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Near-BPS Skyrmions: recent developments

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We present our most recent results regarding near-BPS Skyrmions and argue that they provide an improved description of nucleons and nuclei. For some years now, the Skyrme Model has been considered a natural candidate for a low-energy effective theory of QCD, a point of view supported by results coming from $1/N_c$ expansion and holographic QCD. This framework leads to an attractive picture where baryons (and nuclei) emerge as topological solitons with a topological number identified to the baryon number. But even the most naive Skyrme Model extensions have been plagued with the same problem: they predict large binding energies for the nuclei. On the other hand, the solutions that arise from the more recently proposed near-BPS Skyrme model nearly saturate the Bogomol'nyi bound which means that by construction they must have small binding energies. We address here the issue regarding the energy minimizer which remains unknown for $A > 1$ by proposing a more appropriate ansatz than the usual axially symmetric solution.

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1. Introduction

Despite the successes of the Standard Model, there remain unanswered questions among which one of the most important is due to our inability to provide a clear explanation on confinement of quarks and gluons from QCD. Yet there are indications coming from the $1/N_c$ expansion approach and more recently from holographic QCD (1) that the low-energy limit of QCD should lead to an effective theory of mesons in which nucleons and nuclei emerge as topological solitons. It is somewhat ironic that such a theory, the Skyrme Model (2), was already proposed and almost forgotten due to the advent of QCD. The model, in its original form, provides a direct link between baryons and soft-pion physics. It succeeds in predicting the properties of the nucleon within a 30% accuracy which is considered a rather good agreement for a model involving only two free parameters. Some attempts to improve the model have given birth to a number of extensions relying, to some extent, on our ignorance of the exact form of the low-energy effective Lagrangian of QCD namely, the structure of the potential term, the contribution of other vector mesons or simply the addition of higher-order terms in derivatives of the pion fields (3). This generated extensive applications to low energy phenomenology of baryons. Unfortunately, in its most naive versions, the model fails to give an appropriate account of multibaryon physics or nuclei. Among the most common problems are large binding energies, shell-like baryon density configurations with unexpected discrete symmetries, as well as nuclear radius that grows as $\sqrt{A}$ instead of the usual $|A|^{1/3}$ mass number dependence.

In a recent attempt to improve the model, it was pointed out in refs. (4; 5) that if the model was to be constructed along the lines of a BPS model it would have zero or small binding energies. A BPS soliton saturates the Bogomol'nyi bound leading to a static energy $E_{\rm BPS}(A) = E_{\rm BPS}(1)\,|A|$ and no binding energy which come close to what is observed experimentally. The present work is based on the more realistic extension of the original Skyrme Model called near-BPS Skyrme Model (5; 6). The model has been shown to replicates some basic relations for nuclei: (a) a small but non zero binding energies, (b) a nuclear radius that grows $|A|^{1/3}$, (c) solutions that possess the symmetries of incompressible fluid and more. But the most interesting improvements are with regards to the baryon density configurations and binding energies per nucleon B/A which are in much better agreement with experimental data. For example, whereas the values of B/A predicted by the Skyrme Model are too large by at least an order of magnitude, near-BPS Skyrme models give more realistic values as can be seen from the results of ref. (6) shown in Fig. 1.

There remains however an open question: what is the energy minimizing solution and does it affect the nuclei properties? The pure BPS Skyrme Model possesses an infinite number of such solutions with the same energy so one usually resort to the simpler axially symmetric (AS) ansatz to perform calulations (4). For the near-BPS models, lowest energy solutions are unknown for $A > 1$ and using the usual AS solution as an approximation leads to a potentially problematic behavior as $E_{\rm nBPS}$ grow as $A^{7/3}$ for large A. On the other hand, a complete analysis of this class of models is numerically difficult so here, in the absence of an exact energy minimizer, we aim for a simpler prospective analysis and propose to extend the usual axially symmetric ansatz to "multilayer" solutions. It turns out that distributing the energy among several layers is energically favored and modifies how nuclei masses depend on A to a more acceptable behavior, i.e. $E_{\rm nBPS}$ grows roughly as A for large A.

