Scaling behaviour of the $p_T$ spectra for pions, kaons and protons in $pp$ collisions

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We present our recent study [1] on the scaling behaviour of the pions, kaons and protons $p_T$ spectra in proton-proton ($pp$) collisions with center of mass energies ($\sqrt{s}$) at 0.9, 2.76 and 7 TeV. This study shows that the scaling behaviour arises when a linear transformation, $p_T \rightarrow p_T/K$, is applied on these spectra. The scaling parameter $K$ depends on $\sqrt{s}$ and is determined the quality factor method. It also illustrates that the pions, kaons and protons originate from different distributions of clusters which are formed by strings overlapping, and the scaling behaviour of these identified particles $p_T$ spectra could be understood with the colour string percolation model in a quantitative way simultaneously.

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1. Introduction

One of the most important observations in high energy collisions is the $p_T$ spectra of final state particles, such as pions, kaons and protons. From the spectra, we can learn a lot about the particle production mechanism. In many studies, searching for a scaling behaviour of the $p_T$ spectra is helpful to reveal the mechanism. In [2], a scaling behaviour was exhibited in the pion $p_T$ spectra with different collision centralities at midrapidity in Au+Au collisions at the Relativistic Heavy Ion Collider (RHIC). This scaling behaviour was extended to noncentral regions in Au+Au and d+Au collisions [3]. Similar scaling behaviour was found in the proton and anti-proton $p_T$ spectra with different collision centralities at midrapidity in Au+Au collisions at RHIC [4].

Recently, we observed a scaling behaviour in the $p_T$ spectra of inclusive charged hadrons in proton-proton (proton-antiproton) collisions at $\sqrt{s} = 0.9, 2.76$ and 7 (0.63, 1.8 and 1.96) TeV [5]. This scaling behaviour arises when the spectra are presented in terms of the scaling variable $z = p_T/K$, where $K$ is the scaling parameter and depends on $\sqrt{s}$. Here, we will show this scaling behaviour could be extended to the $p_T$ spectra of pions, kaons and protons in pp collisions at 0.9, 2.76 and 7 TeV. The quality factor method [6, 7] is utilized to determine the scaling parameter $K$. We will argue that the pions, kaons and protons originate from different distributions of clusters which are formed by strings overlapping, and the colour string percolation model can describe the scaling behaviour not only in a qualitative way but also in a quantitative way simultaneously.

2. Method to search for the scaling behaviour

Let us take the pion $p_T$ spectra as an example, we define a scaling variable, $z = p_T/K$, and a scaled $p_T$ spectra, $\Phi(z) = A \cdot (2\pi p_T)^{-1} d^2N/dp_Tdy|_{p_T=Kz}$. Here the parameters $K$ and $A$ depend on the collision energy. By choosing proper $K$ and $A$, the data points of the pion $p_T$ spectra in pp collisions at 0.9, 2.76 and 7 TeV will migrate into one curve. As a convention, we usually set both $K$ and $A$ of the highest energy collisions to be 1. However, the $p_T$ coverages of the pion spectra at 0.9 and 7 TeV are much smaller than the coverage at 2.76 TeV. In order to make the scaling function $\Phi(z)$ to describe the pion spectra in the large $p_T$ region faithfully, we choose both $K$ and $A$ at 2.76 TeV as 1. The ways to search for the scaling behaviour in the $p_T$ spectra of kaons and protons are identical to the one for pions.

3. Scaling behaviour of identified pions, kaons and protons

The $p_T$ spectra of identified pions, kaons and protons in pp collisions at 0.9, 2.76 and 7 TeV were published by the ALICE collaboration [8, 9, 10]. Here the pion, kaon and proton $p_T$ spectra refer to the spectra of $(\pi^+ + \pi^-)/2$, $(K^+ + K^-)/2$ and $(p + \bar{p})/2$. As described in section 2, both $K$ and $A$ at 2.76 TeV are set as 1, thus $\Phi(z)$ is nothing but the $p_T$ spectra of identified particles at this energy. Since the threshold of the $p_T$ range is 0.1 (0.1 and 0.1) GeV/c at 2.76 TeV [10], which is below the mass of pions (kaons and protons) 0.14 (0.494 and 0.938) GeV/c$^2$ [11], we write $\Phi(z)$ for pions, kaons and protons with the modified Tsallis form

