PROCEEDINGS OF SCIENCE

The curvature of the chiral pseudo-critical line from lattice QCD

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The study of the temperature - baryon chemical potential $T-\mu_B$ phase diagram of strongly interacting matter is being performed both experimentally and by theoretical means. The comparison between the experimental chemical freeze-out line and the crossover line, corresponding to chiral symmetry restoration, is one of the main issues. At present it is not possible to perform lattice simulations at real μ_B because of the sign problem. In order to circumvent this issue, we make use of analytic continuation from an imaginary chemical potential: this approach makes it possible to obtain reliable predictions for small real μ_B. By using a state-of-the-art discretization, we study the phase diagram of strongly interacting matter at the physical point for purely imaginary baryon chemical potential and zero strange quark chemical potential μ_s. We locate the pseudocritical line by computing two observables related to chiral symmetry, namely the chiral condensate and the chiral susceptibility. We then perform a continuum limit extrapolation with N_t =6,8,10 and 12 lattices, obtaining our final estimate for the curvature of the pseudocritical line $\kappa = 0.0135(20)$. Our study includes a thorough analysis of the systematics involved in the definition of $T_c(\mu_B)$, and of the effect of a nonzero μ_s.

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1. The critical line of QCD and the method of analytic continuation

The study of strongly interacting matter in extreme conditions is of great interest for several reasons, due to its relation with, e.g., the physics of compact stars and of the early universe. From the experimental side, data coming from heavy ion experiments can be used to probe the phase diagram of QCD (see, e.g., [1]). As a matter of fact little is known from a theoretical standpoint about the QCD phase diagram at finite baryon density, at least as far as deductions from first principle are concerned: the infamous *sign problem* hinders numerical simulations in that regime. On the other hand, at zero density the picture looks pretty clear: lattice simulations have reliably confirmed the existence of a smooth crossover between the confined low-temperature region, where chiral symmetry is broken, and the deconfined high-temperature one where chiral symmetry is restored. The position of the crossover between the two phases can be then identified with a pseudocritical temperature T_c, which is around 155 MeV as far as chiral symmetry is concerned.

For small values of the baryon chemical potential μ_B, this picture is expected to remain valid: we can thus define the *pseudocritical line* of QCD as the set of points in the phase diagram for which $T = T_c(\mu_B)$. It has been possible to study $T_c(\mu_B)$ in QCD with physical quark masses with two approaches: *Taylor expansion* [2, 3, 4] and *analytic continuation* from imaginary μ_B [5], which was recently employed in [6, 7] and in the work presented here [8]. Analytic continuation relies on the fact that for a purely imaginary chemical potential the Dirac operator retains the property that guarantees its determinant at $\mu_B = 0$ to be real, thus removing the sign problem present at real μ_B.

Given the parity of the QCD partition function in μ_B, the behaviour of T_c with μ_B should be of the form

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B)}\right)^2 + O(\mu_{B,I}^4) = 1 + \kappa \left(\frac{\mu_{B,I}}{T_c(\mu_{B,I})}\right)^2 + O(\mu_{B,I}^4)\,, \tag{1.1}$$

assuming analyticity for $\mu_B = 0$. In the last part of the equation, the imaginary baryon chemical potential $\mu_{B,I} = -i\mu_B$ is introduced. The present work is aimed to determine the coefficient κ, i.e. the curvature of the crossover line of QCD, performing a careful analysis of the systematic errors involved in order to obtain a reliable continuum extrapolated estimate.

The bulk of our simulations is performed with the strange quark chemical potential μ_s set to zero. This is not a priori the right setup to compare with heavy ion collision experiments where *the strangeness S* is zero, since $\partial n_s/\partial \mu_l$ is not null. We should in fact fine-tune the values of μ_l and μ_s to reach *strangeness neutrality*: for example at $T = T_c$, $n_S = 0$ would require $\mu_s \simeq 0.25\mu_l$ [9]. For these considerations, we scheduled a number of simulations to study the setup with $\mu_s = \mu_l$ as well, in order to obtain data in a range which is supposed to cover also the $n_s = S = 0$ case.

