

## The Fate of the Higgs Vacuum

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This talk reviews our recent work showing how tiny black holes can act as nucleation sites for the decay of the metastable Higgs vacuum [1, 2, 3, 4]. We start by discussing the formation of thin wall bubbles of true vacuum inside a false vacuum, and show how adding a black hole lowers the action of the Euclidean tunneling solution, thus strongly enhancing the probability of vacuum decay. We then review numerical results for the Higgs vacuum showing that the decay rate is even higher for these “thick wall” bubbles. The results imply either tiny black holes are not a component of our universe, or BSM corrections to the Higgs potential must stabilise our vacuum.

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## 1. Introduction

One of the more curious aspects about discovery of the Higgs [5, 6], is that its mass suggests that our universe is metastable. The running of the Higgs self-coupling indicates the true Higgs vacuum lies at large expectation values of the Higgs and negative vacuum energy [7]. Although a metastable vacuum is somewhat disconcerting, the key factor is the lifetime for its decay: a region of the universe must tunnel through a sizeable energy barrier, with a typical probability dominated by an exponential factor [8]

$$\Gamma \sim Ae^{-B/\hbar} \quad (1.1)$$

where  $B$  is the action of a solution to the Euclidean field equations interpolating from the metastable (false) to the true vacuum. If this action is large, then the lifetime of our vacuum can be many orders of magnitude greater than the age of our universe, and therefore not necessarily a problem.

The Euclidean solution, or “bounce”, was analysed by Coleman and collaborators [9, 10, 11], whereby the decay was understood as bubble nucleation. The idea is that a bubble of true vacuum forms within the false, and typically there is an energy balance between the ‘cost’ of the bubble wall (the energy barrier between false and true vacua) and the ‘gain’ from the interior of the bubble now being at lower energy. Optimising this energy pay-off gives the critical bubble size that corresponds to the Euclidean solution – the instanton – that drives vacuum decay. Once formed, the bubble expands, and we have a first order phase transition from the false to the true vacuum.

This picture, while intuitive, is incomplete, as we must take into account gravity: a false vacuum energy will gravitate, and we must now consider bubbles between false vacua of one cosmological constant to true vacua of a lower cosmological constant. In [11], Coleman and de Luccia (CDL) did precisely this. It turns out that gravity completely fixes the bubble radius from the energy of the wall. The CDL instanton action is simply a curved space generalisation of the flat space bubble.

All of these calculations however are rather idealised, they refer to a single bubble in a completely pure and featureless universe. In reality however, our universe is not featureless, and phase transitions are rarely clean, indeed, they are often catalysed by the presence of an impurity. How dependent are the results of Coleman et al. on the assumptions of homogeneity and isotropy? Here, we briefly review our work [1, 2, 3, 4] exploring this issue by introducing a simple gravitational impurity, a black hole, and showing how even a single tiny black hole can overturn our picture of how stable our universe is.

## 2. Thin Wall Tunneling

In thin wall tunneling [11, 1, 2, 3], the physical input is that we have a potential with two local minima (false and true vacuum) with a sufficient energy barrier that a Euclidean solution interpolating between the two will be very thin in comparison to the radius of the instanton bubble so that we can use the Israel formalism [12]. While this requires a putative quantum gravity correction to the Higgs potential to ensure a second, stable, true vacuum (see next section) it allows us to easily analytically explore the impact of a black hole, and provides a proof of principle of black hole catalysis of decay.

In [11], CDL showed how to construct the gravitational version of Coleman's thin wall calculation [9] that extracted the key physics from the decay process. In gravity, provided we have enough symmetry, we can solve the Euclidean Einstein equations for a thin wall analytically, and CDL found an explicit expression for the action of a bubble between different vacua in terms of  $\sigma$ , the energy per unit area of the wall, and  $\mathcal{E}$ , the difference in vacuum energy.

Adding a black hole to the system turns out to be straightforward, the relevant results for black holes and walls were derived in [13], which proved that the general solution is a bubble wall separating two black hole spacetimes

$$ds^2 = f_{\pm}(r)d\tau_{\pm}^2 + \frac{dr^2}{f_{\pm}(r)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \quad \text{where} \quad f_{\pm}(r) = 1 - \frac{2GM_{\pm}}{r} + \frac{\Lambda_{\pm}r^2}{3} \quad (2.1)$$

Inside the bubble, we have a vacuum energy  $\mathcal{E}_- = \Lambda_-/8\pi G$ , and possibly a remnant black hole of mass  $M_-$ . Outside the bubble, the vacuum has energy  $\Lambda_+/8\pi G$ , and  $M_+$  is the mass of the seed black hole catalysing the decay. The bubble lies at some  $r = R(\tau)$ , satisfying a Friedmann-like equation

$$\left(\frac{\dot{R}}{R}\right)^2 = (2\pi G\sigma)^2 - \frac{f_+ + f_-}{2R^2} + \frac{(f_+ - f_-)^2}{16R^4(2\pi G\sigma)^2}. \quad (2.2)$$

For very small mass black holes, the solution is time dependent, and the lowest action instanton has no remnant black hole – the instanton is thus a perturbed CDL bubble. For larger seed masses however the nature of the instanton changes, and the bubble is now 'static' and contains a remnant black hole.

