

## Lepton Flavor Violation in B Decays

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The  $R_K$  measurement by LHCb suggests non-standard lepton non-universality (LNU) to occur in  $b \rightarrow s\ell^+\ell^-$  transitions, with effects in muons rather than electrons. A number of other measurements of  $b \rightarrow s\ell^+\ell^-$  transitions by LHCb and  $B$ -factories display disagreement with the Standard-Model predictions and, remarkably, these discrepancies are consistent in magnitude and sign with the  $R_K$  effect. Non-standard LNU suggests non-standard lepton flavor violation (LFV) as well. We discuss several  $B$ -physics observables to measure such LFV effects.

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## 1. Introduction

The LHCb experiment as well as  $B$  factories measured several key  $b \rightarrow s$  and  $b \rightarrow c$  modes, and agreement with the Standard Model (SM) is less than perfect. Among these measurements, the elephant in the room is the ratio known as  $R_K$  [1]

$$R_K \equiv \frac{\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)}{\mathcal{B}(B^+ \rightarrow K^+ e^+ e^-)} = 0.745_{-0.074}^{+0.090} (\text{stat}) \pm 0.036 (\text{syst}), \quad (1.1)$$

as measured in the di-lepton invariant-mass-squared range  $[1, 6] \text{ GeV}^2$ . The SM predicts unity with percent-level corrections [2, 3, 4, 5], implying a  $2.6\sigma$  discrepancy. The electron-channel measurement would be an obvious culprit, because of bremsstrahlung and lower statistics with respect to the muon channel. On the other hand, disagreement is rather in the muon channel [6, 7]. A systematic effect there is less likely than in the electron channel, as muons are among the most reliable objects within LHCb.

Additional measurements support the above picture:

- The very same pattern, with data lower than the SM prediction, is also observed in the  $B_s \rightarrow \phi \mu^+ \mu^-$  channel and in the same range  $m_{\mu\mu}^2 \in [1, 6] \text{ GeV}^2$ , as initially found in 1/fb of LHCb data [8] and then confirmed by a full run-1 analysis [9]. This discrepancy is estimated to be more than  $3\sigma$  [9].
- Additional support comes from the  $B \rightarrow K^* \mu \mu$  decay, for which LHCb can perform a full angular analysis. The quantity known as  $P_5'$ , designed to have reduced sensitivity to form-factor uncertainties [10], exhibits a discrepancy in two bins, again in the low- $m_{\mu\mu}^2$  range. The effect was originally found in 1/fb of LHCb data [11], and confirmed by a full run-1 analysis [12] as well as, very recently, by a Belle analysis [13]. The  $P_5'$  discrepancy as estimated by LHCb amounts to  $3.4\sigma$ , and is in the  $2\sigma$ -ballpark from Belle ( $2.1\sigma$  as compared to [14] and  $1.7\sigma$  as compared to [15, 16, 17]). The theoretical error is, however, still debated, see in particular [18, 19, 17, 20].

Further interesting results come from measurements of the ratios  $R(D^{(*)}) \equiv \mathcal{B}(B \rightarrow D^{(*)} \tau \nu) / \mathcal{B}(B \rightarrow D^{(*)} \ell \nu)$ , but they will not be covered in this contribution due to lack of time.

## 2. Theory considerations

It is clear that each of the mentioned effects needs confirmation from LHCb's run 2 to be taken seriously. Yet, focusing for the moment on the  $b \rightarrow s$  discrepancies, we can ask ourselves two questions: whether we can (easily) make theoretical sense of the above data; and what are the most immediate signatures to expect in case the above discrepancies are real.

Let us actually start from the second question, by putting forward one basic observation: if  $R_K$  is signaling lepton non-universality (LFNU) at a beyond-SM level, we may also expect lepton flavor violation (LFV) at a beyond-SM level. In fact, consider a new, LFNU interaction introduced to explain  $R_K$ , and defined above the electroweak symmetry breaking (EWSB) scale. Such interaction would be of the kind  $\bar{\ell} Z' \ell$ , with  $Z'$  a new vector boson, or  $\bar{\ell} \phi q$ , with  $\phi$  a leptoquark. The question

arises, in what basis are quarks and leptons in the above interaction. Generically, it is not the mass eigenbasis – this basis does not yet even exist, as we are above the EWSB scale. Then, rotating the  $q$  and  $\ell$  fields to the mass eigenbasis generates LFV effects, although the initial interaction was introduced to produce only LFNU ones.