2. Observables

For the determination of $T_c(\mu_B)$, we used 2 different prescriptions based on 3 quantities related to chiral symmetry in the up and down quark sector. We looked at the light chiral condensate, renormalized in two different ways $\langle\bar{\psi}\psi\rangle^r_{(1)}$ and $\langle\bar{\psi}\psi\rangle^r_{(2)}$ (introduced in [10] and [4], respectively) and at the full renormalized chiral susceptibility $\chi^r_{\bar{\psi}\psi}$ [11]. We define the crossover temperature as the abscissa of the inflection point for the condensates, and as the abscissa of the maximum in the case of the chiral susceptibility, locating these points by a fitting procedure (see Fig.1). The data for the renormalized chiral condensate(s) has been fitted with an arctangent function, while for the renormalized chiral susceptibility we opted for a Lorentzian form:

$$\langle\bar{\psi}\psi\rangle^r(T) = A_1 + B_1 \arctan\left[C_1\left(T - T_c\right)\right] \quad , \quad \chi^r_{\bar{\psi}\psi}(T) = \frac{A_2}{(T-T_c)^2 + B_2^2} \ . \tag{2.1}$$

These definitions of $T_c(\mu)B)$ can be considered *faithful*, as for a *proper* phase transition (i.e., not the analytical crossover present with physical quark masses) they would give the corresponding critical temperature.

Our strategy is to determine T_c for a number of values of $\mu_{B,I}$ and fit the data points with the expression in Eq.(1.1) to obtain κ.

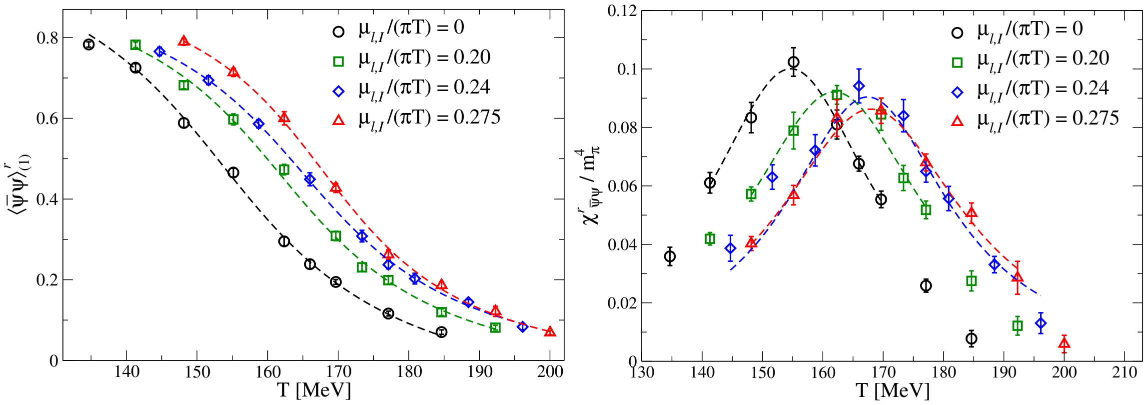


Figure 1: Dependence of the renormalized observables on the temperature. The results of the fits are also plotted. (from [8]). **Left:** renormalized chiral condensate $\langle\bar{\psi}\psi\rangle^r_{(1)}$. **Right:** renormalized chiral susceptibility $\chi^r_{\bar{\psi}\psi}$.

3. Numerical setup

Our lattice simulations were performed making use of a tree level Symanzik improved gauge action with root-staggered stout-improved fermions, *at the physical point*, which means we tuned the quark masses with β following the *line of constant physics*[1] obtained in [12] (see [8] for details).

An analysis of finite size effects was done in [8], where we found that for lattice aspect ratio L/T equal to 4 such effects were negligible. This translates to a lattice spatial size of about 5 fm (for temperatures around $T_c(0)$). For this reason, we have since then run simulations on $32^3 \times 8$, $40^3 \times 10$ and $48^3 \times 12$ lattices in order to perform a continuum limit extrapolation of κ in the setup $\mu_s = 0$.

To compute renormalized observables we also had to measure bare quantities in zero temperature setups (on 32^4 and $48^3 \times 96$ lattices).

The effect of a nonzero μ_s has been studied with simulations in which $\mu_s = \mu_l$, in addition to the "standard" ones at $\mu_s = 0$, for several values of μ_l (on the $32^3 \times 8$ lattice only: again, see [8] for details).

[1] This means tuning the parameters in the lattice action so that a set of quantities, e.g. m_π, have the physical value.

