We present updated results of $\epsilon_K$ evaluated directly from the standard model with lattice QCD inputs. Here, we use the lattice QCD inputs for $\hat{B}_K$, $|V_{cb}|$, $\xi_0$, $\xi_2$, $|\nu|$, and $m_c(m_c)$. Recently, FLAG has updated $\hat{B}_K$. RBC-UKQCD has also updated $\xi_0$ and $\xi_2$. Exclusive $|V_{cb}|$ has been updated with new lattice data in the $\bar{B} \to D\ell\bar{\nu}$ decay mode, too. We find that the theoretical value of $\epsilon_K$ with exclusive $|V_{cb}|$ (lattice QCD inputs) evaluated directly from the standard model is $3.2\sigma$ lower than the experimental value, while that with inclusive $|V_{cb}|$ (heavy quark expansion) has no tension.
1. Introduction

We have been monitoring $\epsilon_K$ since 2012, which is the indirect CP violation parameter in neutral kaons calculated directly from the standard model (SM) using lattice QCD inputs. The parameter $\epsilon_K$ is very precisely measured in experiment. From the theoretical point of view, it comes from the FCNC loop effects of box diagrams in the SM, and so provide a direct probe of CP violation in the neutral kaon system. Hence, naturally it is sensitive to physics models beyond the standard model (BSM). In this paper, we present results of $\epsilon_K$ evaluated directly from the SM with lattice QCD inputs. We also compare them with the experimental results. This paper is an update of our previous paper [1, 2].

2. Input parameters

The master formula for $\epsilon_K$ in the SM is

$$\epsilon_K = e^{\theta} \sqrt{2} \sin \theta \left( C_e X_{SD} \xi_0 + \xi_{\text{LD}} \right) + O(\omega e') + O(\xi_0 \Gamma_2/\Gamma_1). \quad (2.1)$$

Here, the short distance contribution proportional to $\xi_0$ gives a contribution of about 105% of $\epsilon_K$. The long distance effect, $\xi_{\text{LD}}$, from the absorptive part gives about $-5\%$ correction. The long distance effect, $\xi_{\text{LD}}$ from the dispersive part gives about $\pm 1.6\%$ correction. Details on remaining input parameters such as $C_e$, $X_{SD}$, $\xi_0$, and $\xi_{\text{LD}}$ are given in Ref. [1]. We need 18 input parameters to determine $\epsilon_K$ in the SM. Six of them can, in principle, be obtained from lattice QCD: $\hat{B}_K$, $V_{cb}$, $V_{us}$, $\xi_0$, $\xi_{\text{LD}}$, and $m_c(m_c)$. Here, we address recent progress on determining those input parameters.

| Decay mode $|V_{ab}|$ | Ref. |
|------------------|------|
| $\bar{B} \to \pi^0\bar{\nu}$ | 3.72(16) | [3] |
| $\bar{B} \to \pi^0\bar{\nu}$ | 3.61(32) | [4] |
| ex-combined | 3.70(14) | this paper |
| $\bar{B} \to X_u \ell\bar{\nu}$ | 4.45(16)/22 | [5] |

Table 1: Results for $|V_{ab}|$

| Decay mode $|V_{cb}|$ | Ref. |
|------------------|------|
| $\bar{B} \to D^*\ell\bar{\nu}$ | 39.04(99)(19) | [6] |
| $\bar{B} \to D\ell\bar{\nu}$ | 40.7(10)(2) | [7] |
| ex-combined | 39.62(60) | this paper |
| $\bar{B} \to X_c \ell\bar{\nu}$ | 42.00(64) | [8] |

Table 2: Results for $|V_{cb}|$

Recent results for $|V_{ab}|$ and $|V_{cb}|$ are presented in Tables 1 and 2, respectively. Recently, DeTar has collected the lattice QCD results of FNAL/MILC [9] and HPQCD [10], and the experimental results of Babar [11] and Belle [12] for the $\bar{B} \to D^*\ell\bar{\nu}$ decay mode. He has made combined fit of all of them simultaneously to determine $|V_{cb}|$ [7]. The “ex-combined” result in Table 2 corresponds to a weighted average of the $V_{cb}$ results from the $\bar{B} \to D^*\ell\bar{\nu}$ and $\bar{B} \to D\ell\bar{\nu}$ decay channels. Similarly, the “ex-combined” result in Table 1 corresponds to a weighted average of the two $V_{ab}$ results from $\bar{B} \to \pi^0\bar{\nu}$ decay. In Fig. 1, we show all the results simultaneously.\footnote{The plot is based on that by Andreas Kronfeld in Ref. [7].} We find that the inclusive results show about $3\sigma$ tension with those from exclusive $B$ meson decays respectively as well as from the LHCb results for $|V_{ab}|/|V_{cb}|$, which corresponds to the magenta band in Fig. 1.