Turning to the question whether we can easily make theoretical sense of the experimental anomalies, the answer is yes, already within an effective-theory framework. Consider in fact the following Hamiltonian:

$$\mathcal{H}_{\text{SM+NP}}(\bar{b} \rightarrow \bar{s}\mu^+\mu^-) = -\frac{4G_F}{\sqrt{2}}V_{tb}^*V_{ts}\frac{\alpha_{em}(m_b)}{4\pi}\left[\bar{b}_L\gamma^\lambda s_L\bar{\mu}\left(C_9^{(\mu)}\gamma_\lambda + C_{10}^{(\mu)}\gamma_\lambda\gamma_5\right)\mu\right] + \text{H.c.}, \quad (2.1)$$

where the index  $(\mu)$  indicates that the Wilson coefficients of the corresponding operators (denoted as  $\mathcal{O}_9$  and  $\mathcal{O}_{10}$ ) distinguish among different lepton flavors, as the Hamiltonian on the l.h.s. includes new-physics (NP) contributions as well. The SM contributions for these Wilson coefficients are flavor universal, and such that  $C_9 \simeq -C_{10}$  at the  $m_b$  scale, yielding an approximate  $(V-A) \times (V-A)$  structure. Advocating the same structure also for the corrections to  $C_9^{\text{SM}}$  and  $C_{10}^{\text{SM}}$  – in the  $\mu$ -channel only! – would account at one stroke for  $R_K$  lower than 1,  $\mathcal{B}(B \rightarrow K\mu\mu)$  (and  $\mathcal{B}(B_s \rightarrow \mu\mu)$ ) data below predictions, and the  $P'_5$  anomaly in  $B \rightarrow K^*\mu\mu$  data. A fully quantitative test of this statement requires a global fit, see in particular [21, 22].

In short, and as stated before, all  $b \rightarrow s$  data can be explained if  $C_9^{(\ell)} \approx -C_{10}^{(\ell)}$  and  $|C_9^{(\mu),\text{NP}}| \gg |C_9^{(e),\text{NP}}|$ . This pattern can be generated from a purely third-generation interaction of the kind [23]

$$\mathcal{H}_{\text{NP}} = G\bar{b}'_L\gamma^\lambda b'_L\bar{\tau}'_L\gamma_\lambda\tau'_L, \quad (2.2)$$

with  $G = 1/\Lambda_{\text{NP}}^2$  a new Fermi-like coupling, corresponding to a NP scale  $\Lambda_{\text{NP}}$  in the TeV ballpark. The interaction in eq. (2.2) is expected, e.g., in partial-compositeness frameworks. The prime on the fields indicates that they are in the “gauge” basis, i.e. that below the EWSB scale they need to be rotated to the mass eigenbasis by usual chiral unitary transformations of the form

$$b'_L \equiv (d'_L)_3 = (U_L^d)_{3i}(d_L)_i, \quad \tau'_L \equiv (\ell'_L)_3 = (U_L^\ell)_{3i}(\ell_L)_i, \quad (2.3)$$

whereby the r.h.s. fields represent the mass eigenbasis. These rotations induce LFNU and LFV effects, as previously mentioned.

With the above ingredients we can straightforwardly explain  $b \rightarrow s$  data. In fact, recalling our full Hamiltonian eq. (2.1), and denoting  $k_{\text{SM}} \equiv -\frac{4G_F}{\sqrt{2}}V_{tb}^*V_{ts}\frac{\alpha_{em}(m_b)}{4\pi}$  the Wilson-coefficient normalization factor within the SM, the shift to the  $C_9^{(\mu)}$  Wilson coefficient becomes

$$k_{\text{SM}}C_9^{(\mu)} = k_{\text{SM}}C_9^{\text{SM}} + \frac{G}{2}(U_L^d)_{33}^*(U_L^d)_{32}|(U_L^\ell)_{32}|^2. \quad (2.4)$$

For the shift on the r.h.s. to explain the  $R_K$  discrepancy, one needs destructive interference between the SM and NP contributions to  $C_9^{(\mu)}$ . This occurs for  $G(U_L^d)_{32} < 0$ , assuming  $(U_L^d)_{33} \approx 1$ . On the other hand, in the  $ee$ -channel one has

$$k_{\text{SM}}C_9^{(e)} = k_{\text{SM}}C_9^{\text{SM}} + \frac{G}{2}(U_L^d)_{33}^*(U_L^d)_{32}|(U_L^\ell)_{31}|^2, \quad (2.5)$$

whereby the last term on the r.h.s. is negligible by assumption, as  $|(U_L^\ell)_{31}|^2 \ll |(U_L^\ell)_{32}|^2$ .