4. Numerical results

The continuum limit has been computed in two ways, with the aim of assessing the systematics involved. In the first method, we extrapolated directly κ to the continuum, while in the second method observables and pseudocritical temperatures were extrapolated to continuum limit and then κ was computed from these.

It has to be remarked that the results of the fit used for locating $T_c(\mu_B)$ is affected both by the form of the function used the choice of the fit range. Our estimate of the the systematic errors on $T_c(\mu_B)$ is based on the differences obtained by varying the fit range and changing the function used in the fit with reasonable alternatives. Statistical uncertainties were instead computed making use of a bootstrap analysis.

4.1 The effect of a nonzero μ_s

Our results about the effect of a nonzero μ_s are shown in Fig. (2, left). The result of our analysis

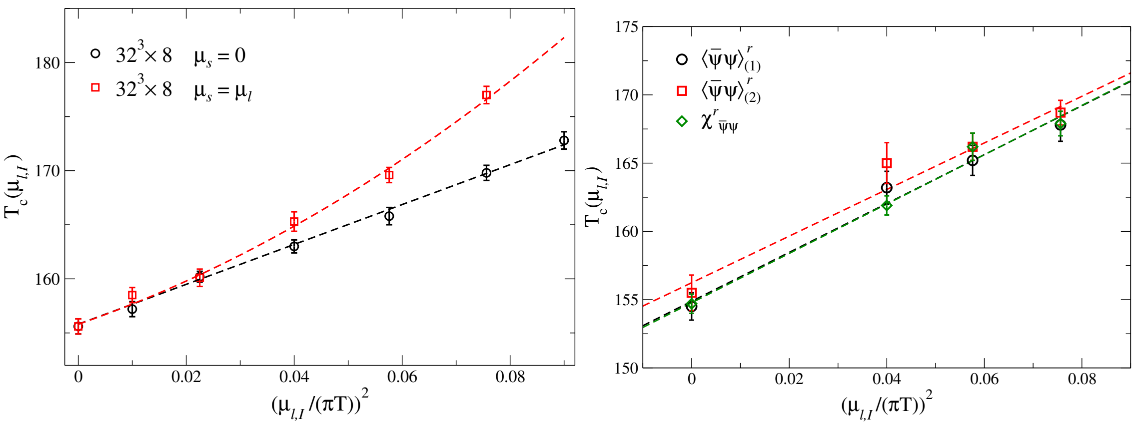
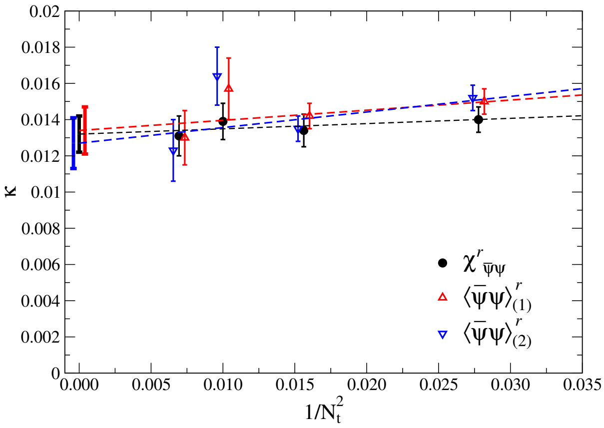


Figure 2: Left: The effect of the inclusion of a nonzero chemical potential: the critical line on the $\mu_{l,I}$-T plane in the two cases, $\mu_s = 0$ (red) and $\mu_s = \mu_l$ (black) [from the normalized chiral susceptibility]. **Right:** Critical lines from the $48^3 \times 12$ lattice, from different chiral observables.

is that, up to the present level of accuracy, the values of κ obtained in the $\mu_s = 0$ and $\mu_s = \mu_l$ cases are compatible if a quartic term in $\mu_{l,I}$ is taken into consideration on the latter situation. In fact, omitting that term and fitting the data for $\mu_s = \mu_l$ just with a quadratic expression, the obtained χ^2/n_{dof} is 2.4, while including that term we obtain $\chi^2/n_{dof} \simeq 1$. Given the results of this analysis, we expect the value of κ measured in the strangeness neutrality condition not to be significantly different from the value obtained here.

4.2 The continuum limit

In the first method κ is computed for each value of N_t (6,8,10 and 12) and the obtained curvatures are extrapolated to the continuum limit, assuming corrections of order $1/N_t^2$. The resulting values are $\kappa = 0.0134(13), 0.0127(14), 0.0132(10)$, respectively from the renormalized chiral condensate I, II and the renormalized chiral susceptibility. The results of this method are shown in Fig. (3, left).



