Direct CP violation in $K \rightarrow \pi\pi$ decays and supersymmetry

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The quantities $\epsilon'_K$ and $\epsilon_K$ measure the amount of direct and indirect CP violation in $K \rightarrow \pi\pi$ decays, respectively. Using the recent lattice results from the RBC and UKQCD Collaborations and a new compact implementation of the $\Delta S = 1$ renormalization group evolution we predict

$$\text{Re} \frac{\epsilon'_K}{\epsilon_K} = (1.06 \pm 5.07) \times 10^{-4}$$

in the Standard Model. This value is $2.8\sigma$ below the experimental value of

$$\text{Re} \frac{\epsilon'_K}{\epsilon_K} = (16.6 \pm 2.3) \times 10^{-4}.$$

In generic models of new physics the well-understood $\epsilon_K$ precludes large contributions to $\epsilon'_K$, if the new contributions enter at loop level. However, one can resolve the tension in $\epsilon'_K/\epsilon_K$ within the Minimal Supersymmetric Standard Model. To this end two features of supersymmetry are crucial: First, one can have large isospin-breaking contributions (involving the strong instead of the weak interaction) which enhance $\epsilon'_K$. Second the Majorana nature of gluinos permits a suppression of the MSSM contribution to $\epsilon_K$, because two box diagrams interfere destructively.

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1. Formalism and Standard-Model prediction

Flavour-changing neutral current (FCNC) transitions of Kaons are extremely sensitive to new physics and probe mass scales far above the reach of current high-$p_T$ experiments. $K \to \pi\pi$ decays give access to two CP-violating quantities, which are related to FCNC amplitudes changing strangeness $S$ by one or two units, respectively. To define these quantities $\varepsilon'_K$ and $\varepsilon_K$ one first combines the decay amplitudes $A(K^0 \to \pi^+\pi^-)$ and $A(K^0 \to \pi^0\pi^0)$ into $A_0 \equiv A(K^0 \to (\pi\pi)_{I=0})$ and $A_2 \equiv A(K^0 \to (\pi\pi)_{I=2})$ where $I$ denotes the strong isospin. Indirect CP violation (stemming from the $\Delta S = 2$ box diagrams) is quantified by

$$
\varepsilon_K \equiv \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} = (2.228 \pm 0.011) \cdot 10^{-3} \cdot e^{i(0.97 \pm 0.02)\pi/4}
$$

(1.1)

and was discovered in 1964 [1]. The measure of direct CP violation, which originates from the $\Delta S = 1$ Kaon decay amplitude, is

$$
\varepsilon'_K \simeq \frac{\varepsilon_K}{\sqrt{2}} \left[ \frac{A(K_L \to (\pi\pi)_{I=2})}{A(K_L \to (\pi\pi)_{I=0})} \cdot \frac{A(K_S \to (\pi\pi)_{I=2})}{A(K_S \to (\pi\pi)_{I=0})} \right] = (16.6 \pm 2.3) \cdot 10^{-4} \cdot \varepsilon_K.
$$

(1.2)

This experimental result was established in 1999 and constituted the first measurement of direct CP violation in any decay [2]. Adopting the standard phase convention for the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, the real parts of the isospin amplitudes are experimentally determined as

$$
\text{Re} A_0 = (3.3201 \pm 0.0018) \times 10^{-7} \text{ GeV}, \quad \text{Re} A_2 = (1.4787 \pm 0.0031) \times 10^{-8} \text{ GeV}.
$$

(1.3)

The master equation for $\varepsilon'_K/\varepsilon_K$ (see e.g. Ref. [3]) reads:

$$
\frac{\epsilon'_K}{\epsilon_K} = \frac{\omega_+}{\sqrt{2} \varepsilon_K} \frac{\text{exp} \text{Re} A_0}{\text{exp} \text{Re} A_0} \left\{ \frac{\text{Im} A_2}{\omega_+} - (1 - \hat{\Omega}_{\text{eff}}) \text{Im} A_0 \right\}.
$$

(1.4)

Here $\omega_+ \simeq \frac{\text{Re} A_2}{\text{Re} A_0} = (4.53 \pm 0.02) \cdot 10^{-2}$ is determined from the charged counterparts of $\text{Re} A_{0,2}$ and $\hat{\Omega}_{\text{eff}} = (14.8 \pm 8.0) \cdot 10^{-2}$ quantifies isospin breaking. The quantities $|\varepsilon'_K|\text{exp}$ and $\text{Re} A_0\text{exp}$ are also taken from experiment, as indicated.