$$\Phi(z) = C_q \left[ 1 - (1-q) \frac{\sqrt{m^2 + z^2} - m}{z_0} \right]^{\frac{1}{1-q}}, \quad (3.1)$$
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where $C_{q}$, $q$ and $z_{0}$ are free parameters, $m$ is the mass of the particle species, and $1-q$ is a measure of the non-extensivity. These free parameters are given by fitting equation 3.1 to the $p_{T}$ spectra of identified species at 2.76 TeV with the least $\chi^{2}$s method. Table 1 lists $C_{q}$, $q$, $z_{0}$ as well as their uncertainties returned by the fit. The last column of this table shows the $\chi^{2}$s per degrees of freedom (dof), named reduced $\chi^{2}$s, for these fits.

Table 1: $C_{q}$, $q$ and $z_{0}$ of the scaling functions $\Phi(\nu)$ for pions, kaons and protons. The uncertainties quoted are due to the statistical plus systematic errors of the data points added in quadrature.

<table>
<thead>
<tr>
<th></th>
<th>$C_{q}$</th>
<th>$q$</th>
<th>$z_{0}$</th>
<th>$\chi^{2}$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pions</td>
<td>6.01±0.08</td>
<td>1.1416±0.0007</td>
<td>0.1308±0.0008</td>
<td>0.49</td>
</tr>
<tr>
<td>Kaons</td>
<td>0.214±0.002</td>
<td>1.1402±0.0004</td>
<td>0.193±0.001</td>
<td>0.23</td>
</tr>
<tr>
<td>Protons</td>
<td>0.0546±0.0008</td>
<td>1.115±0.002</td>
<td>0.220±0.002</td>
<td>0.40</td>
</tr>
</tbody>
</table>

For $K$ and $A$ at 0.9 and 7 TeV, we adopt the quality factor (QF) method to determine them. In this method, the QF is defined in terms of a set of data points $(u_{i}, v_{i})$ [6, 7]

$QF(K, A) = \left[ \sum_{i} \frac{(v_{i}-v_{i-1})^{2}}{(u_{i}-u_{i-1})^{2} + \epsilon^{2}} \right]^{-1}$,

(3.2)

where $u_{i} = p_{T}^{i}/K$, $v_{i} = \log(A \cdot (2\pi p_{T}^{i}d^{2}N_{i}/dp_{T}^{i}dy^{i}))$, the small constant $\epsilon$ (0.01) is utilized to keep the sum being finite when two points have the same $u$ value. Before entering the QF formula, $(u_{i}, v_{i})$ has been rescaled so that $0 \leq u_{i}, v_{i} \leq 1$, and $u_{i}$ are ordered. Obviously, two successive data points being close in $u$ and far in $v$ will give a large contribution to the sum in equation 3.2. As a result, a set of data points with a small sum (thus a large QF) is expected to lie close to a unique curve. For $K$ and $A$ at 0.9 (7) TeV, we utilize the data points at 0.9 (7) and 2.76 TeV to determine them. The best set of $(K, A)$ is chosen to be the one which globally maximizes the QF. Table 2 tabulates $K$ and $A$ for pions, kaons and protons in $pp$ collisions at 0.9, 2.76 and 7 TeV.