So, in the above setup one would have

$$R_K \approx \frac{|C_9^{(\mu)}|^2 + |C_{10}^{(\mu)}|^2}{|C_9^{(e)}|^2 + |C_{10}^{(e)}|^2} \simeq \frac{2|C_{10}^{\text{SM}} + C_{10}^{(\mu),\text{NP}}|^2}{2|C_{10}^{\text{SM}}|^2}, \quad (2.6)$$

where the factors of 2 on the r.h.s. are due to the contributions from  $|C_9|$  and  $|C_{10}|$  being equal by assumption. The above expression is approximate as, in particular, phase-space factors are slightly different between the muon and the electron channels. Note as well that

$$0.77 \pm 0.20 = \frac{\mathcal{B}(B_s \rightarrow \mu\mu)_{\text{exp}}}{\mathcal{B}(B_s \rightarrow \mu\mu)_{\text{SM}}} = \frac{\mathcal{B}(B_s \rightarrow \mu\mu)_{\text{SM+NP}}}{\mathcal{B}(B_s \rightarrow \mu\mu)_{\text{SM}}} = \frac{|C_{10}^{\text{SM}} + C_{10}^{(\mu),\text{NP}}|^2}{|C_{10}^{\text{SM}}|^2}, \quad (2.7)$$

implying, within the model in ref. [23], the correlations (see also [24])

$$\frac{\mathcal{B}(B_s \rightarrow \mu\mu)_{\text{exp}}}{\mathcal{B}(B_s \rightarrow \mu\mu)_{\text{SM}}} \simeq R_K \simeq \frac{\mathcal{B}(B^+ \rightarrow K^+ \mu\mu)_{\text{exp}}}{\mathcal{B}(B^+ \rightarrow K^+ \mu\mu)_{\text{SM}}}. \quad (2.8)$$

According to the above relation, the measurement-over-SM ratio for  $\mathcal{B}(B_s \rightarrow \mu\mu)$  provides a proxy for  $R_K$ . This is one more good reason to pursue accuracy in the  $\mathcal{B}(B_s \rightarrow \mu\mu)$  measurement. Provided that the central value on the l.h.s. of eq. (2.7) does not increase, this test will be a sensitive one already by the end of run 2, as the  $\mathcal{B}(B_s \rightarrow \mu\mu)$  total error (dominated by the experimental component) is anticipated to be around 10% [25].

### 3. Experimental signatures

From the argument made above it is clear that, if  $R_K$  is signaling beyond-SM LFNU, then we may expect measurable LFV as well. This expectation holds true, barring further theoretical assumptions preventing LFV in the presence of LFNU. As a general rule, the two types of effects go hand in hand. Assuming the interaction (2.2), the amount of LFNU pointed to by  $R_K$  actually allows to quantify rather generally [23] the expected amount of LFV. In fact,  $R_K$  yields the ratio

$$\rho_{\text{NP}} = -0.159_{-0.070}^{+0.060} \quad (3.1)$$

between the NP and the SM+NP contribution to  $C_9^{(\mu)}$ . Then, for example,

$$\frac{\mathcal{B}(B \rightarrow K \ell_i^\pm \ell_j^\mp)}{\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)} \simeq 2\rho_{\text{NP}}^2 \frac{|(U_L^\ell)_{3i}|^2 |(U_L^\ell)_{3j}|^2}{|(U_L^\ell)_{32}|^4}, \quad (3.2)$$

implying

$$\mathcal{B}(B \rightarrow K \ell_i^\pm \ell_j^\mp) \simeq 5\% \cdot \mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-) \cdot \frac{|(U_L^\ell)_{3i}|^2 |(U_L^\ell)_{3j}|^2}{|(U_L^\ell)_{32}|^4} \simeq 2.2 \times 10^{-8} \cdot \frac{|(U_L^\ell)_{3i}|^2 |(U_L^\ell)_{3j}|^2}{|(U_L^\ell)_{32}|^4}, \quad (3.3)$$

where we used  $\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-) \simeq 4.3 \times 10^{-7}$  [6], and neglected all terms proportional to the different masses of the final-state leptons.<sup>1</sup> Eq. (3.3) tells us that LFV  $B \rightarrow K$  decays are

<sup>1</sup>Because of this approximation, eqs. (3.2)-(3.3) provide only crude estimates in the case of decays involving a  $\tau$  lepton. However, this approximation does not change the argument of the present paragraph.