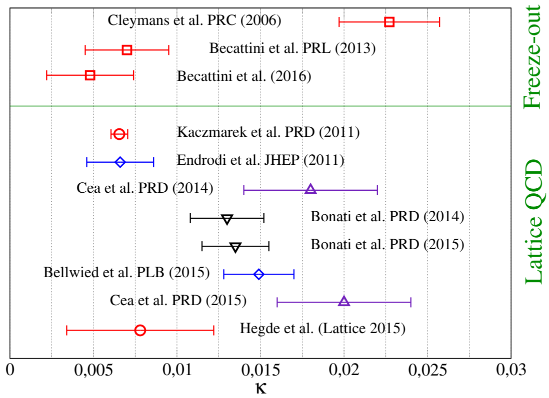
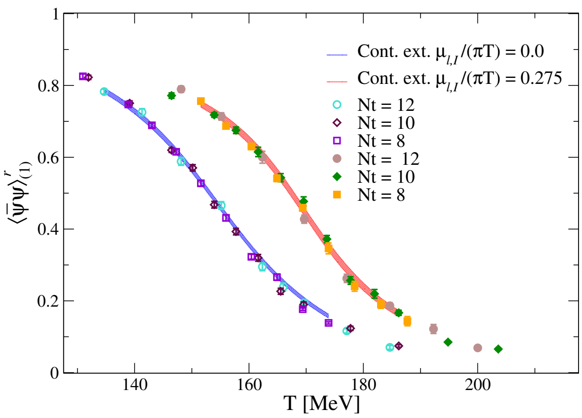


Figure 3: Estimates of the curvature of the pseudocritical line. **Left:** our continuum extrapolated results (first method) for κ. **Right:** values of κ from the literature.

In the second method we made an ansatz on the combined dependence of the observables on the temperature and on the lattice spacing. By making use of data from $N_t = 8$,10 and 12 lattices we obtained the continuum extrapolated values of the observables as a function of T, as shown in Figs. (4, left), as well as a continuum extrapolated estimate of $T_c(\mu_B)$. We then fitted the $T_c(\mu_B)$ with the form of Eq.(1.1) and obtained $\kappa = 0.0145(11), 0.0138(10), 0.0131(12)$ from $\langle\bar\psi\psi\rangle^r_{(1)}$, $\langle\bar\psi\psi\rangle^r_{(2)}$, and $\chi^r_{\bar\psi\psi}$ respectively: this procedure is shown in Fig. 4, and we notice that these results are compatible with the ones obtained with the first method.



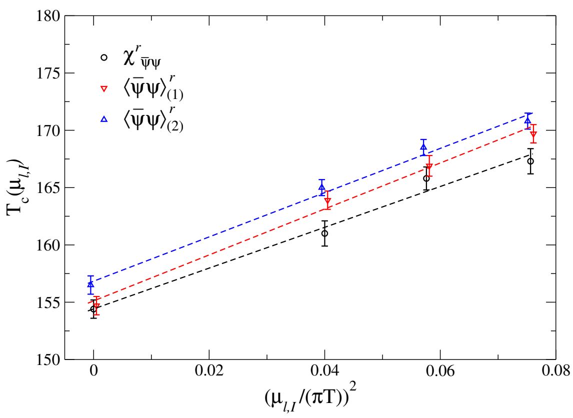


Figure 4: Continuum limit of the observables. **Left:** renormalized chiral condensate $\langle\bar\psi\psi\rangle^r_{(1)}$. **Right :** critical lines in the $\mu_{l,I}$ versus T plane obtained with the method of continuum extrapolated observables.

5. Conclusions

Considering also the uncertainty derived from the strangeness neutrality issue, we give $\kappa = 0.0135(20)$ as an estimate of the curvature of the chiral crossover line. In Fig. 3 we show the comparison of our value with the ones obtained in the recent literature at the physical point. While old determinations lacked a careful assessment of the systematics, we can state the most recent determinations of κ are in agreement within errors.

