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Scaling behaviour of the $p_{\rm T}$ spectra for pions, kaons and protons in pp collisions

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We present our recent study [1] on the scaling behaviour of the pions, kaons and protons p_T spectra in proton-proton (pp) collisions with center of mass energies ($\sqrt{s}$) at 0.9, 2.76 and 7 TeV. This study shows that the scaling behaviour arises when a linear transformation, $p_{\rm T} \to p_{\rm T}/K$, is applied on these spectra. The scaling parameter K depends on $\sqrt{s}$ and is determined the quality factor method. It also illustrates that the pions, kaons and protons originate from different distributions of clusters which are formed by strings overlapping, and the scaling behaviour of these identified particles $p_{\rm T}$ spectra could be understood with the colour string percolation model in a quantitative way simultaneously.

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1. Introduction

One of the most important observations in high energy collisions is the p_T spectra of final state particles, such as pions, kaons and protons. From the spectra, we can learn a lot about the particle production mechanism. In many studies, searching for a scaling behaviour of the p_T spectra is helpful to reveal the mechanism. In [2], a scaling behaviour was exhibited in the pion p_T spectra with different collision centralities at midrapidity in Au+Au collisions at the Relativistic Heavy Ion Collider (RHIC). This scaling behaviour was extended to noncentral regions in Au+Au and d+Au collisions [3]. Similar scaling behaviour was found in the proton and anti-proton p_T spectra with different collision centralities at midrapidity in Au+Au collisions at RHIC [4].

Recently, we observed a scaling behaviour in the p_T spectra of inclusive charged hadrons in proton-proton (proton-antiproton) collisions at $\sqrt{s}$ = 0.9, 2.76 and 7 (0.63, 1.8 and 1.96) TeV [5]. This scaling behaviour arises when the spectra are presented in terms of the scaling variable $z = p_T/K$, where K is the scaling parameter and depends on $\sqrt{s}$. Here, we will show this scaling behaviour could be extended to the p_T spectra of pions, kaons and protons in pp collisions at 0.9, 2.76 and 7 TeV. The quality factor method [6, 7] is utilized to determine the scaling parameter K. We will argue that the pions, kaons and protons originate from different distributions of clusters which are formed by strings overlapping, and the colour string percolation model can describe the scaling behaviour not only in a qualitative way but also in a quantitative way simultaneously.

2. Method to search for the scaling behaviour

Let us take the pion p_T spectra as an example, we define a scaling variable, $z = p_T/K$, and a scaled p_T spectra, $\Phi(z) = A\cdot(2\pi p_T)^{-1}d^2N/dp_Tdy|_{p_T=Kz}$. Here the parameters K and A depend on the collision energy. By choosing proper K and A, the data points of the pion p_T spectra in pp collisions at 0.9, 2.76 and 7 TeV will migrate into one curve. As a convention, we usually set both K and A of the highest energy collisions to be 1. However, the p_T coverages of the pion spectra at 0.9 and 7 TeV are much smaller than the coverage at 2.76 TeV. In order to make the scaling function $\Phi(z)$ to describe the pion spectra in the large p_T region faithfully, we choose both K and A at 2.76 TeV as 1. The ways to search for the scaling behaviour in the p_T spectra of kaons and protons are identical to the one for pions.

3. Scaling behaviour of identified pions, kaons and protons

The p_T spectra of identified pions, kaons and protons in pp collisions at 0.9, 2.76 and 7 TeV were published by the ALICE collaboration [8, 9, 10]. Here the pion, kaon and proton p_T spectra refer to the spectra of $(\pi^+ + \pi^-)/2$, $(K^+ + K^-)/2$ and $(p + \bar{p})/2$. As described in section 2, both K and A at 2.76 TeV are set as 1, thus $\Phi(z)$ is nothing but the p_T spectra of identified particles at this energy. Since the threshold of the p_T range is 0.1 (0.1 and 0.1) GeV/c at 2.76 TeV [10], which is below the mass of pions (kaons and protons) 0.14 (0.494 and 0.938) GeV/c^2 [11], we write $\Phi(z)$ for pions, kaons and protons with the modified Tsallis form

$$\Phi(z) = C_q\left[1-(1-q)\frac{\sqrt{m^2+z^2}-m}{z_0}\right]^{\frac{1}{1-q}}, \tag{3.1}$$

where C_q, q and z_0 are free parameters, m is the mass of the particle species, and $1-q$ is a measure of the non-extensivity. These free parameters are given by fitting equation 3.1 to the $p_{\rm T}$ spectra of identified species at 2.76 TeV with the least χ^2s method. Table 1 lists C_q, q, z_0 as well as their uncertainties returned by the fit. The last column of this table shows the χ^2s per degrees of freedom (dof), named reduced χ^2s, for these fits.