The important theoretical ingredients encoding potential new-physics effects are $\text{Im} A_0$ and $\text{Im} A_2$, which are calculated from the effective hamiltonian $H^{[\Delta S=1]}$ describing $s \to dq\bar{q}$ decays. This hamiltonian is known for a while at the level of next-to-leading-order (NLO) in QCD [4] and a precise prediction of $\varepsilon'_K/\varepsilon_K$ is challenged by the difficulty to calculate the hadronic matrix elements of the operators in $H^{[\Delta S=1]}$. Within the Standard Model (SM) $\text{Im} A_0$ is dominated by gluon penguins, with roughly $2/3$ stemming from the matrix element $\langle (\pi\pi)_{I=0} | Q_6 | K^0 \rangle$ with the operator

$$
Q_6 = \sum_q \gamma_\mu d^\dagger_q \bar{q}_R^\gamma_5 \gamma_\mu q^\dagger_R.
$$

(1.5)

About $3/4$ of the contribution to $\text{Im} A_2$ stems from $\langle (\pi\pi)_{I=2} | Q_8 | K^0 \rangle$ with

$$
Q_8 = \sum_q \gamma_\mu d^\dagger_q \bar{q}_R^\gamma_5 \gamma_\mu q^\dagger_R.
$$

(1.6)
The Wilson coefficient of $Q_8$ stems from electroweak penguins and box diagrams (Fig. 1). Lattice-gauge theory has $\langle (\pi\pi)_{I=2}\vert Q_8\vert K^0 \rangle$ (and thereby $\text{Im}A_2$) under good control for some time [5], while lattice calculations of $\langle (\pi\pi)_{I=0}\vert Q_6\vert K^0 \rangle$ and the other matrix elements entering $\text{Im}A_0$ are new [6]. The results are consistent with earlier analytic calculations in the large-$N_c$ “dual QCD” approach [7]. Using these matrix elements from lattice QCD we find [8]

$$\frac{\varepsilon'_{K}}{\varepsilon_{K}} = (1.06 \pm 4.66_{\text{Lattice}} \pm 1.91_{\text{NNLO}} \pm 0.59_{\text{IV}} \pm 0.23_{m_t}) \times 10^{-4}.$$  \hspace{1cm} (1.7)

The various sources of errors are indicated in the subscripts, with “NNLO” referring to unknown higher-orders of the perturbative expansion and “IV” meaning isospin violation. Adding the errors in quadrature gives the result in the abstract. We use the methodology of [9], which exploits the CP-conserving data of Eq. (1.3) to constrain the matrix elements. To arrive at Eq. (1.7) we have implemented a novel compact solution of the renormalization group equations; the result is in full agreement with the calculation in Ref. [3]. Eq. (1.7) disagrees with the experimental number in Eq. (1.2) by 2.8 standard deviations. The original lattice paper, Ref. [6], quotes a smaller discrepancy. The discussion at this conference has indicated that the combination of Eq. (1.3) with Fierz identities between different matrix elements has lead to the sharper prediction in Refs. [3, 8].

2. A supersymmetric solution

The large factor $1/\omega_t$ multiplying $\text{Im}A_2$ in Eq. (1.4) renders $\varepsilon'_{K}/\varepsilon_{K}$ especially sensitive to new physics in the $\Delta I = 3/2$ decay $K \to (\pi\pi)_{I=2}$. This feature makes $\varepsilon'_{K}/\varepsilon_{K}$ special among all FCNC processes. However, it is difficult to place a large effect into $\varepsilon'_{K}$ without overshooting $\varepsilon_{K}$: The SM contributions to both quantities are governed by the CKM combination

$$\tau = -\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*} \sim (1.5 - i0.6) \cdot 10^{-3}.$$  \hspace{1cm} (2.1)

Our quantities scale as

$$\varepsilon_{K}^{SM} \propto \text{Im} \frac{\tau}{M_W^2} \quad \text{and} \quad \varepsilon_{K}^{SM} \propto \text{Im} \frac{\tau^2}{M_W^4}.$$  \hspace{1cm} (2.2)

