

Three-dimensional fragmentation function studies in e^+e^- -annihilations at high energies

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The presentation is a summary of a series of papers [1-4] on fragmentation function studies in e^+e^- -annihilation at high energies published recently by the SDU group on QCD and hadron physics. After a brief description of the three-dimensional fragmentation functions defined via quark-quark correlator for hadrons with different spins (0, 1/2 and 1) at leading and higher twists, we discuss how to study them in hadron production processes in e^+e^- -annihilation at high energies. We first present the general framework to express the cross section in terms of the corresponding structure functions based on the general kinematic analysis then give the parton model results for the differential cross sections and/or different spin asymmetries up to twist-3 in terms of gauge invariant fragmentations. The results can serve as a basis for experimental and phenomenological studies in this reaction.

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1. Introduction

In describing high energy reactions, we need two sets of important quantities, the parton distribution functions (PDFs) and the fragmentation functions (FFs). The former are used to describe the hadron structure and the latter describe the hadronization process. In a quantum field theoretical formulation, both PDFs and FFs are defined via the corresponding quark-quark correlator. The quark-quark correlator is defined as a matrix in the Dirac space depending on the hadron states. It is then decomposed into different components expressed in terms of the basic Lorentz covariants and the scalar functions. These scalar functions contain the information of the hadron structure and/or hadronization mechanism and are called the corresponding PDFs or FFs.

Since usually different types of hadrons with different flavors and spins are produced in a high energy reaction, FFs are therefore more involved and perhaps even more interesting but less studied yet. However, because the polarizations can be measured via the angular distributions of the decay products, polarization dependent FFs can be studied and may give even deeper insight in QCD and are necessary for the precise description of high energy reactions.

It is well-known that e^+e^- -annihilation at high energies is the best place to study FFs because there is no hadron is involved in initial states thus only FFs (no PDFs) are involved in this process. The most convenient place to study the spin independent three-dimensional FFs is perhaps $e^+e^- \rightarrow \pi\pi X$, for the vector polarization dependent FFs, $e^+e^- \rightarrow H\pi X$, while for the tensor polarization dependent FFs of vector mesons it is $e^+e^- \rightarrow V\pi X$, where H is a hyperon and V denotes a vector meson. Formally, $e^+e^- \rightarrow \pi\pi X$ and $e^+e^- \rightarrow H\pi X$ are completely the same as the unpolarized and vector polarization dependent part of $e^+e^- \rightarrow V\pi X$. We therefore take $e^+e^- \rightarrow V\pi X$ as example and summarize the main conclusions in connection with the kinematic analysis and parton model results up to twist-3. The main results can be found e.g. in [2].

2. Fragmentation functions defined via the quark-quark correlator

A systematic study is given in a recent paper [2]. Similar to PDFs, in quantum field theory, the quark fragmentation is defined via the quark-quark correlator given by,

$$\hat{\Xi}_{ij}^{(0)}(k_F; p, S) = \frac{1}{2\pi} \sum_X \int d^4\xi e^{-ik_F\xi} \langle 0 | \mathcal{L}^\dagger(0; \infty) \psi_i(0) | p, S; X \rangle \langle p, S; X | \bar{\psi}_j(\xi) \mathcal{L}(\xi; \infty) | 0 \rangle, \quad (2.1)$$

which can be expressed as the sum of a polarization independent part $\hat{\Xi}^{U(0)}$, a vector polarization dependent part $\hat{\Xi}^{V(0)}$ and a tensor polarization dependent part $\hat{\Xi}^{T(0)}$, i.e.,

$$\hat{\Xi}^{(0)}(z, k_{F\perp}; p, S) = \hat{\Xi}^{U(0)}(z, k_{F\perp}; p) + \hat{\Xi}^{V(0)}(z, k_{F\perp}; p, S) + \hat{\Xi}^{T(0)}(z, k_{F\perp}; p, S). \quad (2.2)$$

The FFs are obtained by expanding Ξ in terms of the Γ -matrices followed by Lorentz decompositions of the coefficients. The main results are summarized as follows.

The unpolarized part: For the unpolarized part $\hat{\Xi}^{U(0)}(z, k_{F\perp}; p)$, we obtain 8 TMD FFs, 2 of them contribute at twist-2, 4 at twist-3 and the other 2 at twist-4 level. The 2 twist-2 FFs are the well-known number density D_1 and H_1^\perp that corresponds to the Boer-Mulders function in PDFs.

