ and (Fig1-b) trace anamoly $((\varepsilon - 3p)/T^4)$ (Fig 1-b) for $\mu = 0$ MeV and $\mu = 100$ MeV

Next, we discuss the estimation of averaged relaxation time from all the scatterings in the present approach as a function of temperature. Let us recall that this quantity is inversely related to the transition rate $W_{a,b|c,d}$ as in Eq.(3.4) summed over all the particles other than 'a'. In general, the dominant contribution here comes from quark-antiquark scattering from the s channel through propagation of the resonance states, the pions and the sigma. The mass of the sigma meson decreases with an increase in temperature, becoming a minimum at the Mott transition temperature T_M and leading to an enhancement of the cross section. This, in turn, leads to a minimum in the relaxation time. Beyond the transition temperature the resonance masses increase with temperature linearly leading to a smaller cross section and hence an increase in the relaxation time beyond the Mott temperature. This generic feature is observed in Fig.2-a.

In Fig(2-b) we have plotted the shear viscosity to entropy ratio ($\frac{\eta}{s}$) as a function of temperature for $\mu = 0$ MeV and $\mu = 100$ MeV . As expected from the τ behavior with temperature, η/s has a minimum with $\eta/s|_{min} \sim 0.24$ at the critical temperature beyond which it increases slowly. This behavior of having a minimum around the Mott transition due to the suppression of scattering cross sections at higher temperatures is in contrast to results of Ref.[21] where it shows a monotonic decrease with the value of the ratio going below the KSS bound. At finite μ, the ratio η/s is larger as compared to vanishing μ. This is due to two reasons. Firstly, τ at finite μ is larger and, further, the quark density is also larger as compared to the antiquarks at finite density.

In Fig. (3-a) we have plotted the specific bulk viscosity normalized to entropy density as a function of temperature. We have also shown here the results of earlier calculations, based on

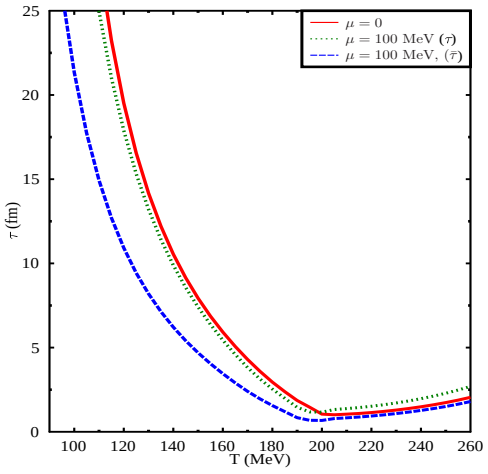

Fig. 2-a

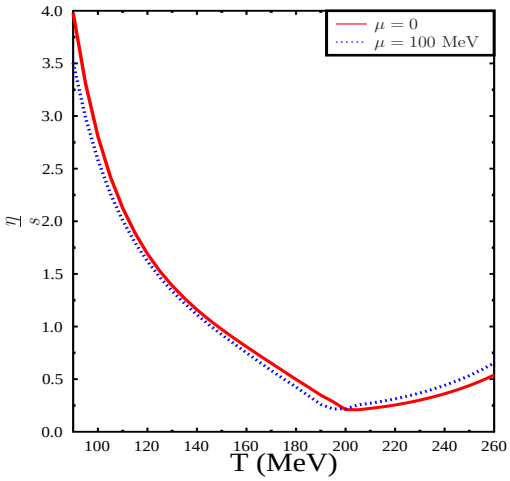

Fig. 2-b

Figure 2: Relaxation time as a function of temperature for $\mu = 0$ MeV and for $\mu = 100$MeV (Fig 2-a). In Fig (2-b), shear viscosity to entropy density ratio is shown for $\mu = 0$ MeV and $\mu = 100$ MeV.

the linear sigma model [17], NJL model [21] and SHMC model [22]. The ratio of bulk viscosity to entropy density increases rapidly near the critical temperature as temperature decrease from high temperature beyond the critical temperature to temperatures below it. However, it is not a maximum at the critical temperature. After the rapid rise near the critical temperature it increases slowly. As may be observed, in all these calculations the ratio ζ/s decreases monotonically with temperature. We might mention here that, such a behavior of decreasing bulk viscosity to entropy ratio was also observed in estimations based on PHSD transport codes [24] as well as in the linear sigma model in the large N limit [25]. On the other hand, when only ζ is plotted as a function of T, there is a maximum at the Mott temperature. Such a peak in ζ was also observed in Ref. [26] within a chiral perturbation theory framework with a maximum value of about $\zeta \sim 0.008$ GeV^3 as compared to $\zeta \sim 0.01$GeV^3 in the NJL model here. However, the ratio ζ/s does not show such a peak, probably because of the fact that the entropy of the system with massive constituent quarks becomes rather small to mask the peak structure in ζ.Beyond the Mott transition temperature the ratio ζ/s vanishes.

Finally, in Fig. 4 we have plotted thermal conductivity of quark matter at $\mu = 100$ MeV in units of T^2. Let us note that thermal conduction, which involves the relative flow of energy and baryon number, vanishes at zero baryon density. In fact, λ diverges as $\lambda \sim 1/n^2$ as may be observed in Eq.(3.3). Such a divergence, however, is inconsequential because, e.g., in the dissipative current, it enters as λn^2 [27, 16] and the heat conduction vanishes for $\mu = 0$ [28]. We have therefore shown the results for thermal conductivity for non vanishing μ arising from quark scatterings. As may be noted, the ratio λ/T^2 shows a non monotonic behavior with a minimum at the critical temperature.