We have two independent methods to determine $\xi_0$ in lattice QCD: the indirect and direct methods. In the indirect method, we determine $\xi_0$ from the experimental values of $\Re(e'/e)$, $\omega$, \ldots
The yellow band represents $|V_{cb}|$ determined from the $\bar{B} \rightarrow D^* \ell \nu$ decay, and the yellow-green band $|V_{cb}|$ determined from the $\bar{B} \rightarrow D \ell \nu$ decay. The yellow band represents $|V_{ub}|$ determined from the $\bar{B} \rightarrow \pi \ell \nu$ decay, and the magenta band $|V_{ub}/V_{cb}|$ determined from the LHCB data of the $\Lambda_b \rightarrow \Lambda_c \ell \nu$ and $\Lambda_b \rightarrow p \ell \nu$ decays. The orange circle represents the combined results for exclusive $|V_{cb}|$ and $|V_{ub}|$ from the $B$ meson decays, and the black cross $\times$ the inclusive $|V_{cb}|$ and $|V_{ub}|$ (heavy quark expansion).

$|V_{cb}| \times 10^3$

and $\varepsilon_K$ using the lattice QCD input $\xi_2$. The master formulas are

$$\xi_0 = \frac{\text{Im} A_0}{\text{Re} A_0}, \quad \xi_2 = \frac{\text{Im} A_2}{\text{Re} A_2}, \quad \text{Re} \left( \frac{\varepsilon}{\varepsilon_K} \right) = \frac{\omega}{\sqrt{2}|\varepsilon_K|} (\xi_2 - \xi_0).$$

(2.2)

Recently, RBC-UKQCD reported updated results for $\xi_2$ [13]. The results for $\xi_0$ from the indirect method are presented in Table 3.

<table>
<thead>
<tr>
<th>Input</th>
<th>Method</th>
<th>Value</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_0$</td>
<td>indirect</td>
<td>$-1.63(19) \times 10^{-4}$</td>
<td>[13]</td>
</tr>
<tr>
<td>$\xi_0$</td>
<td>direct</td>
<td>$-0.57(49) \times 10^{-4}$</td>
<td>[14]</td>
</tr>
<tr>
<td>$\xi^\text{LD}_0$</td>
<td>—</td>
<td>$(0 \pm 1.6)%$</td>
<td>[15]</td>
</tr>
</tbody>
</table>

Table 3: Long distance effects: $\xi_0$ and $\xi^\text{LD}_0$.

Recently, RBC-UKQCD has reported new lattice QCD results for Im$A_0$ [14]. Combining them with the experimental value of Re$A_0$, we can determine $\xi_0$ directly from the lattice input Im$A_0$ using the master formula in Eq. (2.2). This is the direct method. In Ref. [14], RBC-UKQCD has also reported the S-wave $\pi - \pi$ scattering phase shift with isospin $I=0$: $\delta_0 = 23.8(49)(12)$. This value is $3.0\sigma$ lower than the conventional value of $\delta_0$ in Refs. [16] (KPY-2011) and [17, 18] (CGL-2001). KPY-2011 used a singly subtracted Roy-like equation and CGL-2001 used a doubly subtracted Roy equation (CGL-2001) to do the interpolation around $\sqrt{s} = m_K \approx 500$MeV. The values for $\delta_0$ are summarized in Table 4. The KPY-2011 fits to the experimental data work well from the $\pi - \pi$ threshold ($\approx 280$MeV) to $\sqrt{s} = 800$MeV. In addition, KPY-2011 is highly consistent with CGL-2001 in the interpolating region around $\sqrt{s} = m_K \approx 500$MeV.

For $\delta_0$ (S-wave, $I=0$), we plot the results of RBC-UKQCD together with those of KPY-2011 and CGL-2001 in Fig. 2. We find that there is essentially no difference between KPY-2011 and CGL-2001 in the region near $\sqrt{s} = m_K \approx 500$MeV. Here, we observe the $3.0\sigma$ gap between RBC-UKQCD and KPY-2011. In contrast, for $\delta_2$ (S-wave, $I=2$), we observe no tension between RBC-UKQCD and KPY-2011, as one can see in Fig. 3.