The vector polarization dependent part: There are 24 vector polarization dependent TMD FFs, 6 of them contribute at twist-2, 12 at twist-3 and the other 6 at twist-4 level. Among them, 8 are

naive T-odd and the other 16 are T-even. At leading twist, we have a D_{1T}^\perp for induced polarization, a longitudinal (G_{1L}), two transverse (H_{1T}, H_{1T}^\perp), a longitudinal to transverse (G_{1T}^\perp) and a transverse to longitudinal (H_{1L}^\perp) spin transfer.

The tensor polarization dependent part: The tensor polarization of a hadron is described by a Lorentz scalar S_{LL} , a transverse Lorentz vector S_{LT} and a transverse Lorentz tensor S_{TT} . There are totally 40 tensor polarization dependent TMD FFs, 10 contribute at twist-2, 20 at twist-3 and the other 10 at twist-4. Among them, 24 are T-odd and the other 16 are T-even. We note in particular the following. (1) Since S_{LL} is a Lorentz scalar, the S_{LL} -dependent TMD FFs have exactly one to one correspondence to the unpolarized part. (2) S_{LT} and S behave differently under space reflection, hence there is one-to-one correspondence for S_{LT} - to S_T -dependent FFs with the replacement of $S_{T\alpha}$ by $\varepsilon_{\perp\alpha\beta} S_{LT}^\beta$ in the Lorentz decomposition. (3) There is a direct one to one correspondence between S_{LT} - and S_{TT} -dependent FFs with the replacement of $S_{LT\alpha}$ by $S_{TT\alpha\beta} k_F^\beta$ in the Lorentz decomposition.

Totally there are 72 TMD FFs, 8 for spin independent, 24 for the vector polarization dependent and the other 40 for the tensor polarization dependent part. Among them, 18 contribute at leading twist, 36 at twist-3 and the other 18 at twist-4; half of them are T -odd, the other half are T -even; also half are χ -odd and the other half are χ -even.

3. Kinematic analysis of $e^+e^- \rightarrow V\pi X$

The basic Lorentz tensors for the hadronic tensor $W^{\mu\nu}(q, p_1, S, p_2)$: After a general kinematic analysis, [2] reaches a very interesting conclusion, i.e., for the polarized part, the complete set of basic Lorentz tensors (BLT) can be expressed by a polarization dependent scalar (or pseudo-scalar) times the unpolarized set. For the unpolarized part, there are 9 such BLTs for $e^+e^- \rightarrow V\pi X$; 27 for the vector polarization dependent part, and 45 for the tensor polarization dependent part. Totally we have 81 such independent BLTs for $e^+e^- \rightarrow V\pi X$, 42 are parity conserving (i.e. space reflection even) and 39 are parity violating.

The general structure for the cross section: The best frame we recommend is helicity Gottfried-Jackson frame that is the center-of-mass frame of e^+e^- where the direction of motion of V is chosen as the z direction and the lepton-hadron (vector meson) plane as the Oxz plane. There are totally 81 structure functions corresponding to the 81 independent BLTs.

The azimuthal asymmetries and hadron polarizations are in general coupled with each other and are described by the corresponding structure functions. In practice, it is much simpler to study the azimuthal asymmetries in the unpolarized case and hadron polarizations averaged over the azimuthal angle φ . For unpolarized hadrons, there are 4 azimuthal asymmetries i.e. $\langle \cos \varphi \rangle_U$, $\langle \sin \varphi \rangle_U$, $\langle \cos 2\varphi \rangle_U$, and $\langle \sin 2\varphi \rangle_U$. The two cosine asymmetries are parity conserving while the two sine asymmetries are parity violating.

The hadron polarizations are most conveniently studied in the helicity Gottfried-Jackson frame. Here, we have two longitudinal components $\langle \lambda \rangle$ and $\langle S_{LL} \rangle$ defined in the helicity basis, and 6 transverse components that can be defined either w.r.t. the lepton-hadron plane, i.e., $\langle S_T^x \rangle$, $\langle S_T^y \rangle$, $\langle S_{LT}^x \rangle$, $\langle S_{LT}^y \rangle$, $\langle S_{TT}^{xx} \rangle$ and $\langle S_{TT}^{xy} \rangle$, or w.r.t the hadron-hadron plane i.e. $\langle S_T^n \rangle$, $\langle S_T^t \rangle$, $\langle S_{LT}^n \rangle$, $\langle S_{LT}^t \rangle$, $\langle S_{TT}^{nn} \rangle$ and $\langle S_{TT}^{nt} \rangle$. Half of them are parity conserving while the other half are parity violating.

