

Fluctuations and correlations in finite temperature QCD

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We calculate fluctuations and correlations of conserved charges in finite temperature QCD. We also present the equation up to order $(\mu_B/T)^6$. This allows reliable calculations of thermodynamic quantities up to $\mu_B/T \approx 2$, which covers most of the Beam Energy Scan program at RHIC. Our simulations use staggered quarks with physical quark masses. All of our results are extrapolated to the continuum limit. This conference contribution is mainly based on the papers [1] and [2].

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1. QCD in the grand canonical ensemble

We calculate fluctuations and correlations of conserved charges in finite temperature QCD at zero chemical potential using lattice QCD simulations. These are interesting for their sensitivity to criticality, for probing the composition and distribution of the conserved charges in the QCD medium, and for providing stringent tests on the hadron resonance gas model at low and resummed perturbation theory at high temperatures. They can also be used for extrapolation to small finite chemical potentials. Moreover there is an ongoing effort in the Heavy Ion Experimental community to measure the moments of conserved charge distributions, which can be related one-to-one to fluctuations (with some known caveats). For the fluctuations, or generalized susceptibilities, we use the following notation:

$$\chi_{i,j,k,l}^{u,d,s,c} = \frac{\partial^{i+j+k+l} (p/T^4)}{(\partial \hat{\mu}_u)^i (\partial \hat{\mu}_d)^j (\partial \hat{\mu}_s)^k (\partial \hat{\mu}_c)^l} \quad \chi_{i,j,k}^{B,S,Q} = \frac{\partial^{i+j+k} (p/T^4)}{(\partial \hat{\mu}_B)^i (\partial \hat{\mu}_S)^j (\partial \hat{\mu}_Q)^k}$$

where $\hat{\mu} = \mu/T$. The relationship between the chemical potentials:

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \quad \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \quad \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$$

allows one to convert the two types of derivatives into each other and write say the $\chi_{i,j,k}^{B,S,Q}$ as linear combinations of the $\chi_{i,j,k}^{u,d,s}$. These fluctuations then can be used to extrapolate the equation of states to small but general values of μ_B, μ_Q, μ_S , where we have:

$$\frac{p}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BSQ}(T) \hat{\mu}_B^i \hat{\mu}_S^j \hat{\mu}_Q^k$$

If we restrict ourselves to conditions present in Heavy Ion Collisions, namely $\langle n_S \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$, we get an expansion of the form:

$$\frac{p}{T^4} = c_0(T) + c_2(T) \cdot \hat{\mu}_B^2 + c_4(T) \cdot \hat{\mu}_B^4 + c_6(T) \cdot \hat{\mu}_B^6 + \dots$$

This gives the finite chemical potential equation of state relevant to hydrodynamic simulations of heavy ion collisions. We will also present results on these coefficients. The equation of state of QCD at $\mu_B = 0$ is known for a while [3, 4, 5], and is a standard component of hydrodynamic description of heavy ion collisions. The corrections at finite density are much harder to calculate [6, 7]. Our work is an important step in the finite μ direction.

2. The two methods to calculate fluctuations

We will use two distinct method to calculate the fluctuations. First, we can calculate the μ derivatives directly at zero chemical potentials [8, 9]. This has the advantage that there is no additional systematic error coming from fitting procedure. However, higher derivatives are very noisy. This traditional technique finds the c_4 and c_6 coefficients from the non-Gaussianity of the fluctuations of conserved charges. In large simulation volumes, however, the central limit theorem reduces these below the level of detection. With physical quark masses, fine and large lattices one must seek for an alternative technique.

The method we use is to simulate at imaginary chemical potential [10, 11, 12, 13, 14], do a fit for the μ dependence of different observables, and deduce the derivatives at 0 that way. This makes higher accuracy possible with the same amount of computer time, however it has systematic errors coming from fitting, as at imaginary μ you have an exact result, containing all orders of the Taylor expansion. This is illustrated on Figure 1. Such systematic errors are included in all of our result plots.