1 Accidentally, $\varepsilon'_{K}/\varepsilon_{K}$ is essentially real.
In new-physics scenarios $\tau$ is replaced by some new $\Delta S = 1$ parameter $\delta$ and $M_W$ is replaced by some particle mass $M \gg M_W$. The new-physics contributions scale as
\[
\varepsilon^{\text{NP}}_K \propto \text{Im} \frac{\delta}{M^2}, \quad \text{and} \quad \varepsilon^{\text{NP}}_K \propto \text{Im} \frac{\delta^2}{M^2}.
\]
(2.3)

If new-physics enters through a loop, the only chance to have a detectable effect in $\varepsilon'_K$ is a scenario with $|\delta| \gg |\tau|$. Using Eqs. (2.2) and (2.3) the experimental constraint $|\varepsilon^{\text{NP}}_K| \leq |\varepsilon^{\text{SM}}_K|$ entails
\[
|\varepsilon^{\text{NP}}_K| \leq \left| \frac{\varepsilon^{\text{NP}}_K}{\varepsilon^{\text{SM}}_K} \right| = \mathcal{O} \left( \frac{\text{Re } \tau}{\text{Re } \delta} \right).
\]
(2.4)

Thus large effects in $\varepsilon'_K$ from loop-induced new physics are seemingly forbidden. Many studies of $\varepsilon'_K$ indeed involve new-physics scenarios with tree-level contributions to $\varepsilon'_K$ [10], in which the requirement $|\delta| \gg |\tau|$ can be relaxed.

Here we present an explanation of the measurement in Eq. (1.2) by a supersymmetric loop effect [11]. We circumvent the argument in Eq. (2.4) by exploiting two special features of the Minimal Supersymmetric Standard Model (MSSM): Firstly, the MSSM permits large $\Delta I = 3/2$ transitions mediated by the strong interaction (“Trojan penguins”) [12]. These enhanced amplitudes occur if the mass splitting between the right-handed up and down squarks is sizable (see Fig. 2). Secondly, the Majorana nature of the gluino permits the suppression of $\varepsilon_K$, which receives contributions from two squark-gluino box diagrams (“regular” and “crossed”). These diagrams cancel each other efficiently, once the gluino mass $m_{\tilde{g}}$ and the squark mass $m_{\tilde{Q}}$ in the loop satisfy $m_{\tilde{g}} \geq 1.5 m_{\tilde{Q}}$ [13]. In our scenario, the mass scale $M_S$ of the supersymmetric particles is large, of order 3–7 TeV. Squark flavour mixing appears only among the left-handed doublets. We choose the CP-violating phase of the (2,1) element $\Delta_{sd}^{LL}$ of the left-handed squark mass matrix equal to $\arg(\Delta_{sd}^{LL}) = \pi/4$. The results are shown in Fig. 3.

3. Summary

Novel lattice results reveal a tension between the measured value of $\varepsilon'_K$ in Eq. (1.2) and the SM prediction in Eq. (1.7). Within the MSSM one can simultaneously enhance $\varepsilon'_K$ and suppress unwanted effects in $\varepsilon_K$. Our MSSM scenario works with large superpartner masses in the 3–7 TeV range and thereby comply with bounds from collider searches. Crucial elements are $m_{\tilde{g}} \geq 1.5 M_{\tilde{Q}}$, a sizable mass splitting between right-handed up and down squarks, and flavour mixing among left-handed squarks.
Figure 3: Left: $\epsilon_K^{\text{SUSY}}/\epsilon_K^{\text{SM}}$ as a function of $m_{\tilde{g}}/M_S$ for a common mass $M_S = 10$ TeV of all superpartners except the gluino. Right: Parameter region explaining $\epsilon_K$ while complying with the measured $\epsilon_K$ for the point $m_{\tilde{g}} = 1.5 M_S$ and $M_S = m_{\tilde{Q}} = m_{\tilde{D}}$. The lines labeled with negative values of the MSSM contribution $\epsilon_K^{\text{SUSY}}/\epsilon_K$ correspond to correct (positive) solutions if the CP phase is appropriately adjusted. The SM prediction for $\epsilon_K$ strongly depends on $|V_{cb}|$. The blue (red) lines in both plots delimit the region which complies with $\epsilon_K$ if $|V_{cb}|$ is determined from exclusive (inclusive) $b \to c\ell\nu$ decays. If the exclusive determination is correct, some new physics in $\epsilon_K$ is welcome. In the inclusive case the forbidden region is marked with the red shading. For more details see Ref. [11], from which the plots are taken.

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