4. Parton model results up to twist-3

Structure functions: At leading twist there are 27 non-vanishing structure functions 19 correspond to parity conserving and 8 are parity violating. We have also 36 structure functions that have twist-3 as leading power contributions. We note that they are all the leading power contributions i.e. all these 36 structure functions are zero at twist-2.

Azimuthal asymmetries: At leading twist and for unpolarized V , there is only one azimuthal asymmetry $\langle \cos 2\varphi \rangle_U^{(0)}$ due to Collins effect and transverse spin correlation c_{nm}^q for $q\bar{q}$ produced via e^+e^- annihilation. Up to twist-3, there is another asymmetry $\langle \cos \varphi \rangle_U^{(1)}$ similar as Cahn effect in SIDIS and a parity violating asymmetry $\langle \sin \varphi \rangle_U^{(1)}$.

Hadron polarizations: Longitudinal components of hadron polarization $\langle \lambda \rangle$ and $\langle S_{LL} \rangle$ exist at leading twist. While the former depends on the initial polarization P_q of the quark produced at the e^+e^- annihilation vertex and exists only in weak interaction processes, the latter is independent of P_q and exists also in electromagnetic processes.

Transverse components $\langle S_T^n \rangle$, $\langle S_T^t \rangle$, $\langle S_{LT}^n \rangle$, $\langle S_{LT}^t \rangle$, $\langle S_{TT}^{nn} \rangle$ and $\langle S_{TT}^{tt} \rangle$ w.r.t. the hadron-hadron plane exist at leading twist. Among them $\langle S_T^n \rangle$, $\langle S_{LT}^t \rangle$ and $\langle S_{TT}^{nn} \rangle$ are parity conserving and $\langle S_T^t \rangle$, $\langle S_{LT}^n \rangle$ and $\langle S_{TT}^{nt} \rangle$ are parity violating.

There are also twist-3 transverse components $\langle S_T^x \rangle$, $\langle S_T^y \rangle$, $\langle S_{LT}^x \rangle$, $\langle S_{LT}^y \rangle$, $\langle S_{TT}^{xx} \rangle$ and $\langle S_{TT}^{xy} \rangle$ w.r.t. lepton-hadron plane. They are determined by the corresponding twist-3 FFs. Similarly, $\langle S_T^y \rangle$, $\langle S_{LT}^x \rangle$ and $\langle S_{TT}^{xx} \rangle$ are parity conserving and $\langle S_T^x \rangle$, $\langle S_{LT}^y \rangle$ and $\langle S_{TT}^{xy} \rangle$ are parity violating.

5. Summary and discussion

A systematic study has been made for $e^+e^- \rightarrow V\pi X$. With a general kinematic analysis we obtain the general form of the cross section in terms of the structure functions and the calculations have been carried out that lead to the complete twist-3 result in QCD parton model. The results provide a general framework for future experimental and phenomenological studies on three-dimensional FFs in e^+e^- -annihilation at high energies. We emphasize in particular that although the number of FFs and that of the independent structure functions are very large, the results show great regularities that allow us to study them case by case. Such studies are not only necessary for a precise description of hadron production in high energy collisions but also could provide us a more sensitive window to study polarization effects in high energy reactions in particular and to develop QCD theory in general.

References

- [1] K. b. Chen, W. h. Yang, Y. j. Zhou and Z. t. Liang, arXiv:1609.07001 [hep-ph].
- [2] K. b. Chen, W. h. Yang, S. y. Wei and Z. t. Liang, Phys. Rev. D **94**, no. 3, 034003 (2016) doi:10.1103/PhysRevD.94.034003 [arXiv:1605.07790 [hep-ph]].
- [3] S. Y. Wei, K. b. Chen, Y. k. Song and Z. t. Liang, Phys. Rev. D **91**, no. 3, 034015 (2015) doi:10.1103/PhysRevD.91.034015 [arXiv:1410.4314 [hep-ph]].
- [4] S. y. Wei, Y. k. Song and Z. t. Liang, Phys. Rev. D **89**, no. 1, 014024 (2014) doi:10.1103/PhysRevD.89.014024 [arXiv:1309.4191 [hep-ph]].