Table 1: C_q, q and z_0 of the scaling functions $\Phi(z)$ for pions, kaons and protons. The uncertainties quoted are due to the statistical plus systematic errors of the data points added in quadrature.

	C_q	q	z_0	χ^2/dof
Pions	6.01±0.08	1.1416±0.0007	0.1308±0.0008	0.49
Kaons	0.214±0.002	1.1402±0.0004	0.193±0.001	0.23
Protons	0.0546±0.0008	1.115±0.002	0.220±0.002	0.40

For K and A at 0.9 and 7 TeV, we adopt the quality factor (QF) method to determine them. In this method, the QF is defined in terms of a set of data points (u^i, v^i) [6, 7]

$$QF(K,A) = \left[\sum_i \frac{(v^i - v^{i-1})^2}{(u^i - u^{i-1})^2 + \varepsilon^2}\right]^{-1}, \tag{3.2}$$

where $u^i = p_{\rm T}^i/K$, $v^i = \log(A \cdot (2\pi p_{\rm T}^i)^{-1} d^2N^i/dp_{\rm T}^i dy^i)$, the small constant ε (0.01) is utilized to keep the sum being finite when two points have the same u value. Before entering the QF formula, (u^i, v^i) has been rescaled so that $0 \leq u^i, v^i \leq 1$, and u^i are ordered. Obviously, two successive data points being close in u and far in v will give a large contribution to the sum in equation 3.2. As a result, a set of data points with a small sum (thus a large QF) is expected to lie close to a unique curve. For K and A at 0.9 (7) TeV, we utilize the data points at 0.9 (7) and 2.76 TeV to determine them. The best set of (K, A) is chosen to be the one which globally maximizes the QF. Table 2 tabulates K and A for pions, kaons and protons in pp collisions at 0.9, 2.76 and 7 TeV.

Table 2: K and A for pions, kaons and protons at 0.9, 2.76 and 7 TeV. The errors quoted are due to the statistical uncertainties of the data points.

$\sqrt{s}$ (TeV)	Pions		Kaons		Protons	
	K	A	K	A	K	A
0.9	0.93±0.04	1.11±0.14	0.913±0.007	1.05±0.01	0.926±0.001	1.08±0.02
2.76	1	1	1	1	1	1
7	1.06±0.02	0.89±0.07	1.14±0.04	1.05±0.14	1.108±0.006	1.04±0.02

With K and A in table 2, we plot the scaled pion $p_{\rm T}$ spectra at 0.9, 2.76 and 7 TeV in terms of z in the upper panel of figure 1(a). In the log scale, all data points at different energies now are shifted to the same curve within error bars. This curve is described by the pion scaling function $\Phi(z)$ in equation 3.1 with parameters in the second row of table 1. In order to see the agreement between the experimental data and fitted results, we evaluate the ratios, $R = (\text{data} - \text{fitted})/\text{data}$, at 0.9, 2.76 and 7 TeV. The lower panel of figure 1(a) shows the R distribution as a function of z. It is obvious the R values of all data points are in the range between -0.2 and 0.2, which implies that the data and fitted curve agreement is within 20%. Considering the fact that the data in the pion p_{T} spectra cover about 10 orders of magnitude, the fit performed on the pion p_{T} spectra is good.

The scaling behaviour of the kaon and proton p_{T} spectra at 0.9, 2.76 and 7 TeV are presented in the upper panels of figures 1(b) and 1(c). As done for pions, we show the R distributions in the lower panels of these two figures. For kaons, except for the last data point at 7 TeV, all the data points and the fitted curve agree within 20%. For protons, the data points are consistent with the curve within 20% when $z < 10$ GeV/c. When $z > 10$ GeV/c, the yield and experimental errors for protons at 2.76 TeV have the same order. Thus the values of R are large in this high p_{T} region.