To find the action, we have to calculate the difference between the background black hole solution and the black hole with bubble solution. One of the key technicalities resolved in [1] is the treatment of conical singularities that can arise in the Euclidean solution, we refer the reader to [1] for full detail. The static tunnelling instanton is the one which is most relevant for vacuum decay of our universe, and for this case a remarkable simplification results in the action depending only on the areas of the event horizons,

$$B = \frac{\mathcal{A}_+}{4G} - \frac{\mathcal{A}_-}{4G} \quad (2.3)$$

We recognise these terms as the black hole entropies, and the vacuum decay rate is therefore consistent with the entropy-fluctuation formula first discovered by Einstein:  $\Gamma \propto e^{\Delta S}$ . The rate is considerably larger than the CDL vacuum tunnelling rate, however, there is another quantum decay channel for a small black hole: Hawking evaporation.

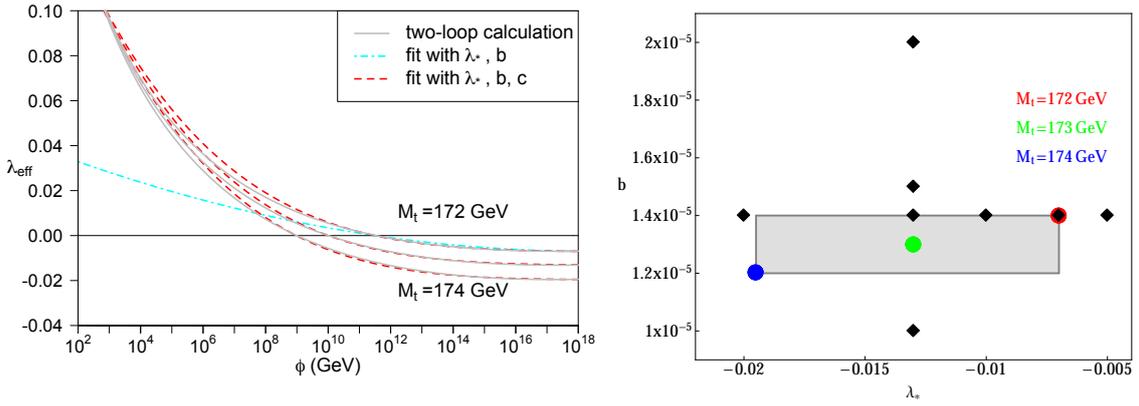
Black holes emit Hawking radiation, and in consequence lose mass, leading to a finite lifetime of order  $\Gamma_H \approx 3.6 \times 10^{-4} (G^2 M_+^3)^{-1}$  [16]. To estimate our vacuum decay, we need both the instanton action and an estimate for the prefactor  $A$ . It turns out that for seed masses sufficiently above the Planck scale that we trust our approximations, we are well into the static instanton branch, for which the action simplifies to the entropic form:  $B = \pi(r_+^2 - r_-^2)/G$ , and following Callan and Coleman, we determine  $A$  by taking a factor  $(B/2\pi)^{1/2}$  for the translational zero mode of the instanton, estimating the determinant piece at  $(GM_+)^{-1}$  by dimensional analysis. Putting together, we obtain the branching ratio for tunneling over evaporation as:

$$\frac{\Gamma_D}{\Gamma_H} \approx 40 \frac{M_+^2}{M_P^2} \sqrt{B} e^{-B} \quad (2.4)$$

This equation will be a key factor in the determination of the relevance of vacuum tunneling. It is correct whether or not the thin wall approximation is used, and simply requires knowledge of the bounce action,  $B$ . For the thin wall, it turns out (roughly) that  $B \propto M_+/M_p$ , leading to a branching ratio greater than one for a range of thin wall data. Whether or not this is relevant to the Higgs potential we now determine.

### 3. Higgs metastability

In order to decide whether enhanced vacuum decay is relevant for the Higgs, we must explore the instantons for the actual Higgs potential. This requires a full numerical analysis of the instantons for a range of parameter space relevant to the standard model (SM), and we now review [2, 4], first discussing our modelling of the running of the coupling, then the bubble solutions. The high energy effective potential for the Higgs field within the standard model has been determined by a two-loop calculation [7], and is conventionally written in terms of an effective coupling, as  $V(\phi) = \frac{1}{4}\lambda_{\text{eff}}(\phi)\phi^4$ . The main uncertainty in the potential comes from the top quark mass uncer-



**Figure 1:** Left: The analytic modelling of the Higgs coupling demonstrating the fit with the two-loop calculation over a wide range of scales. Right: The parameter space explored numerically. The shaded box representing the SM range, and diamond markers the specific parameter values for we used.