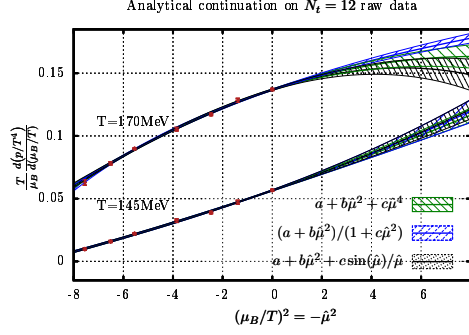


Figure 1: Illustration of the systematic error coming from analytic continuation with different fit ansatzes. The systematic error coming from the choice of these 3 ansatzes is included in our results.

3. Lattice details

The susceptibilities can be expressed as fermionic matrix traces, that can be evaluated on the lattice by utilizing a large number of Gaussian random sources. Our continuum extrapolation for the imaginary chemical potential data is based on the following lattices: $40^3 \times 10$, $48^3 \times 12$ and $64^3 \times 16$, in the 4stout staggered discretization. For the details of the lattice action and ensembles see Ref. [1, 15]. For the zero chemical potential data in the cross-over region we use aspect ratios between 3 and 4 and temporal extent $N_t = 8 \dots 24$. For temperatures $T \geq 300 \text{ MeV}$ we do not keep the lattice geometry constant in our temperature scan, but keep the physical volume constant, with $LT_c > 2$. So e.g. for $N_t = 16$ we have the lattices:

$64^3 \times 16$	$80^3 \times 16$	$96^3 \times 16$	$112^3 \times 16$	$128^3 \times 16$
300 MeV	360 MeV	440 MeV	520 MeV	600 MeV

Keeping the volume large is important for testing perturbation theory.

For some observables, like χ_2^B , there is a long range of safe linear extrapolation, but observables that are related to pion physics, like χ_2^Q , show a strong non-linear $1/N_t$ dependence, and only for very fine lattices, $N_t \geq 16$ we see a linear regime. No such problem appears for the observables considered in our imaginary chemical potential runs.

For the imaginary μ runs, the tuning of the chemical potentials such that $\langle n_S \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$ are satisfied is an additional issue. The method used for tuning the strangeness neutrality condition was first introduced in [16].

4. Results

On Figure 2 and Figure 3 (left) we show several generalized susceptibilities in finite temperature QCD together with some model predictions. All the observables we consider show an agreement with the HRG model up to $T \sim 150 - 155$ MeV. The diagonal second derivatives start to agree with the perturbative results [17, 18] earlier than the off-diagonal derivatives. In the high temperature range a similar calculation to ours was also presented in [19].

In addition to the HRG we show a naive quasiparticle estimate for the charm susceptibility. The mass of the charm quark was fitted to the last points ($m_c^{QP} = 1430$ MeV). The charm quark mass ($m_c^{QP} = 1430$ MeV) is empirical, and may depend on the range of the matching to the data. The quasiparticle model's results are overestimating the lattice data below approx. 350 MeV. This leaves room for multiple interpretations, like a T -dependent m_c^{QP} , limitations of the quasiparticle model or bound states absorbing free quarks.

The coefficients c_n of the expansion of the pressure in μ_B , satisfying $\langle n_S \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$ can be seen on the left side of Figure 3.

In ideal hydrodynamics, we have $S/N = \text{fixed}$. These trajectories can be readily calculated from the equation of state given by the c_n coefficients, once the value of S/N is fixed. To fix these value for the value relevant for the RHIC beam energy scan we used the freeze-out temperature and chemical potential estimates from Ref. [20]. The resulting trajectories can be seen in the left hand side of Figure 4. The equation of state along these trajectories is on the right side of Figure 4. The equation of state can be reliably calculated for $\hat{\mu}_B \leq 2$, or in terms of the RHIC energy scan, for the center of mass energies:

$$\sqrt{s} = 200, 62.4, 39, 27, 19.6, 14.5 \text{ GeV}$$

For the use of μ_S and a function of μ_B and T , determined from our simulations, to Kaon freeze-out, see Ref. [21].