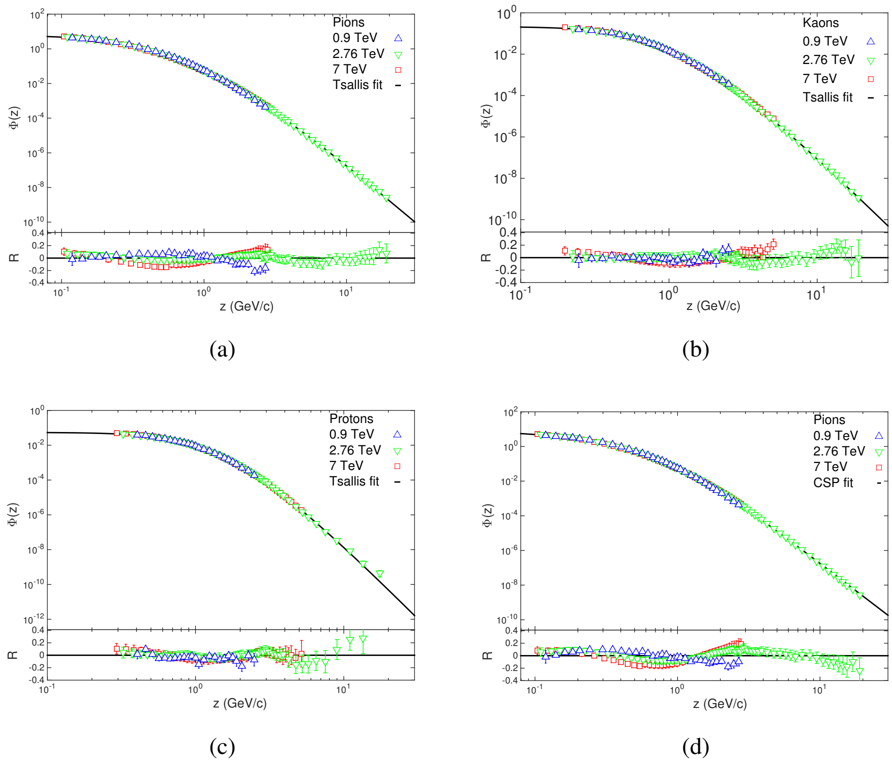

Figure 1: Upper panel in (a) ((b), (c), (d)) : the scaling behaviour of the pion (kaon, proton, pion) p_{T} spectra presented in z for pp collisions at 0.9, 2.76 and 7 TeV. The solid curve in (a) ((b), (c)) is described by equation 3.1 with parameters in the second (third, fourth) row of table 1. The solid curve in (d) is described by the CSP fit in equation 4.1 with parameters in the second row of table 3. The data points are taken from [8, 9, 10]. Lower Panel in (a) ((b), (c), (d)): the R distribution for pions (kaon, proton, pion).

4. Colour String Percolation Model

Now we will show that the scaling behaviour could be explained by the colour string percolation (CSP) model [12, 13] in a quantitative way simultaneously. In this model, colour strings are stretched between the partons of protons in pp collisions. These strings then decay into new ones with the emission of $q\bar{q}$ pairs. Identified hadrons are produced through the hadronization of these new strings. The strings are viewed as small areas of $S_1 = \pi r_0^2$ with $r_0 \approx 0.2$ fm in the transverse plane. The number of strings grows with the increase of the collision energy. When there are n strings, they start to overlap and form clusters with transverse areas of S_n. The mean transverse momentum squared $\langle p_{\mathrm{T}}^2 \rangle_{ni}$ of identified particles produced by a cluster can be written as $\langle p_{\mathrm{T}}^2 \rangle_{ni} = \sqrt{nS_1/S_n}\langle p_{\mathrm{T}}^2 \rangle_{1i}$, where $\langle p_{\mathrm{T}}^2 \rangle_{1i}$ is the mean p_{T}^2 of identified particles produced by a single string, and nS_1/S_n is the degree of string overlap. The p_{T} spectra of identified particles is deemed as a superposition of the spectra produced by each cluster, $f(x, p_{\mathrm{T}})$, weighted with the distribution of the cluster's size, $W(x)$. Here the cluster size x is equivalent to the inverse of $\langle p_{\mathrm{T}}^2 \rangle_{ni}$. In order to describe the spectra at the high p_{T} region, the Schwinger formula $f(x, p_{\mathrm{T}}) = \exp(-p_{\mathrm{T}}^2 x)$ in [12, 13] should be replaced by $f(x, p_{\mathrm{T}}) = \exp(-\sqrt{2x}p_{\mathrm{T}})$ [14, 15]. $W(x) = \frac{\gamma}{\Gamma(k)}(\gamma x)^{k-1}\exp(-\gamma x)$, where γ and k are free parameters. Thus the p_{T} distribution of identified particles is