Table 2: $K$ and $A$ for pions, kaons and protons at 0.9, 2.76 and 7 TeV. The errors quoted are due to the statistical uncertainties of the data points.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (TeV)</th>
<th>Pions</th>
<th></th>
<th>Kaons</th>
<th></th>
<th>Protons</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K$</td>
<td>$A$</td>
<td>$K$</td>
<td>$A$</td>
<td>$K$</td>
<td>$A$</td>
</tr>
<tr>
<td>0.9</td>
<td>0.93±0.04</td>
<td>1.11±0.14</td>
<td>0.913±0.007</td>
<td>1.05±0.01</td>
<td>0.926±0.001</td>
<td>1.08±0.02</td>
</tr>
<tr>
<td>2.76</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>1.06±0.02</td>
<td>0.89±0.07</td>
<td>1.14±0.04</td>
<td>1.05±0.14</td>
<td>1.108±0.006</td>
<td>1.04±0.02</td>
</tr>
</tbody>
</table>

With $K$ and $A$ in table 2, we plot the scaled pion $p_{T}$ spectra at 0.9, 2.76 and 7 TeV in terms of $z$ in the upper panel of figure 1(a). In the log scale, all data points at different energies now are shifted to the same curve within error bars. This curve is described by the pion scaling function $\Phi(z)$ in equation 3.1 with parameters in the second row of table 1. In order to see the agreement
between the experimental data and fitted results, we evaluate the ratios, \( R = (\text{data} - \text{fitted}) / \text{data} \), at 0.9, 2.76 and 7 TeV. The lower panel of figure 1(a) shows the \( R \) distribution as a function of \( z \). It is obvious the \( R \) values of all data points are in the range between -0.2 and 0.2, which implies that the data and fitted curve agreement is within 20%. Considering the fact that the data in the pion \( p_T \) spectra cover about 10 orders of magnitude, the fit performed on the pion \( p_T \) spectra is good.

The scaling behaviour of the kaon and proton \( p_T \) spectra at 0.9, 2.76 and 7 TeV are presented in the upper panels of figures 1(b) and 1(c). As done for pions, we show the \( R \) distributions in the lower panels of these two figures. For kaons, except for the last data point at 7 TeV, all the data points and the fitted curve agree within 20%. For protons, the data points are consistent with the curve within 20% when \( z < 10 \text{ GeV}/c \). When \( z > 10 \text{ GeV}/c \), the yield and experimental errors for protons at 2.76 TeV have the same order. Thus the values of \( R \) are large in this high \( p_T \) region.

4. Colour String Percolation Model

Now we will show that the scaling behaviour could be explained by the colour string percolation (CSP) model [12, 13] in a quantitative way simultaneously. In this model, colour strings are stretched between the partons of protons in \( pp \) collisions. These strings then decay into new
ones with the emission of $q\bar{q}$ pairs. Identified hadrons are produced through the hadronization of these new strings. The strings are viewed as small areas of $S_1 = \pi r_0^2$ with $r_0 \approx 0.2$ fm in the transverse plane. The number of strings grows with the increase of the collision energy. When there are $n$ strings, they start to overlap and form clusters with transverse areas of $S_n$. The mean transverse momentum squared $\langle p_T^2 \rangle_m$ of identified particles produced by a cluster can be written as $\langle p_T^2 \rangle_m = \sqrt{nS_1/S_n} \langle p_T^2 \rangle_1$, where $\langle p_T^2 \rangle_1$ is the mean $p_T^2$ of identified particles produced by a single string, and $nS_1/S_n$ is the degree of string overlap. The $p_T$ spectra of identified particles is deemed as a superposition of the spectra produced by each cluster, $f(x, p_T)$, weighted with the distribution of the cluster’s size, $W(x)$. Here the cluster size $x$ is equivalent to the inverse of $\langle p_T^2 \rangle_m$. In order to describe the spectra at the high $p_T$ region, the Schwinger formula $f(x, p_T) = \exp(-p_T^2x)$ in [12, 13] should be replaced by $f(x, p_T) = \exp(-\sqrt{x}p_T)$ [14, 15]. $W(x) = \frac{\gamma}{\Gamma(k)}(\gamma x)^{k-1}\exp(-\gamma x)$, where $\gamma$ and $k$ are free parameters. Thus the $p_T$ distribution of identified particles is

$$\frac{d^2N}{2\pi p_T dp_T dy} = C \int_0^\infty W(x)f(x, p_T)dx,$$

(4.1)

where $C$ is a normalization constant for the total number of clusters formed for identified particles before hadronization. In order to see whether the CSP model can describe the scaling behaviour, we fit equation 4.1 to the $p_T$ spectra of identified particles at 2.76 TeV. The parameters $C, r$ and $k$ returned by the fits as well as the reduced $\chi^2$'s are tabulated in table 3. As shown in the upper panel of figure 1(d), the data points for pions agree well with the CSP fit in log scale. The accuracy of this agreement is around 20%, which can be seen from the $R$ distribution in the lower panel of the figure. Similar results are obtained for kaons and protons. As the $k$ parameters are different for the identified particles, pions, kaons and protons originate from different distributions of clusters.