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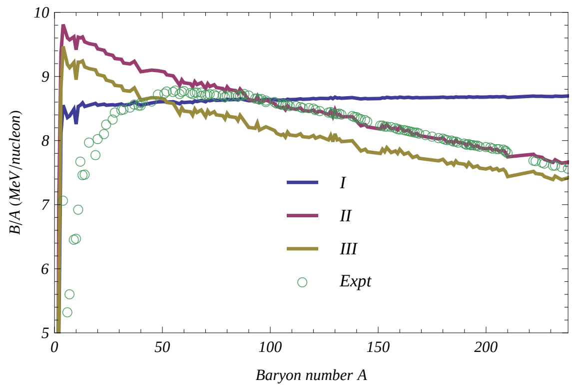

Figure 1: Binding energy per nucleon B/A for the near-BPS Skyrme model. Experimental data are compared with model predictions coming from different sets of fitted parameters μ,α,β,λ (ref. (6)). The original Skyrme Model predictions are at least an order of magnitude too large.

2. Near-BPS Skyrme Model

In extending the Skyrme Model (2) to a regime where the solutions become near-BPS solitons so that $M_{\text{nuclei}} \approx A \cdot M_{\text{nucleon}}$, we consider the Lagrangian density

$$\mathscr{L}_{\text{nBPS}} = \underbrace{\mathscr{L}_2 + \mathscr{L}_4}_{\mathscr{L}_{\text{Skyrme}}} + \underbrace{\mathscr{L}_0 + \mathscr{L}_6}_{\mathscr{L}_{\text{BPS}}} \tag{2.1}$$

The first part, $\mathscr{L}_{\text{Skyrme}}$, is the original Skyrme Model[1] consisting of the nonlinear-σ and the Skyrme terms, which are written respectively as,

$$\mathscr{L}_2 = -\alpha \text{Tr}\left[L_\mu L^\mu\right] \quad \text{and} \quad \mathscr{L}_4 = \beta \text{Tr}\left(\left[L_\mu, L_\nu\right]^2\right).$$

The pion fields π_i are introduced through in $U = \exp(-2i(\vec{\tau}\cdot\vec{\pi})/F_\pi)$ where $U \in SU(2)$, $L_\mu = U^\dagger \partial_\mu U$ and F_π is the pion decay constant. The second part, $\mathscr{L}_{\text{BPS}}$, corresponds to the BPS Skyrme Model proposed by Adam et al. (4). It contains a potential term V and the term of order six in derivative of the pion fields respectively

$$\mathscr{L}_0 = -\mu^2 V(U) \quad \text{and} \quad \mathscr{L}_6 = -\frac{3}{2}\frac{\lambda^2}{16^2}\text{Tr}\left(\left[L_\mu, L_\nu\right]\left[L^\nu, L^\lambda\right]\left[L_\lambda, L_\mu\right]\right) = -\lambda^2\pi^4 \mathscr{B}^\mu \mathscr{B}_\mu \tag{2.2}$$

where $\mathscr{B}^\mu = \frac{1}{24\pi^2}\varepsilon^{\mu\nu\rho\sigma}\text{Tr}\left(L_\nu L_\rho L_\sigma\right)$ is the baryon (or topological) current. So here, $\mathscr{L}_6$ has a special meaning: it is the square of the pullback of the volume form in target space. Finite energy solutions require a conserved topological charge identified with the nuclear mass number which also corresponds to the baryon number

$$A = \int d^3 r \mathscr{B}^0 \tag{2.3}$$

The BPS model energy minimizer saturates of the Bogomol'nyi bound so the static energy $E_{\text{BPS}} \propto A$. Using the general form $U = \cos F + i\hat{n}\cdot\tau \sin F$ with $\hat{n} = (\sin\Theta\cos\Phi, \sin\Theta\sin\Phi, \cos\Theta)$

[1] A potential term $\mathscr{L}_0$ such as in eq. (2.2) is sometimes added.

where $F = F(\mathbf{r})$, $\Theta = \Theta(\mathbf{r})$ and $\Phi = \Phi(\mathbf{r})$, ones finds an infinite number solutions with same lowest energy $E_{\text{BPS}} = 2\mu\lambda\pi^2 \left\langle \sqrt{V} \right\rangle_{S^3} |A|$ provided $\mu\sqrt{V} = \mp\lambda\pi^2 \left(\sin^2 F \sin\Theta \nabla F \cdot (\nabla\Theta \times \nabla\Phi)\right)$. This cannot be achieved in the Skyrme Model whose minimizing energy exceeds the bound by as much as 23% leading eventually to unsatisfactory large binding energies for nuclei. On the other hand, in the BPS model, the binding energies are completely absent at the level of the static energy. Moreover, the model also lacks the kinetic term in $\mathscr{L}_2$ that would define a proper propagator.