tainty, and for given  $m_t, m_H$  can be computed by direct numerical integration of the  $\beta$ -functions [7]. Rather than compute the precise potential for each possible value of  $(m_t, m_H)$ , we instead take an analytic three-parameter fit to the potential

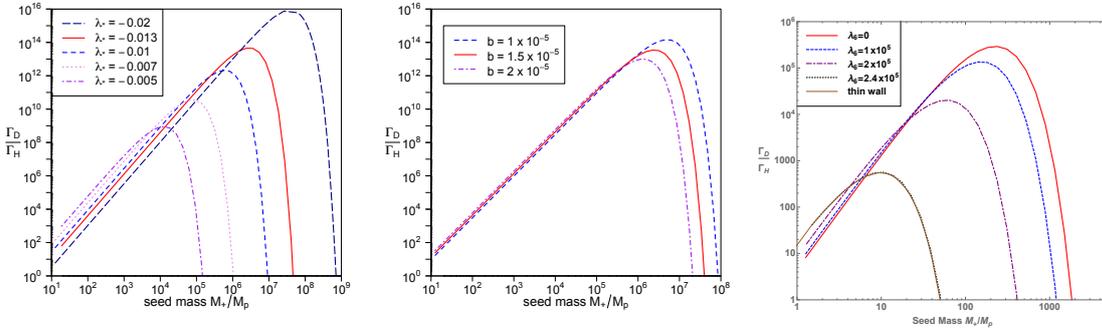
$$\lambda_{\text{eff}}(\phi) = \lambda_* + b \left( \ln \frac{\phi}{M_p} \right)^2 + c \left( \ln \frac{\phi}{M_p} \right)^4. \quad (3.1)$$

that gives a much better fit over the large range of  $\phi$  relevant for tunnelling phenomena, and allows us to explore parameter space beyond the standard model. In practise, we fix the value of  $\lambda_{\text{eff}}$  at the electroweak scale, which leaves two fitting parameters that we take to be  $\lambda_*$  and  $b$ . Figure 1 shows the fit to  $\lambda_{\text{eff}}$ , and the range of parameter space we explore relative to the standard model.

For the pure SM running, there is no thin wall bubble, and instantons must be found by numerical integration. In order to get a second minimum and make contact with the thin wall results of the previous section, one typically adds quantum gravity motivated terms such as  $\frac{\lambda_6 \phi^2}{6M_p^2}$  [14, 15], with increasing  $\lambda_6$  taking us to the thin wall limit (see figure 2). Motivated by the thin wall results, we search for a static bounce solution to the Euclidean Einstein-scalar equations on a black hole background. These are found by using a spherically symmetric Ansatz for the metric

$$ds^2 = f(r)e^{2\delta(r)}d\tau^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad \text{where} \quad f = 1 - \frac{2G\mu(r)}{r}. \quad (3.2)$$

and integrating the scalar and metric equations numerically (see [4] for full details). Given that the instanton is static, the action is given by the difference in area between the seed and remnant black holes. We then input this action in the branching ratio (2.4) to determine the relative risk for vacuum decay as shown in figure 2.



**Figure 2:** The dependence of the branching ratio on the various model parameters. On the left, the dependence of ratio on  $\lambda_*$ , in the middle, on  $b$  and on the right the dependence on a quantum gravity motivated  $\lambda_6$  term showing the transition from thick to the analytic thin wall.

What figure 2 shows is that for small black holes that are nonetheless sufficiently larger than the Planck mass so that the semiclassical approximation is valid, vacuum decay is enormously dominant over evaporation.

#### 4. Conclusion

We have demonstrated by analytical modelling backed up by numerical results, that the lifetime of our universe is crucially dependent on the existence of tiny inhomogeneities: the presence of even a single tiny black hole in our Hubble volume could trigger nucleation to a universe with a very different “standard model”. Such small black holes could arise as relics from the primordial phase of the universe [17]. Although black holes produced in the early universe start out with masses many orders of magnitude higher than  $M_p$ , they gradually evaporate until they come into the range shown in figure 2. At this point, the tunneling half life becomes smaller than the (instantaneous) Hawking lifetime of the black hole:  $\sim 10^{-28}$ s for a  $10^5 M_p$  mass black hole! It is clear that once a primordial black hole nears the end of its life cycle, it *will* seed vacuum decay.

Another scenario where small black holes arise is as a by-product of collisions at the LHC [18] in some large extra dimension models. Such black holes have some higher-dimensional features, notably a different entropic relation, and a preliminary analysis [4] suggests they may be less problematic, though we are exploring this in more detail.

Our conclusions of course depend on the existence of small black holes, and on the running of the Higgs coupling beyond standard model scales – thus, our continued existence suggests either that there are no primordial black holes, or that physics beyond the SM stabilises the Higgs potential. Either way, these results show that the issue of metastability of our universe may not be as simple as was initially thought.

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