Table 3: $C, r$ and $k$ of the CSP fits on the spectra of pions, kaons and protons at 2.76 TeV. The uncertainties quoted are due to the statistical plus systematic errors of the data points added in quadrature.

<table>
<thead>
<tr>
<th></th>
<th>$C$</th>
<th>$\gamma$</th>
<th>$k$</th>
<th>$\chi^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pions</td>
<td>0.91±0.2</td>
<td>0.140±0.003</td>
<td>3.15±0.02</td>
<td>1.53</td>
</tr>
<tr>
<td>Kaons</td>
<td>0.53±0.01</td>
<td>0.342±0.008</td>
<td>3.33±0.02</td>
<td>0.91</td>
</tr>
<tr>
<td>Protons</td>
<td>0.160±0.007</td>
<td>0.91±0.07</td>
<td>5.0±0.2</td>
<td>1.91</td>
</tr>
</tbody>
</table>

Now we would like to see for the reason why the CSP model can describe the scaling behaviour of the identified $p_T$ spectra. Under $x \rightarrow x’ = \lambda x$, $\gamma \rightarrow \gamma’ = \gamma/\lambda$ and $p_T \rightarrow p_T’ = p_T/\sqrt{\lambda}$, both $W(x)$ and $f(x)$ are invariant. So the $p_T$ spectra of identified particles in equation 4.1 are also invariant. This invariance is the scaling behaviour we are looking for. Comparing the $p_T’$ transformation in the CSP model $p_T’ \rightarrow p_T’/\sqrt{\lambda}$ with the one used to search for the scaling behaviour $p_T \rightarrow p_T/K$, we know $K$ is proportional to $1/\sqrt{\lambda}$. As described in [13, 15], $\lambda = \langle S_n/nS_1 \rangle^{1/2}$, where the average is taken over all clusters for identified particles, thus $K$ should be proportional to $\langle nS_1/S_n \rangle^{1/4}$. Since the degree of string overlap $nS_1/S_n$ grows with energy, $K$ should also increase with energy. That’s indeed what we see for the identified particles in table 2. Therefore the CSP model can explain the scaling behaviour for pions, kaons and protons separately. However, the rate of the $K$ values increasing with energy for pions is different with the one for kaons and protons.
This could also be explained by the CSP model. As $K = \langle p_T \rangle / \langle z \rangle$, and the values of $\langle z \rangle$ are the same for pions (kaons or protons) at 0.9, 2.76 and 7 TeV, the ratio between the values of $K$ at different energies should be equal to the ratio between the values of $\langle p_T \rangle$ at different energies. $\langle p_T \rangle$ is evaluated in terms of the CSP model as $\langle p_T \rangle = \int_0^\infty \int_0^\infty W(x) f(x, p_T) p_T^2 dxd p_T$. $f(x, p_T)$ are the same for pions (kaons or protons) at different energies. However, $W(x)$ are different for different particles, which can be seen from the comparison among the $k$ values for pions, kaons and protons in table 3. As a result, for different species of particles, the values of $K$ change in a different rate with energy.

5. Conclusions

We have extended the scaling behaviour in the inclusive charged hadron $p_T$ spectra to the spectra of identified particles at 0.9, 2.76 and 7 TeV. This scaling behaviour is exhibited when $p_T$ is replaced by $z = p_T / K$. The scaling parameter $K$ is determined by the QF method. We have argued that the pions, kaons and protons originate from different distributions of clusters formed by strings overlapping, and the scaling behaviour of these identified particles could be explained by the colour string percolation model in a quantitative way at the same time.

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References