$$\frac{d^2N}{2\pi p_{\mathrm{T}} dp_{\mathrm{T}} dy} = C\int_0^{\infty} W(x) f(x, p_{\mathrm{T}}) dx, \tag{4.1}$$

where C is a normalization constant for the total number of clusters formed for identified particles before hadronization. In order to see whether the CSP model can describe the scaling behaviour, we fit equation 4.1 to the p_{T} spectra of identified particles at 2.76 TeV. The parameters C, r and k returned by the fits as well as the reduced χ^2s are tabulated in table 3. As shown in the upper panel of figure 1(d), the data points for pions agree with the CSP fit well in log scale. The accuracy of this agreement is around 20%, which can be seen from the R distribution in the lower panel of the figure. Similar results are obtained for kaons and protons. As the k parameters are different for the identified particles, pions, kaons and protons originate from different distributions of clusters.

Table 3: C, r and k of the CSP fits on the spectra of pions, kaons and protons at 2.76 TeV. The uncertainties quoted are due to the statistical plus systematic errors of the data points added in quadrature.

	C	γ	k	χ^2/dof
Pions	9.1±0.2	0.140±0.003	3.15±0.02	1.53
Kaons	0.53±0.01	0.342±0.008	3.33±0.02	0.91
Protons	0.160±0.007	0.91±0.07	5.0±0.2	1.91

Now we would like to seek for the reason why the CSP model can describe the scaling behaviour of the identified p_{T} spectra. Under $x \to x' = \lambda x$, $\gamma \to \gamma' = \gamma/\lambda$ and $p_{\mathrm{T}} \to p'_{\mathrm{T}} = p_{\mathrm{T}}/\sqrt{\lambda}$, both $W(x)$ and $f(x)$ are invariant. So the p_{T} spectra of identified particles in equation 4.1 are also invariant. This invariance is the scaling behaviour we are looking for. Comparing the p'_{T} transformation in the CSP model $p'_{\mathrm{T}} \to p'_{\mathrm{T}}\sqrt{\lambda}$ with the one used to search for the scaling behaviour $p_{\mathrm{T}} \to p_{\mathrm{T}}/K$, we know K is proportional to $1/\sqrt{\lambda}$. As described in [13, 15], $\lambda = \langle S_n/nS_1 \rangle^{1/2}$, where the average is taken over all clusters for identified particles, thus K should be proportional to $\langle nS_1/S_n \rangle^{1/4}$. Since the degree of string overlap nS_1/S_n grows with energy, K should also increase with energy. That's indeed what we see for the identified particles in table 2. Therefore the CSP model can explain the scaling behaviour for pions, kaons and protons separately. However, the rate of the K values increasing with energy for pions is different with the one for kaons and protons.

This could also be explained by the CSP model. As $K = \langle p_{\mathrm{T}} \rangle / \langle z \rangle$, and the values of $\langle z \rangle$ are the same for pions (kaons or protons) at 0.9, 2.76 and 7 TeV, the ratio between the values of K at different energies should be equal to the ratio between the values of $\langle p_{\mathrm{T}} \rangle$ at different energies. $\langle p_{\mathrm{T}} \rangle$ is evaluated in terms of the CSP model as $\langle p_{\mathrm{T}} \rangle = \frac{\int_0^\infty \int_0^\infty W(x) f(x, p_{\mathrm{T}}) p_{\mathrm{T}}^2 dx dp_{\mathrm{T}}}{\int_0^\infty \int_0^\infty W(x) f(x, p_{\mathrm{T}}) p_{\mathrm{T}} dx dp_{\mathrm{T}}}$. $f(x, p_{\mathrm{T}})$ are the same for pions (kaons or protons) at different energies. However, $W(x)$ are different for different particles, which can be seen from the comparison among the k values for pions, kaons and protons in table 3. As a result, for different species of particles, the values of K change in a different rate with energy.

5. Conclusions

We have extended the scaling behaviour in the inclusive charged hadron p_{T} spectra to the spectra of identified particles at 0.9, 2.76 and 7 TeV. This scaling behaviour is exhibited when p_{T} is replaced by $z = p_{\mathrm{T}}/K$. The scaling parameter K is determined by the QF method. We have argued that the pions, kaons and protons originate from different distributions of clusters formed by strings overlapping, and the scaling behaviour of these identified particles could be explained by the colour string percolation model in a quantitative way at the same time.

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