The idea behind the near-BPS Skyrme Model (5) in (2.1) is to depart slightly from the BPS model assuming the term $\mathscr{L}_{\text{BPS}}$ dominates while treating $\mathscr{L}_2$ and $\mathscr{L}_4$ as small perturbations. The model then allows for static energies E_{nBPS} nearly proportional to A, small but non-zero binding energies as well as a kinetic term. Unfortunately, the lowest energy solutions for the near-BPS model are not known for $A > 1$ and cannot be guessed from the dominant part $\mathscr{L}_{\text{BPS}}$ since it possesses an infinite number of degenerate solutions. Accordingly, it is both the $\mathscr{L}_2$ and $\mathscr{L}_4$ parts (which are assumed to be perturbations) that fix the shape of the energy minimizing solutions. Because of that even numerical attempts have failed for small values of α and β. One then usually considers an axially symmetric ansatz as in ref. (4) but in the context of the near-BPS model the energies E_{nBPS} grow as $A^{7/3}$ for large nuclei, as opposed to the linear behavior observed experimentally. Furthermore, it was shown in (7) that the solutions should instead be a so-called restricted harmonic. Yet this constraint is not sufficient to determine uniquely the lowest energy solution.

With that in mind, we resort to a more elaborate ansatz. The idea is to allow the axially symmetric solution to apply independently over a number of $L \leq A$ concentric layers each corresponding a segment of length π in $F(r)$ as r goes from 0 to infinity. The solution takes the form

$$U = \cos F(r) + i\hat{\mathbf{n}}_l \cdot \tau \sin F(r) \qquad \text{for} \qquad 0 \leq F \leq L\pi.$$

Each layer is allowed to wind up m_l times around the axis of symmetry

$$\hat{\mathbf{n}}_l = (\sin\theta\cos m_l\varphi, \sin\theta\sin m_l\varphi, \cos\theta) \tag{2.4}$$

thereby carrying a topological charge m_l such that $A = \sum_{l=1}^{L} m_l$. Allowing for multilayer solutions, $L \geq 1$, we get

$$E_{\text{nBPS}} = \sum_{l=1}^{L} [\underbrace{\left(a_0^l + a_6^l\right) m_l}_{E_0+E_6} + \underbrace{m_l^{1/3}\left(a_2^l + b_2^l m_l^2\right)}_{E_2} + \underbrace{m_l^{-1/3}\left(a_4^l + b_4^l m_l^2\right)}_{E_4}] \tag{2.5}$$

where E_i are the respective static energies contribution of the Lagrangians in eq. (2.1). Once the model dependent quantities a_i^l, b_i^l are computed for each layer l, it remains to find which configuration $(m_1, m_2, \ldots, m_L)$ has the lowest energy $E_2 + E_4$. Here all layers contribute equally to E_0 and E_6 making them invariant under the choice of configuration. The axially symmetric ansatz in (4) corresponds to the 1-layer case ($L = 1, m_1 = A$ in (2.5)) and generates terms proportional to $A^{7/3}$ and $A^{5/3}$ which are problematic for large nuclei. It turns out that the distribution of the topological charge over several layers attenuates the large A behavior.

It should be noted that in order that the solution (2.4) remains a minimizing solution of $\mathscr{L}_{\text{BPS}}$, it must saturate the Bogomol'nyi bound requiring that $\left\langle \sqrt{V} \right\rangle_l$ be the same for all layers. Taking this into account, we proposed three simple multilayer near-BPS models using $\beta = 0$: (a) Model

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1 (constant potential): $V_{\mathrm{M1}}(F)=1$, (b) Model 2 (fluctuating potential): $V_{\mathrm{M2}}(F)=\sin^4 F$ and (c) Model 3 (constant potential dropping near surface): $V_{\mathrm{M3}}(F)=1$ for $F\geq F_0$ and $V_{\mathrm{M3}}(F)=\frac{\sin^4 F}{\sin^4 F_0}$ for $0\leq F\leq F_0$ where F_0 is chosen so that $\left\langle\sqrt{V}\right\rangle_l$ are all equal.