expected to be in the ballpark of  $10^{-8}$  times an unknown factor involving  $U_L^\ell$  matrix entries. In the  $\ell_i \ell_j = e\mu$  case, this ratio reads  $|(U_L^\ell)_{31}/(U_L^\ell)_{32}| \lesssim 3.7$  [23], implying that the  $B \rightarrow K\mu e$  rate may be around  $10^{-8}$ , or much less if  $|(U_L^\ell)_{31}/(U_L^\ell)_{32}| \ll 1$ . The latter possibility would suggest  $U_L^\ell$  entries that decrease in magnitude with the distance from the diagonal. But then one may expect the ratio  $|(U_L^\ell)_{33}/(U_L^\ell)_{32}| > 1$ , implying a  $B \rightarrow K\mu\tau$  rate of  $\mathcal{O}(10^{-8})$  or above. In short, assuming the interaction (2.2), one can hope that at least one LFV  $B \rightarrow K$  decay rate be in the ballpark of  $10^{-8}$  [23], which happens to be within reach at LHCb's run 2. An entirely analogous reasoning applies for the purely leptonic modes  $B_s \rightarrow \ell_i^\pm \ell_j^\mp$ , that may well be within reach of LHCb during run 2, if the  $U$ -matrix factor on the r.h.s. is of order unity (or larger!) for at least one LFV mode.<sup>2</sup>

It is worthwhile to open two parentheses on the consequences of the above argument. First, it is an order-of-magnitude argument, and it is worthwhile to speculate on the possibility of more quantitative LFV predictions. This possibility requires knowledge of the  $U_L^\ell$  matrix. One approach towards predicting the  $U_L^\ell$  matrix is the one pursued in ref. [26], whose line of argument goes as follows. A sufficient condition for  $U_L^\ell$  to be predictable is to know the product  $Y_\ell Y_\ell^\dagger$ , with  $Y_\ell$  the charged-lepton Yukawa coupling. To this end, one may start from the ansatz in [27] that the five flavor-SU(3) rotations are not all independent. Choosing three to be the independent ones allows to predict one SM Yukawa coupling in terms of the other two. One can thereby determine  $Y_\ell$  in terms of  $Y_u$  and  $Y_d$ . However, we don't know  $Y_u$  and  $Y_d$  in full. Yet, we can take an independently motivated model for  $Y_u$  and  $Y_d$  textures, such as the one in ref. [28], motivated as a scenario for solving the strong-CP problem in QCD. Another approach [29] starts from the observation that the product  $(U_L^\ell)^\dagger U_L^\nu$  equals a known object, namely the PMNS matrix. Making assumptions about  $U_L^\nu$  then allows to predict  $U_L^\ell$ . In this respect, ref. [29] makes the ansatz  $U_L^\nu = 1$ .

A second parenthesis concerns the observation that the  $B_s \rightarrow e\mu$  is expected to be the most difficult to access among the above-mentioned LFV modes, because it is chirally suppressed, and because the involved lepton combination is the farthest from the third one. It is therefore useful to search for additional decays, that can give access to the same physics, while being comparably (or, hopefully, more) accessible experimentally. As pointed out in ref. [30], in the  $B_s \rightarrow \mu e$  channel one such 'proxy' decay is provided by the inclusion of an additional hard photon in the final state. In fact, the additional photon replaces the chiral-suppression factor, of order  $\max(m_{\ell_1}, m_{\ell_2})^2/m_{B_s}^2$ , with a factor of order  $\alpha_{\text{em}}/\pi$ . The actual enhancement of  $\mathcal{B}(B_s \rightarrow \mu e \gamma)$  is of about 30% [30] over the non-radiative counterpart. Therefore, inclusion of the radiative mode would allow to more-than-double statistics with respect to the non-radiative mode alone.

#### 4. Conclusions

In flavor physics there are by now several persistent discrepancies with respect to the SM. Their most convincing aspects are the following: experimentally, results are consistent between LHCb and  $B$  factories; deviations concern two independent sets of data, namely  $b \rightarrow s$  and  $b \rightarrow c$  decays; discrepancies go in a consistent direction and a beyond-SM explanation is possible already within an effective-theory approach. It is of course far too early to draw conclusions, as the above effects await confirmation from run 2 – but run 2 will provide a definite answer. In the while, it is timely

<sup>2</sup>For a (rough) comparison, we should keep in mind that at run 2 the LHCb is expected [25] to provide a first measurement of  $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$ , which in the SM is about 3% of  $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ .

to propose further tests, one promising direction being that of LFV. The latter offers plenty of channels, many of which largely untested.

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