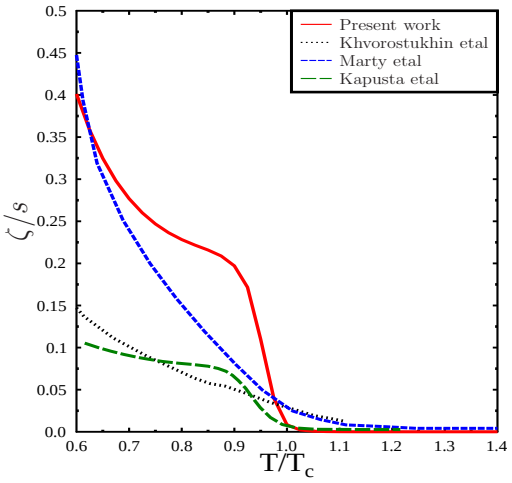

Fig. 3-a

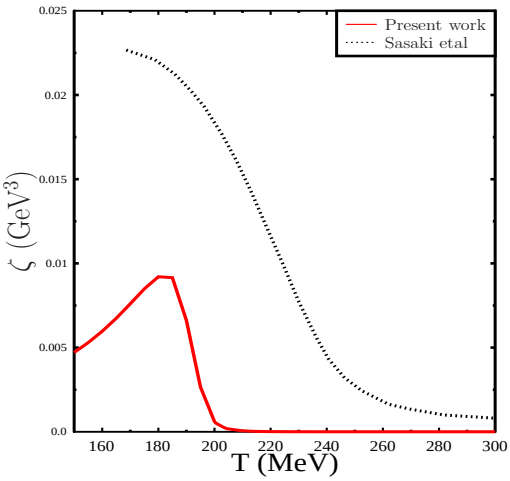

Fig. 3-b

Figure 3: Bulk viscosity to entropy ratio as a function of temperature in units of T_c for zero baryon density (8-a). Also shown results from different models, the SHMC model of Khvosthukin etal[22], Kapusta etal [17], the three flavor NJL results of Marty etal[21]. Bulk viscosity in units of GeV^3 as a function of temperature is shown in Fig. 8-b. Solid red curve corresponds to the present calculations while dotted curve correspond to the results by Sasaki etal [23].

The origin of this again is related to the minimum of the relaxation time at the critical temperature. The present behavior is in contrast to the same obtained in Ref. [21], where, the same ratio shows a monotonically decreasing function of temperature. The behavior of λ/T^2 was also studied in Ref.[29], where, the ratio showed an increasing behavior with temperature with, however, a slower rise with temperature as compared to the results shown in Fig. 6. The reason for a faster rise of λ with temperature beyond T_{mott} is two fold. Firstly, the pre factor in Eq.3.3, $(w/nT)^2$, varies at T^2, because, w rises as T^4, while n varies as μT^2 in the massless limit for small chemical potential. In addition, at large temperature, the integral itself rises as T^3 apart from the temperature dependence of relaxation time, which, again is an increasing function of temperature beyond T_{mott}. Within the Green-Kubo approach, thermal conductivity was estimated for two flavors using NJL model[30] as well as in Ref.[31] within the instanton liquid model where however the thermal conductivity saturates beyond T=150 MeV in contrast to the present result.

To summarize, we have attempted here to compute the transport coefficient in NJL model. The approach uses solving the Boltzmann kinetic equation within relaxation time approximation. To estimate the relaxation time we have considered the quark-antiquark two body scatterings through exchange of pion and sigma resonances. Since the meson masses are minimal at the transition temperatures beyond which they are degenerate and increase linearly with temperature, the meson propagator occurring in the transition amplitude lead to a large contribution to the cross section

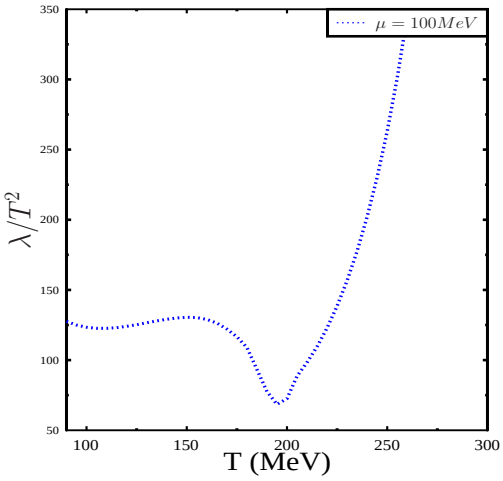

Figure 4: Thermal conductivity(λ) in units of T^2 for $\mu = 100$ MeV.

for the quark- antiquark scattering. This eventually leads to a smaller relaxation time which, in turn, leads to a minimum in the temperature dependence of the relaxation time. We have used the expressions for the transport coefficients that are manifestly positive definite as they should be. The expression for shear viscosity only depends on the relaxation time and the distribution functions. However, the expressions for both the coefficients of bulk viscosity and thermal conductivity involve equation of state. The expressions for the transport coefficients are direct generalization of their counterparts at zero chemical potential [18]. All three transport coefficients are minimal at the Mott temperature.

For the estimation of the relaxation time we have only included two-body scatterings. One can generalize this to include decay processes involving the mesons decaying to a pair of quarks and antiquarks[32]. We have investigated here the temperature dependence of the transport coefficients in relation to the chiral transition in quark matter. It would be interesting to study the interplay of chiral and deconfinement transition using a Polyakov loop to discuss this interdependence. Within NJL model, below the transition temperature, the thermodynamic system is that of a gas of massive constituent quarks as pions are not elementary degrees of freedom. Therefore it is desirable to have a model with mesonic degrees of freedom like Polyakov quark meson coupling model. Some of these investigations is in progress.

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