The apparently simple form of these models however hide a technical difficulty. When the model allows for zeros in the quantity $\frac{\sin^2 F}{\sqrt{2V(F)}}$, the BPS differential equation for F causes F' and E_2 to diverge (for example here V_{M1} and V_{M3} at $F=n\pi$ with $n\in\mathbb{N}^*$). In such cases, one cannot neglect $\mathscr{L}_2$ and F can be obtained from

$$F'(x)=-\left[\frac{1}{V}\left(\left(\frac{m_l}{24A}\frac{\sin^2 F}{x^2}\right)^2+A^{2/3}\gamma^2\right)\right]^{-1/2} \tag{2.6}$$

where $x=ar$ with $a=(\mu/18A\lambda)^{2/3}$. Otherwise the BPS solution (the solution of (2.6) with $\gamma=0$) exists for all F and can be used to estimate E_2. But for such models (V_{M2} for example), the baryon density must be zero at the frontier of each layer where $F=n\pi$ thereby forming distinct concentric shells in the baryon density.

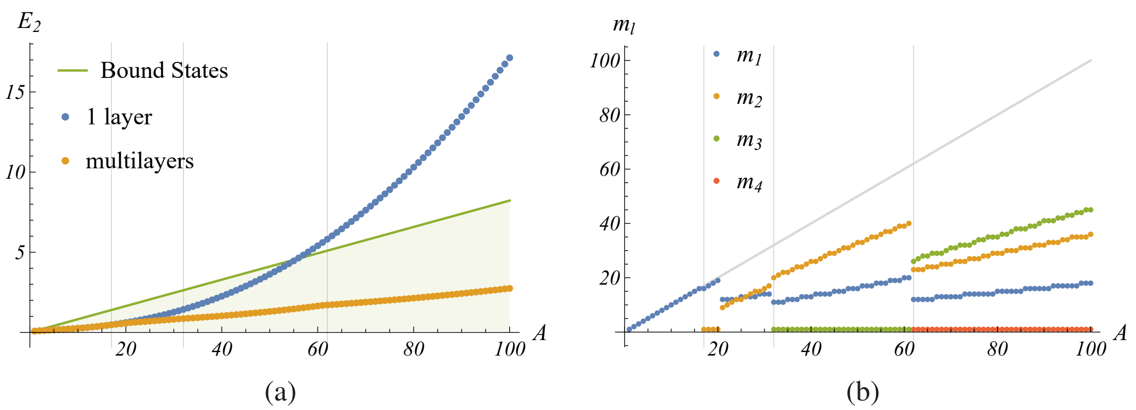

Figure 2: Model 1: (a) Static energy E_2 (in units of $\mu\lambda$) for single and multilayer configurations. Configurations lying in the shaded area are bound states. (b) Distribution of m_l for energy minimizer. The vertical gray lines indicates transitions in the number of layers.

The search for a minimizing configuration requires that we set $\gamma=\sqrt{2\alpha}\left(18\lambda\mu^2\right)^{-1/3}$ whereas the parameters μ and λ may be use to rescale the energyand radial distance in units of $\mu\lambda$ and a respectively. They can be set arbitrarily for our purposes. In Fig.2, we present our results for Model 1 using $\gamma=0.001$, which is typical of the values obtained in previous fit for near-BPS calculations (5). The static energy E_2 is compared for single and multilayer configurations in Fig.2(a). All configurations in the shaded area are lower in energy than A infinitely separated skyrmions and can be considered as bound states. We find that, not only the multilayer configurations are energy favored for large A, but E_2 seems to grow almost linearly with A in that limit. This behavior is certainly more in agreement with experiment than the 1-layer results. Furthermore, Fig.2(b) illustrate how the baryon number is distributed among layers (layer 1 is the innermost layer). It turns out that for all models, the number of layers increases rather slowly with A and one also observe some regularity on how A is distributed among layers. However, this exact pattern of distribution seems to be model dependent. For example in Model 1 (see Fig. 2(b)), the baryon number carried by each layer m_l generally increases as it gets further from the center except for the outermost layer which carry the lowest charge. Model 2 and Model 3 however follow completely different patterns yet multilayers are still energically favored.

3. Conclusion

In this work, we proposed a multilayer axially symmetric ansatz for the near-BPS Skyrme model. Our calculations show that multilayer configurations are energically favored as the baryon number A increases and that they correspond to bound states. This should remains true for even larger A, at least for the prototype models at hand, since their static energy E_{nBPS} only grows as $\sim A$. So, although the exact energy minimizer remains unknown for $A > 1$, we can assume that near-BPS Skyrmions are bound states as well. Further analysis is required to investigate the cases with $\beta \neq 0$, compute rotational and Coulomb contributions to the nuclear masses and verify if the near-BPS models successes (such as the results for B/A shown in Fig. 1) are preserved. More generally, these results support the idea that nuclei could be topological solitons emerging from an effective field theory of mesons, perhaps as near-BPS Skyrmions.

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