References

[1] F. Becattini, J. Steinheimer, R. Stock and M. Bleicher, arXiv:1605.09694 [nucl-th]; F. Becattini, M. Bleicher, T. Kollegger, T. Schuster, J. Steinheimer and R. Stock, Phys. Rev. Lett. **111**, 082302 (2013) [arXiv:1212.2431 [nucl-th]]; J. Cleymans, H. Oeschler, K. Redlich and S. Wheaton, Phys. Rev. C **73**, 034905 (2006) [hep-ph/0511094].

[2] S. Borsanyi, G. Endrodi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti and K. K. Szabo, JHEP **1208**, 053 (2012) [arXiv:1204.6710 [hep-lat]]; O. Kaczmarek, F. Karsch, E. Laermann, C. Miao, S. Mukherjee, P. Petreczky, C. Schmidt, W. Soeldner and W. Unger, Phys. Rev. D **83**, 014504 (2011) [arXiv:1011.3130 [hep-lat]].

[3] P. Hegde *et al.* [Bielefeld-BNL-CCNU Collaboration], arXiv:1511.03378 [hep-lat].

[4] G. Endrodi, Z. Fodor, S. D. Katz and K. K. Szabo, JHEP **1104**, 001 (2011) [arXiv:1102.1356 [hep-lat]].

[5] P. de Forcrand and O. Philipsen, Nucl. Phys. B **642**, 290 (2002) [hep-lat/0205016]; Nucl. Phys. B **673**, 170 (2003) [hep-lat/0307020]; M. D'Elia and M. P. Lombardo, Phys. Rev. D **67**, 014505 (2003) [hep-lat/0209146], Phys. Rev. D **70**, 074509 (2004) [hep-lat/0406012]; V. Azcoiti, G. Di Carlo, A. Galante and V. Laliena, Nucl. Phys. B **723**, 77 (2005) [hep-lat/0503010]; L. K. Wu, X. Q. Luo and H. S. Chen, Phys. Rev. D **76**, 034505 (2007) [hep-lat/0611035]; P. Cea, L. Cosmai, M. D'Elia and A. Papa, Phys. Rev. D **77**, 051501 (2008) [arXiv:0712.3755 [hep-lat]]; K. Nagata and A. Nakamura, Phys. Rev. D **83**, 114507 (2011) [arXiv:1104.2142 [hep-lat]]; P. Cea, L. Cosmai, M. D'Elia, A. Papa and F. Sanfilippo, Phys. Rev. D **85**, 094512 (2012) [arXiv:1202.5700 [hep-lat]]; E. Laermann, F. Meyer and M. P. Lombardo, J. Phys. Conf. Ser. **432**, 012016 (2013).

[6] P. Cea, L. Cosmai and A. Papa, Phys. Rev. D **89**, 074512 (2014) [arXiv:1403.0821 [hep-lat]]; Phys. Rev. D **93**, 014507 (2016) [arXiv:1508.07599 [hep-lat]];

[7] R. Bellwied, S. Borsanyi, Z. Fodor, J. Günther, S. D. Katz, C. Ratti and K. K. Szabo, Phys. Lett. B **751**, 559 (2015) [arXiv:1507.07510 [hep-lat]].

[8] C. Bonati, M. D'Elia, M. Mariti, M. Mesiti, F. Negro and F. Sanfilippo, Phys. Rev. D **90**, 114025 (2014) [arXiv:1410.5758 [hep-lat]]; Phys. Rev. D **92**, 054503 (2015) [arXiv:1507.03571 [hep-lat]].

[9] A. Bazavov *et al.*, Phys. Rev. Lett. **113**, 072001 (2014) [arXiv:1404.6511 [hep-lat]]; S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti and K. K. Szabo, Phys. Rev. Lett. **111**, 062005 (2013) [arXiv:1305.5161 [hep-lat]].

[10] M. Cheng, N. H. Christ, S. Datta, J. van der Heide, C. Jung, F. Karsch, O. Kaczmarek and E. Laermann *et al.*, Phys. Rev. D **77**, 014511 (2008) [arXiv:0710.0354 [hep-lat]].

[11] Y. Aoki, S. Borsanyi, S. Durr, Z. Fodor, S. D. Katz, S. Krieg and K. K. Szabo, JHEP **0906**, 088 (2009) [arXiv:0903.4155 [hep-lat]].

[12] S. Borsanyi, G. Endrodi, Z. Fodor, A. Jakovac, S. D. Katz, S. Krieg, C. Ratti and K. K. Szabo, JHEP **1011**, 077 (2010) [arXiv:1007.2580 [hep-lat]].

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