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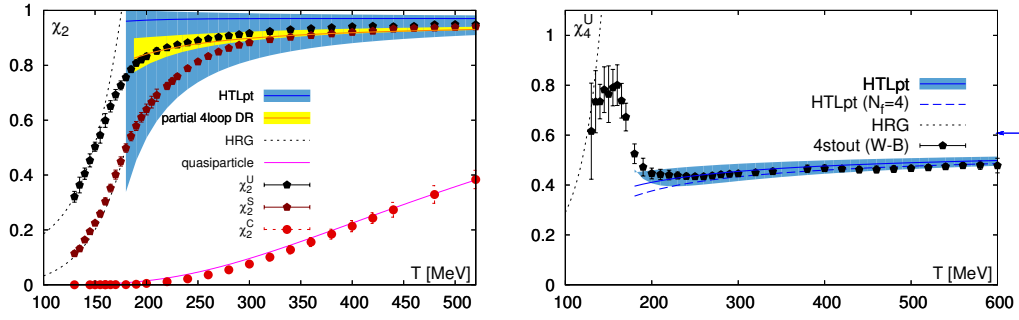


Figure 2: Second order susceptibilities (left) and fourth order diagonal susceptibilities (right). Various model estimates are also shown.

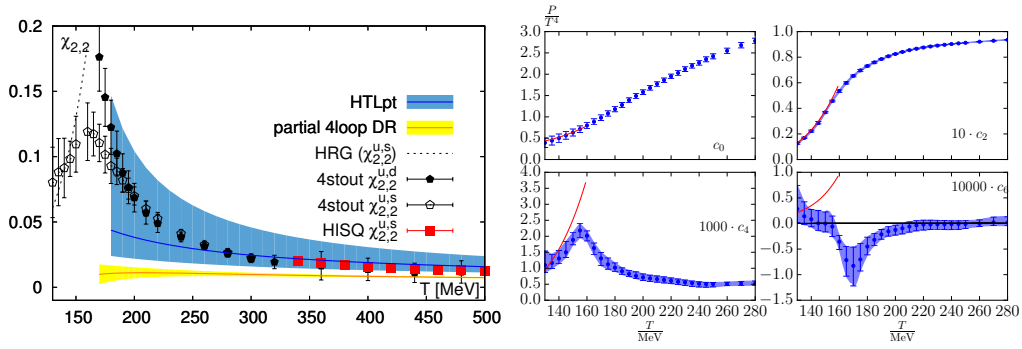


Figure 3: Fourth order diagonal susceptibilities (left) and the coefficients of the equation of state with the constraints $\langle n_S \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$ (right). The hadron resonance gas model predictions are shown in red, for comparison.

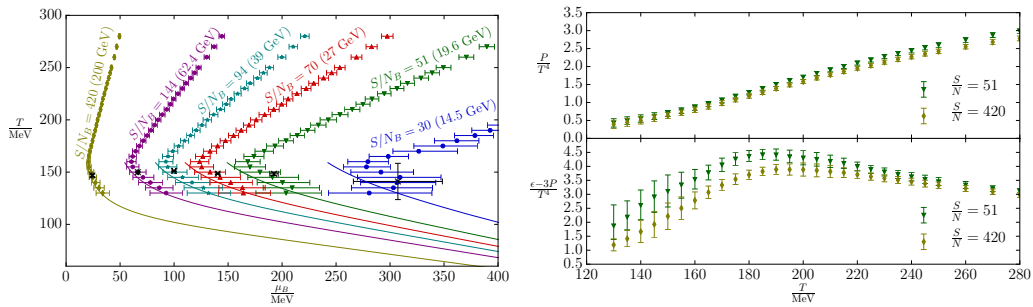


Figure 4: The isentropic trajectories (left) and the equation of state along these isentropic trajectories (right).

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