Therefore, we conclude that the results of the indirect method are more reliable than those of the direct method for $\xi_0$, since the direct calculation of Im$A_0$ by RBC-UKQCD might have unresolved issues. Hence, we use the indirect method to determine $\xi_0$ in this paper.

$\xi^\text{LD}_0$ represents the long distance effect in the dispersive part. Its master formula in the continuum is given in Ref. [1]. A theoretical framework for calculating it on the lattice is well established.
in Ref. [15]. An on-going efforts to calculate it on the lattice can be found in [19]. However, this attempt [20], at present, is in a exploratory stage yet. Hence, we use the rough estimate of $\xi_{\text{LD}}$ given in Ref. [15].

Recent results for $\hat{B}_K$ in lattice QCD market with $N_f = 2 + 1$ flavors are summarized in Table 5. Here, FLAG-2016 represents the global average of the results of BMW-2011 [21], Laiho-2011 [22], RBC-UK-2016 [23], and SWME-2016 [24]. For more details, refer to Ref. [25]. SWME-2014 and RBC-UK-2016 represent the $\hat{B}_K$ results reported in Refs. [26] and [23], respectively. Here we use the FLAG-2016 result for $\hat{B}_K$.

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>Value</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FLAG-2016</td>
<td>0.7625(97)</td>
<td>[25]</td>
</tr>
<tr>
<td>SWME-2014</td>
<td>0.7379(47)(365)</td>
<td>[26]</td>
</tr>
<tr>
<td>RBC-UK-2016</td>
<td>0.7499(24)(150)</td>
<td>[23]</td>
</tr>
</tbody>
</table>

Table 5: $\hat{B}_K$

For the Wolfenstein parameters $\lambda$, $\bar{\rho}$, and $\bar{\eta}$, both CKMfitter and UTfit updated their results in Refs. [27, 28]. However, the angle-only-fit has not been updated since Lattice 2015. The global unitarity triangle (UT) fits of both CKMfitter and UTfit use $\varepsilon_K$ and $|V_{cb}|$ as input parameters to determine Wolfenstein parameters $\bar{\rho}$ and $\bar{\eta}$. Hence, using them to evaluate $\varepsilon_K$ leads to unwanted correlations through $\varepsilon_K$ and $|V_{cb}|$. In contrast, the angle-only-fit (AOF) results have no correlation with $\varepsilon_K$ and $|V_{cb}|$. Hence, we use the AOF results in this paper.

For the QCD corrections $\eta_{cc}$, $\eta_{ct}$, and $\eta_{tt}$, we use the same values as in Ref. [1]. They are collected in Table 7. In particular, we use the SWME value of $\eta_{cc}$ reported in Ref. [1] instead of that in Ref. [31]. This issue is well explained in Ref. [1]. One reason is that the size of the NNLO correction is already a conservative estimate for the truncation error of the NNNLO level in perturbation theory. Another reason is that the SWME result is consistent with that of Ref. [32].

In Table 8, we summarize remaining input parameters. They are the same as those in Ref. [1] except for the charm quark mass $m_c(m_c)$. For the charm quark mass, we use the HPQCD result reported in Ref. [35].

3. **Current status of $\varepsilon_K$**

Here, we present the results for $\varepsilon_K$ evaluated directly from the SM with the lattice QCD inputs.
\[ |\varepsilon_K| = 1.69 \pm 0.17 \text{ for exclusive } V_{cb} \text{ (lattice QCD)} \] (3.1)
\[ |\varepsilon_K| = 2.10 \pm 0.21 \text{ for inclusive } V_{cb} \text{ (heavy quark expansion)} \] (3.2)
\[ |\varepsilon_K| = 2.228 \pm 0.011 \text{ (experimental value)} \] (3.3)

Here, exclusive \( V_{cb} \) represents the theoretical evaluation of \( \varepsilon_K \) with the FLAG-2016 \( B_K \), AOF for the Wolfenstein parameters, and exclusive \( |V_{cb}| \) that corresponds to ex-combined in Table 2. We observe 3.2\( \sigma \) tension in the exclusive \( V_{cb} \) channel (lattice QCD), and no tension in the inclusive \( V_{cb} \) channel (heavy quark expansion; QCD sum rules).

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Weonjong Lee

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